# An Efficient Filter Design via Optimized Rational-Function Fitting, without Similarity Transformation

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## Abstract

An efficient method is presented to design filters without the similarity transform of their coupling coefficient matrix as circuit parameters, which is very tedious due to pivoting and deciding rotation angles needed during the iterations. The transfer function of a filter is directly used for the design and its desired form is derived by the optimized rational-function fitting technique. A 3rd order coaxial lowpass filter is taken as an example to validate the proposed method.


***Key words*** : Passive Components, Microwave Filter, Coupling Coefficient Matrix, Rational Function, Optimization.

## Ⅰ. Introduction

Given the specifications of a filter, its transfer function needs evaluating for its design as a first step, as always. Then the primary coupling matrix $\overline{\overline{M}}$ as its circuit parameters can be obtained following the methods in [1], [2], which describes the entire relation of the resonance modes of the filter network. In most of the design cases, the required coupling structure differs from that of the primary $\overline{\overline{M}}$. For this purpose, the similarity transform is carried out to modify the 1st matrix to the target matrix and is pointed out that entails cumbersome work to decide pivots and rotation-angles[1]~[7].

Finding the transfer function suitable for the target coupling structure is the best way possible to handle the filter design. However, using the conventional fitting techniques with defining many frequency points on the specification mask, it ends up with extended computation time and likelihood of searching the areas away from the decisive points.

The genetic-algorithm(GA)[9] is proposed to optimize the transfer function for meeting the specifications. This will work on finding the critical points firstly, and secondly avoiding the aforementioned pivoting and rotation angles. A coaxial filter[6]~[8] for EMI concerned application is exemplified regarding this technique.

## Ⅱ. Theory

To design a filter, as the first step, the transfer function can be found, expressed as

$$Tran(S) = u_0 \frac{S^m + B_{m-1}S^{m-1} + \cdots + B_2S^2 + B_1S^1 + B_0}{S^n + A_{n-1}s^{n-1} + \cdots + A_2S^2 + A_1S^1 + A_0} \quad (1)$$

where $S=\sigma+j\tau$, $j=\sqrt{-1}$ and $n\rangle\, m$. Coefficients $A_i(i=0,$ $1,\ n-1)$ and $B_h(h=0,\ 1,\ L,\ m-1)$ of the denominator and numerator of $Trans(S)$, respectively, are determined by the reflection zeros and transmission zeros according to the specs on the frequency response. The transfer function can be equated to $S_{21}$ to find the matrix of the unknown initial coupling coefficients shown in the following(denoted as $\overline{\overline{M}}$) and this will avoid a conventional network synthesis where L's and C's are found.

$$\overline{\overline{M}}^{(0)} = \begin{bmatrix} 0 & m_{12} & 0 & 0 & 0 & 0 & 0 & m_{18} \\ m_{12} & 0 & m_{23} & 0 & 0 & 0 & m_{27} & 0 \\ 0 & m_{23} & 0 & m_{34} & 0 & m_{36} & 0 & 0 \\ 0 & 0 & m_{34} & 0 & m_{45} & 0 & 0 & 0 \\ 0 & 0 & 0 & m_{45} & 0 & m_{56} & 0 & 0 \\ 0 & 0 & m_{36} & 0 & m_{56} & 0 & m_{67} & 0 \\ 0 & m_{27} & 0 & 0 & 0 & m_{67} & 0 & m_{78} \\ m_{18} & 0 & 0 & 0 & 0 & 0 & m_{78} & 0 \end{bmatrix} \quad (2)$$

$\overline{\overline{M}}^{(0)}$ as the primary or initial $\overline{\overline{M}}$ is for the case of a canonical 8th order filter. If this matches with the target structure, no more modification on it is needed. The procedure of solving the unknown $\overline{\overline{M}}$ on the basis of the known transfer function, or Eq. (1) goes like this.

A general 2-port filter network is shown in Fig. 1, where the coupling coefficients are related to the resonance modes.

