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A Theoretical Investigation on the Generation of Strength in Staple Yarns

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Abstract: In this article, an attempt has been made to explain the failure mechanism of spun yarns. The mechanism includes the aspects of generation and distribution of forces on a fibre under the tensile loading of a yarn, the free body diagram of forces, the conditions for gripping and slipping of a fibre, and the initiation, propagation, and ultimate yarn rupture in its weakest link. A simple mathematical model for the tenacity of spun yarns has been proposed. The model is based on the translation of fibre bundle tenacity into the yarn tenacity.

Keywords: Critical length, Fibre breakage, Fibre slippage, Spun yarn, Tensile failure, Yarn strength, Weakest link

Introduction

The strength of a spun yarn has great significance in terms of yarn quality. Therefore, the importance of the theoretical explanation of generation of strength in a spun yarn is worth mentioning. In spite of the number of publications [1-13] reported so far in the literature concerning this aspect, the actual failure mechanism of staple yarns is not yet clearly explained. Therefore, in this present article, firstly an attempt has been made to explain the mechanism of spun yarn failure in a simple way, which has validity in explaining some of the anomalies encountered in the previous literatures.

In this paper, another attempt has been made to derive a mathematical model for the spun yarn tenacity, which is based on the concept of translation of fibre bundle tenacity into yarn tenacity. The model is derived from the knowledge of the mechanism of spun yarn failure.

A Theoretical Analysis of Spun Yarn Failure

To begin with the analysis of failure process in a staple yarn, we have primarily adopted Hearle's [1] qualitative approach of the behavior of spun yarns during extension and thereafter we have proposed our approach to the problem. A most desirable description of staple yarn failure mechanism might read as follows:

Twist imparts coherence to the fibres in a staple yarn. Apart from twist, fibre migration, which causes each fibre to move across the yarn cross-section, gives better interlocking of fibres in yarns. Therefore, a staple yarn is a self-interlocking structure. To develop strength in a staple yarn, the individual fibres must grip each other when the strand is stressed. When a load is applied to staple yarn, the fibres are held together by friction, which is mainly derived from geometry, such as twist and migration.

At a given load in the yarn, twist and migration cause an externally imposed tension to be converted internally into lateral pressure σ_l applied by neighboring fibres acting normal to the fibre axis (Figure 1). Due to the lateral pressure the frictional forces between the fibres are generated which build up a resistance against the tension in the yarn body. In order to give a simple treatment of spun yarn failure, the following assumption are made:

- The lateral pressure applied by neighboring fibres is a linearly increasing function of applied tension in the yarn.
- The length of fibre is constant.
- Fibre diameter is constant along the length.
- Fibre-to-fibre friction follows Amonton's law.

Thus,

$$F = \mu N \quad (1)$$

where F: frictional force, N: lateral force, μ: coefficient of fibre-to-fibre friction.

The following notations are used:

L: fibre length
d: fibre diameter
σ_a: axial stress on a fibre
σ_l: lateral pressure on a fibre

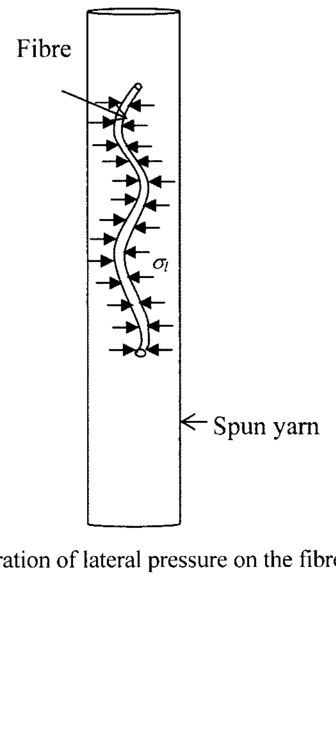

Figure 1. Generation of lateral pressure on the fibre.

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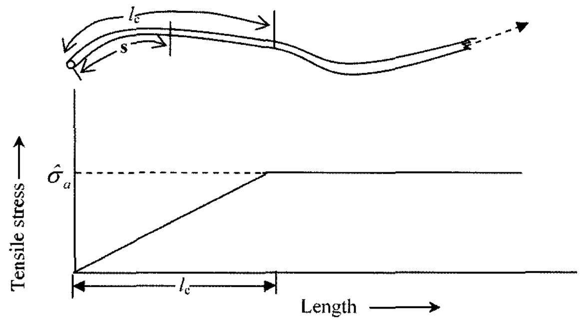

Figure 2. Build-up of tensile stress in a fibre [Hearle (1980)].

l_c: critical length

$\hat{\sigma}_a$: maximum stress of a fibre at a given load in yarn

A simple and linear diagram of the variation of tensile stress along the length of a fibre in a yarn is given in Figure 2. At the fibre end tension is zero, since at the tip of the fibre no force can be applied. But due to generation of lateral pressure on a fibre, there will be gradual build up of the frictional force along the fibre. At a distance s from an end of a fibre (Figure 2), frictional force equals to the cumulative resistance to slip. Therefore, frictional force F_s at a distance s from an end of a fibre can be expressed as:

$$F_s = \mu\pi d\int_0^s \sigma_\ell ds \tag{2}$$

When external tension in the yarn is zero, $\sigma_\ell = 0$. Again, at zero twist, $\sigma_\ell = 0$. Therefore, in these two conditions, there is no frictional resistance and fibres can no longer resist the load.