The coupling coefficient matrix $\overline{\overline{M}}$ as the unkowns can be solved via the generalized impedance and its scattering parameters as follows.

$$\overline{\overline{Z}} \cdot \overline{I} = \overline{e}, \quad (3)$$

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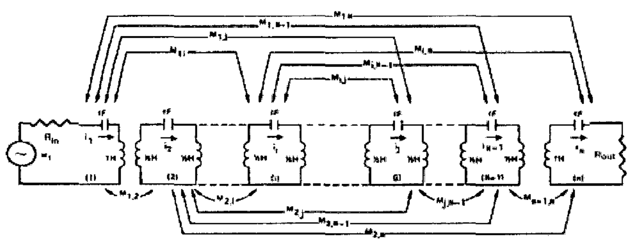


Fig. 1. A 2-port network with all resonance modes coupled.

where $\overline{e}^T=(1,\ 0,\ 0,\cdots,\ 0,\ 0)$ is the voltage excitation vector and $\overline{I}^T=(i_1,\ i_2,\ i_3,\ \ldots,\ i_{n-1},\ i_n)$ is the current vector including all the resonators. Also, the generalized impedance $\overline{\overline{Z}}$

$$\overline{\overline{Z}}=j(\tau\overline{\overline{U}}+\overline{\overline{M}}) \tag{4}$$

has self- and mutual coupling values of the network.

$$\tau=\frac{f_0}{\triangle f}\left(\frac{f}{f_0}-\frac{f_0}{f}\right). \tag{5}$$

$\overline{\overline{U}}$ is the identity matrix. Using the above relation of the current and voltage, major S-parameters can be represented as

$$S_{21}=-2\sqrt{R_{in}R_{out}}\,i_n=Trans(s) \tag{6}$$

and

$$S_{11}=1-2R_{in}i_1. \tag{7}$$

where $R_{in}$ and $R_{out}$ mean input and output port resistance. As is addressed earlier, solving the nonlinear equations from Eq. (6) will find the values for the elements of Eq. (2).

As a next step, Eq. (2) is transformed through the matrix rotation (or similarity transform), because most of practices demand the desired coupling structure that is different from the initial matrix $\overline{\overline{M}}^{(0)}$. The similarity transform is carried out as

$$\overline{\overline{M}}^{(r)}=\overline{\overline{R}}^{(r)}\cdot\overline{\overline{M}}^{(r-1)}\cdot\overline{\overline{R}}^{(r)t}\text{ and } r\geq 1 \tag{8}$$

where

$$\overline{\overline{R}}^{(r)}=\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&\cos\theta_r&0&0&0&-\sin\theta_r&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&\sin\theta_r&0&0&0&\cos\theta_r&0\\0&0&0&0&0&0&0&1\end{bmatrix} \tag{9}$$

Here, $\overline{\overline{M}}^{(r-1)}$ is the r-th rotation of $\overline{\overline{M}}^{(0)}$ and rotation matrix $\overline{\overline{R}}^{(r)}$ and its transpose $\overline{\overline{R}}^{(r)t}$ are also shown. The pivot of $\overline{\overline{R}}^{(r)}$ means $R^r_{ii}=R^r_{jj}=\cos\theta_r$, $R^r_{ij}=-R^r_{ji}=-\sin\theta_r$ with $i,\ j\neq 1$ or $n$. Plus, $\theta_r$ is the angle for rotation. The decision of the pivoting points and angles are made not by a generalized formula but by the trial-and-error based steps until the finalization. The purpose of this work is to suggest an efficient methodology that finds the desired final coupling coefficient matrix directly from the transfer function using a stochastic pole-residue finder(as an optimization technique), with neither undergoing the similarity transform nor L's and C's network synthesis scheme.

Eq. (1) is a rational function and is decomposed as follows to make it convenient form to use in an optimization scheme.

$$Trans(S')=\sum_{i=1}^{N_P}\frac{\mathrm{Res}_i}{s'-p_i}+C_l+Q_hS' \tag{10}$$

Depending on the combination of the three terms, the result becomes bandpass, or lowpass filters. The GA is suggested to figure out the terms including Residues Re $s_i$, poles $p_i$, constants $C_l$ and the coefficients $Q_h$ of the 1st order to optimally fit the given frequency responses[9]. And then the unknowns of the target coupling mechanism make up a rational function, and this is equated to Eq. (5) and directly solved by the GA again.

## Ⅲ. Calculation and Design Results

Firstly, the design problem of a lowpass filter(LPF) is referred to. The specifications are the cutoff frequency of 0.45 GHz, the insertion-loss less than 2 dB, the passband ripple less than 0.5 dB, and attenuation greater than 5 dB at 0.45 GHz, as is often wanted in the cable-type EMI filtering research areas[6]~[8]. Aimed at implementing the coaxial cable-type LPF, the circuit parameters will be evaluated by performing the optimization (GA)-based rational-function fitting technique mentioned before, for the following physical structure.