The Distribution of Tensile Stress on a Fibre

If σ_ℓ is assumed to be constant along the length of a fibre, the equation of frictional resistance of a fibre at a distance s from an end is given by

$$F_s = \mu\pi d\sigma_\ell s \tag{3}$$

The applied tension at that position of the fibre is $\frac{\pi d^2}{4}\sigma_a$. Now, at equilibrium, the applied tension in the fibre is balanced by the frictional resistance. Hence,

$$\frac{\pi d^2}{4}\cdot\sigma_a = \mu\pi d\sigma_\ell s \tag{4}$$

$$\therefore \sigma_a = \frac{4\mu\sigma_\ell s}{d} \tag{5}$$

The equation (5) represents the axial stress value of a fibre at a distance s from its end. But, if σ_ℓ is assumed to vary along the length of a fibre, the axial stress at a distance s is given by

$$\sigma_a = \frac{4\mu}{d}\int_0^s \sigma_\ell ds \tag{6}$$

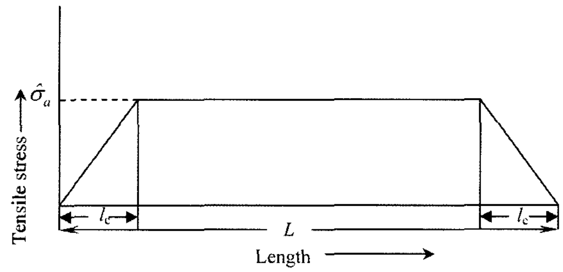

Figure 3. The distribution of tensile stress along the whole length of a fibre.

On a fibre, at a distance l_c from an end, tensile stress reaches maximum value for a given yarn tension. This means that any force greater than this value cannot be applied to the fibre body at a certain level of yarn tension. The distance l_c from the fibre end is known as critical length.

For constant σ_ℓ along the fibre length, the maximum value of stress of a fibre at a given load in yarn is given by the following equation

$$\hat{\sigma}_a = \frac{4\mu\sigma_\ell l_c}{d} \tag{7}$$

For a given load in the yarn, the magnitude of the axial stress in the length $(L - 2l_c)$ is the effective stress value on the fibre. Figure 3 depicts the distribution of tensile stress along the fibre.

The Conditions for Gripping and Slipping of a Fibre

From the foregoing discussion, it is clear that, a fibre cannot develop maximum stress value at a given load in the yarn, if it will satisfy the following condition

$$\ell_c = \frac{L}{2} \tag{8}$$

Therefore, a slipped fibre will also satisfy the following condition

$$\mu\pi dl_c\sigma_\ell < \frac{\pi d^2}{4}\cdot\hat{\sigma}_a \tag{9}$$

$$\therefore l_c < \frac{\hat{\sigma}_a\cdot d}{4\mu\sigma_\ell} \tag{10}$$

A slip fibre never be gripped at any position along its length and it does not fully contributing to the yarn strength. Thus a slipped fibre is also known as the non-load-contributing fibre. From the equation (10) it can be concluded that if the value of σ_ℓ is lower for a fibre, the possibility of a fibre slipping during tensile loading of a yarn increases. Fibres having poor migration such as those in the outer layer of a yarn tend to slip under yarn tension. Further, short fibres in the yarn are prone to slip. Also, a poor degree of fibre interlocking in the yarn causes more fibre to slip. A typical example of the distribution of tensile stress along a slipped

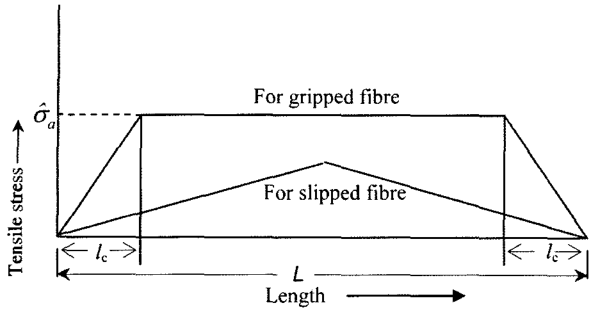

Figure 4. The distribution of tensile stress along the length of gripped and slipped fibres.

fibre is shown in Figure 4.

On the contrary, a gripped fibre will satisfy the following condition

$$(L - 2\ell_c) > 0 \tag{11}$$

$$\therefore l_c < \frac{L}{2} \tag{12}$$

Therefore, for a gripped fibre, a balance will be effectively built up between the maximum values of tension and frictional forces. Hence,

$$\mu \pi d l_c \sigma_\ell = \frac{\pi d^2}{4} \hat{\sigma}_a \tag{13}$$

$$\therefore l_c = \frac{\hat{\sigma}_a \cdot d}{4 \mu \sigma_\ell} \tag{14}$$

A gripped fibre contributes fully to bear the load when a yarn is stressed. At the place of yarn failure, when a gripped fibre reaches its breaking load or extension, it will break. Thus the gripped fibres are known as the contributing fibres. Figure 4 depicts the typical distribution of tensile stress along a gripped fibre.

The Relationship Between Axial and Lateral Stresses

Helical migratory configuration of fibres leads to the conversion of applied load in the yarn to lateral pressure. Let us assume that P be the applied load to the yarn. Now, for a gripped fibre, the maximum stress value is directly proportional to the applied load in the yarn. Figure 5 represents the distribution of tensile stress on a fibre for two different values of the applied load in the yarn. Therefore, we have

$$\hat{\sigma}_a \propto P \tag{15}$$

Also, lateral pressure applied by the neighboring fibres is directly proportional to the applied load in the yarn, which follows

$$\sigma_l \propto P \tag{16}$$

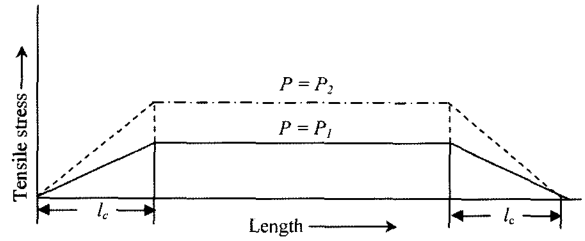

Figure 5. The distribution of tensile stress on a fibre for two different values of the applied load in the yarn, where $P_2 > P_1$.

Therefore, from the above equations, we can write that

$$\sigma_\ell \propto \hat{\sigma}_a \tag{17}$$

$$\therefore \sigma_\ell = \kappa \hat{\sigma}_a \tag{18}$$

where κ = an operational factor which converts the tensile stress to transverse stress.

The Free Body Diagram of Forces

In Figure 6, a yarn AA' is subjected to axial tension by the action of a vertical load P applied in the axis of a yarn, the proper weight of which is neglected. The load on the yarn stretches it to produce rupture. This tendency to rupture is resisted by internal forces within the yarn. To visualize these internal forces, imagine that the yarn is cut at a section *mn* perpendicular to its axis and that the lower portion is isolated as a free body (Figure 6). At the lower end of this portion of the yarn, the external force P is applied. On the upper end are the internal forces representing the actions of the fibres of the upper part of the yarn on those of the lower part. These forces are continuously distributed over the cross-section *mn*. Visualizing each fibre in the yarn carries its fair

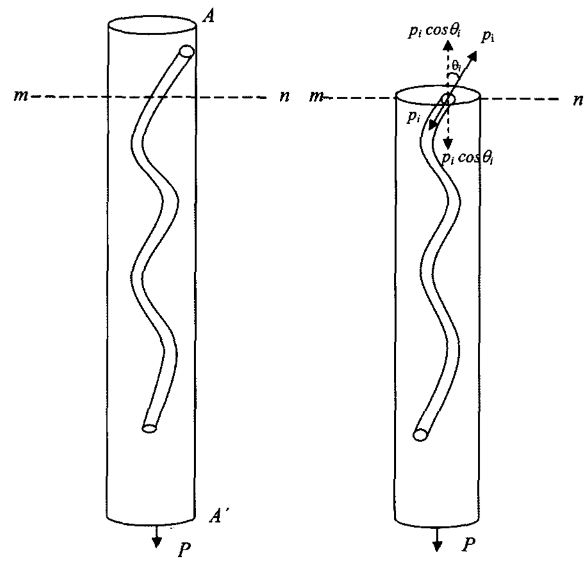

Figure 6. The forces acting in the yarn.