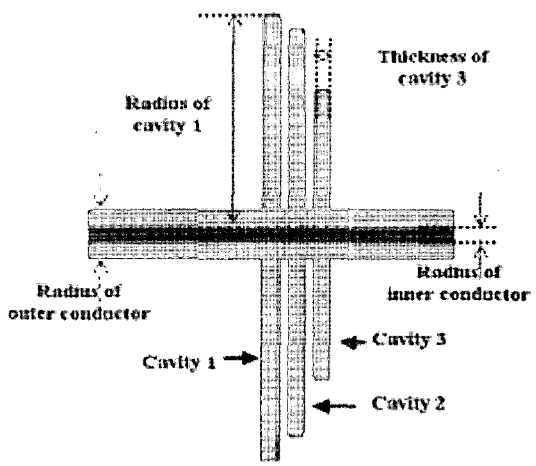



Fig. 2. A coaxial-cable filter with 3 cavities.

Though looking a little complicating, the physical sizes such as radii and thickness of cavities for the fixed outer and inner radii of the coaxial cable, can be easily obtained from the circuit parameters that are directly related to Re $s_i$, $P_i$, $C_{li}$ and $Q_h$. What should be done prior to this is determining G.A. parameters and applying it to Eq. (5). As for G.A., undergoing watching the level of complexity or what not, the present method adopts 8 bits, 10 parameters, 100 individuals, 50 generations and crossover rate of 0.75. All the variables as unknown real and imaginary parts vary from $-1$ to $+1$ in terms of normalized complex frequency. The error function is effectively defined as the locations of poles for the desired frequency response. And physical passivity is enforced on poles in the right half plane of $S$. Fig. 3 shows the error function with three mutation rates and convergence over 35 of the generation number. 0.095 is chosen as the mutation rate.

What must be noted for improving the design method as our contribution is that the attenuation poles are placed as we desire and the optimized rational-function fitting will do good to the finding of the desired poles.

This can not be accomplished by the traditional filter methodology of maximally flat or equi-ripple types. For better sharpness than that of 5 dB at 0.45 GHz, attenuation poles 0.45 GHz and 0.6 GHz near the cutoff frequency are wanted for certain EMI filter design. What follows is the procedure of our work to find the desired frequency response. For the first few stages of searching the values of Eq. (10), their complex poles and residues are tried and produce the frequency points of $S_{21}$ that exceed 0 dB. However, they will disappear by physical constraints. After similar procedures, at the final stage, Eq. (10) is found as in the following table.

This table shows three pairs of poles and residues and a constant in order. Nonetheless, it is found out that is

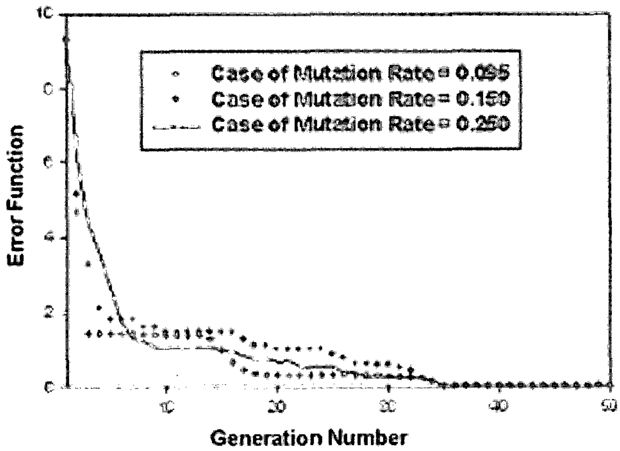



Fig. 3. Error function behavior v.s. mutation rate.

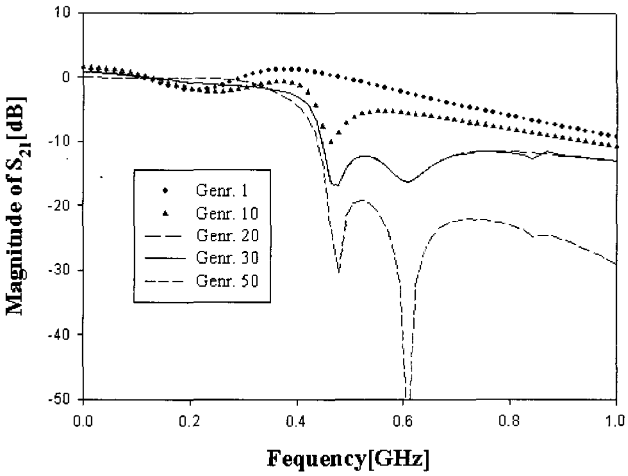



Fig. 4. Magnitude variation during the searching process.

Table 1. Computed unknowns of Eq. (5)

| Computed values | |
|---|---|
| Im($P_1$)= $\omega$ Re $s_i$ | : 2.805G[rad/s]<br>: $(-0.202j*0.060)$G[rad/s] |
| Im($P_2$)= $\omega$ Re $s_2$ | : 3.797G[rad/s]<br>: $(-0.085j*0174)$G[rad/s] |
| Im($P_3$)= $\omega$ Re $s_3$ | : 5.362G[rad/s]<br>: $(0.002j*0.001)$G[rad/s] |
| $C_{li}$ | 0.056 |

not needed for this case. It needs stating that the first two poles almost coincide with the desired ones. And then, the design result of this work is given.

With the computed values of Eq. (10), the circuit parameters have been converted to their corresponding radii and thickness of the coaxial LPF(radii=250 mm,

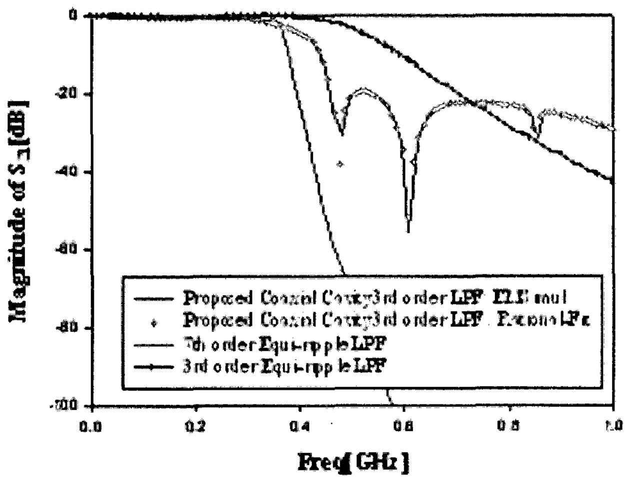



Fig. 5. Design result, compared to conventional Chebyshev cases.

200 mm, 140 mm, and thickness=40 mm, 37 mm, 30 mm for cavities 1, 2 and 3 in order), with reference to [7]. The followings are the realized structure and the tabulated physical sizes.

This proposed RF filter has only three cavities, and has superiority to the 7th ordered equi-ripple LPF from the compactness point of view(more than twice shorter than the total length of 7th ordered Chebyshev LPF). Also, compared to the same order of the Chebyshev LPF, the present structure has far better sharpness with the almost same size. Let alone, the computed data by Eq. (10) overlaps the rigorous evaluation.

Next, two designed filters(named Elliptic Cases 1 and 2) for the aforementioned specs are displayed. The magnitude and phase of $S_{21}$ are compared about the two experimental cases. The number of their transmission zeros is the same, but their locations differ from each other. Because of this, Elliptic Case 1 is slightly inferior to Elliptic Case 2 in damping the 1st peak in the out-of-band.

Lastly, the magnitude and phase of return loss are

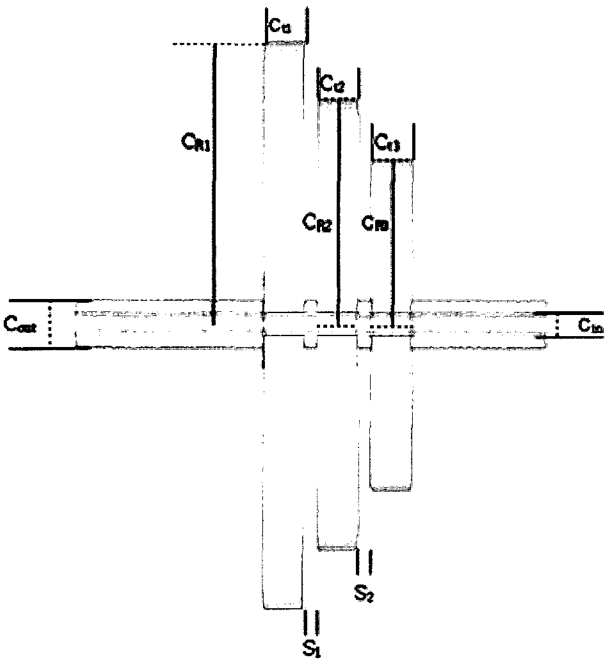
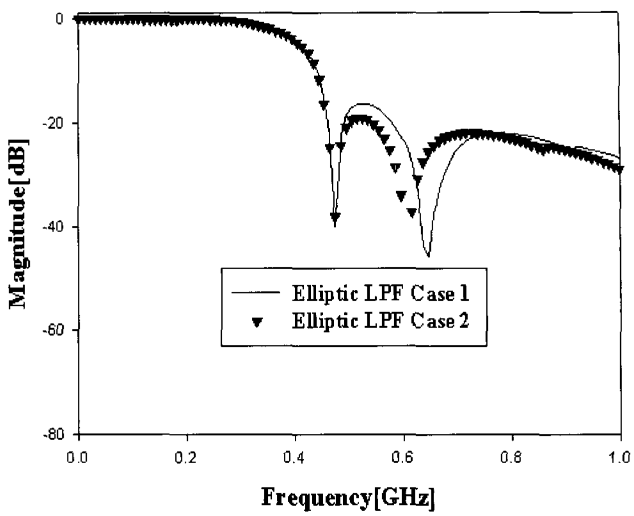


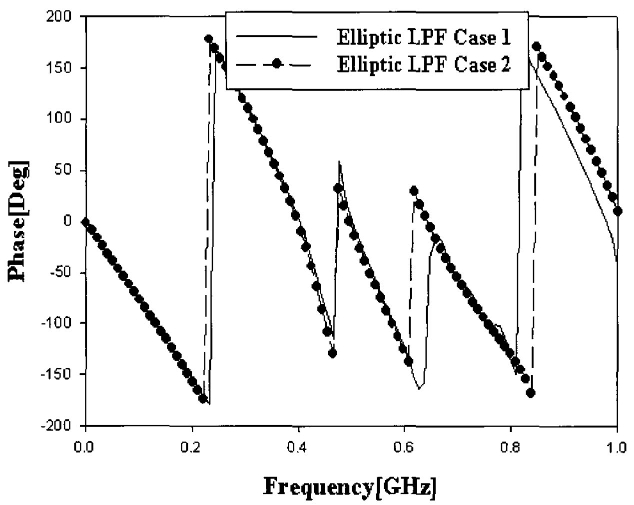

Fig. 6. Realized structure.

Table 2. Physical sizes.

| | Cavity 1 $n$=1 | Cavity 2 $n$=2 | Cavity 3 $n$=3 |
|---|---|---|---|
| Thick($C_{tn}$) | 40 mm | 37 mm | 30 mm |
| Radius($C_{Rn}$) | 250 mm | 200 mm | 140 mm |
| $C_{out}$=20 mm, $C_{in}$=10 mm, $S_1$=15 mm, $S_2$=10 mm | | | |




(a) Magnitude of $S_{21}$ for Elliptic Cases 1 and 2




(b) Phase of $S_{21}$ for Elliptic Cases 1 and 2

Fig. 7. Experimental $S_{21}$ results of Elliptic Cases 1 and 2.

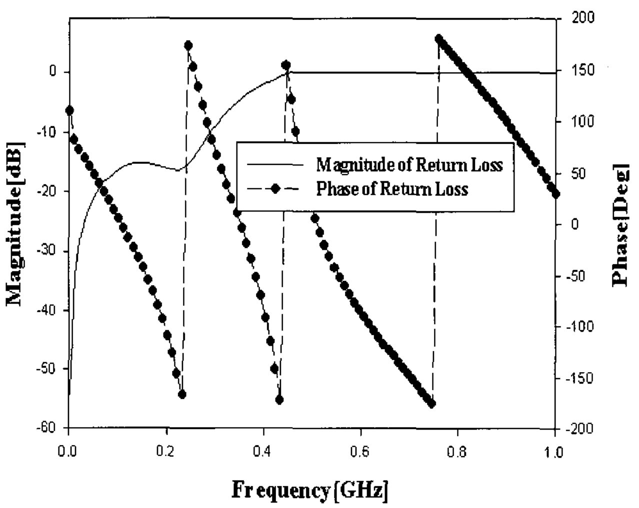



Fig. 8. Magnitude and phase of the experimental return loss of Elliptic Case 2.

shown for Elliptic Case 2. Elliptic Case 2 is excluded this time, since it turned out similar to the other.

In the passband, the return is lower than −15 dB, which can be used for good impedance matching. To improve the matching performance, the aperture of the cavity 1 needs deviating from the uniformity of the apertures of all the cavities that is adopted for the sake of convenience. This will accompany the modification of the value of $S_1$.

## Ⅳ. Conclusion

In this paper, an efficient design method is presented for filters' implementation, avoiding the similarity transform of their coupling coefficient matrices. The transfer function of a target filter is obtained by the GA based rational-function fitting technique with passivity enforcement. A 3rd order coaxial filter for an electromagnetic protection(EMI filter) application has been chosen to validate the proposed method and successful design results have proven it.

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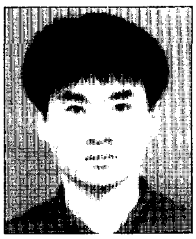

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