share of the load, it appears reasonable to assume, in this case, that the distribution of forces over the yarn cross-section will be uniform. From the condition of equilibrium of the free body (Figure 6), it is seen that the resultant of this uniform distribution of internal forces must be equal to the external load. Thus, if the force, p_i is acting along the i-th fibre axis, the force along the direction of yarn axis will be $p_i \cos\theta_i$, where θ_i is the helix angle made by the i-th fibre with the yarn axis. Therefore, we have

$$P = \sum_{i=0}^{n} (p_i \cos\theta_i) \qquad (19)$$

The Yarn as a Successive Link of Fibre Bundle

Those fibre bundle at the place of yarn break, which themselves break, and therefore determine the yarn strength must obviously have been gripped at two places and are pulled apart. Therefore, the fibre bundle tenacity, which is registered as yarn tenacity, must correspond to that at a certain gauge length and this length can be termed as the effective gripping length. This effective gripping length need not necessarily be either zero, or 1/8-inch. Obviously with increasing twist and increasing degree of fibre bundle interlocking, this effective gripping length will decrease. Apart from that, fibre characteristics such as length, crimp, and friction could influence the degree of interlocking and therefore, the distance at which fibre bundle interlock themselves.

In normal yarns the boundaries of the fracture zone are very nebulous and cannot be specified directly, nevertheless some estimate of the fracture zone length may be obtained by observing the broken portions of a yarn. From the analysis of the two failure ends of yarn one can reconstruct the profile of yarn before break. A typical diagram of the two broken pieces of the yarn is shown in Figure 7, from which

$$l_h = l(1-\varepsilon) \qquad (20)$$

where l_h: actual length of the weakest zone in the yarn before the break, l: length of the failure zone, ε: yarn strain at break.

Therefore, a yarn can be assumed to have successive elements or links in the form of sections of fibre bundles. The links may differ in their length, fibre arrangement, and linear density. The length of any given element is poorly

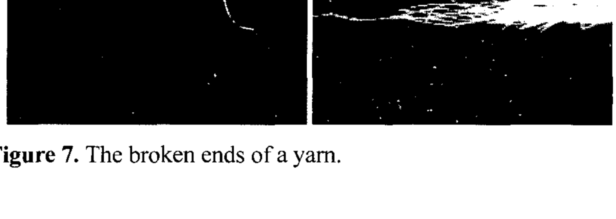

Figure 7. The broken ends of a yarn.

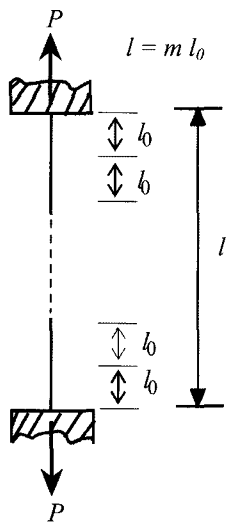

Figure 8. A yarn as a successive link of fibre bundle.

defined, because the elements are not separate entities but each gradually merges into its neighbor at either end. Nevertheless an average element length may be considered to exist [14]. A fibre can pass through several links of fibre bundle. A schematic representation of a yarn as a successive link of fibre bundle is shown in Figure 8, assuming that the length of each link is equal to a length l_0. When a yarn is subjected to tension, the load is uniformly distributed among the links. But the breaking load of a link of fibre bundle can be differed from each other. Yarn breaks at the weakest link and yarn breaking load equals to the breaking strength of the weakest link [15].

Phoenix [16] pointed out that for poor degree of fibre bundle interlocking in the yarn, the equal load sharing would likely to be expected among the surviving fibres and the length of the fibre bundle (l_0) would tend to be large. In this case the appearance of the failure zone would be fraying type. On the other hand, for better degree of fibre bundle interlocking, local load sharing would likely to be taken place among the surviving fibres, thereby, yarn failure would be proceeded by catastrophic crack growth and the length of the link of fibre bundle (l_0) would tend to be small. He also mentioned that the actual load sharing in spun yarns lies somewhere in between these two extremes.

The Condition of Fibre Break in the Weakest Link and Ultimate Yarn Failure

When a yarn is subjected to tensile load, it breaks at the weakest portion, which comprises the minimum number of fibres in the cross-section. The thinnest place having minimum number of fibres has the lowest torsional stiffness so that twist tends to concentrate here, leading to more than optimum twist at this place. Although, higher twist at this place reduces the fibre slippage, but at the same time, increased fibre obliquity owing to higher twist lowers the strength for thinnest

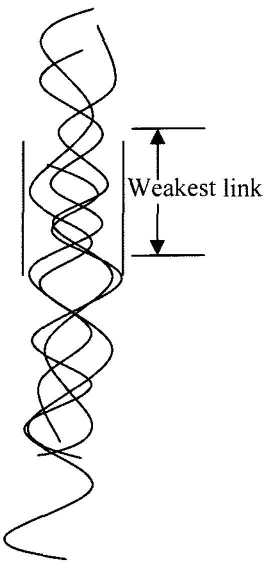

Figure 9. The position of fibres in the weakest link of a yarn.

place. Furthermore, the stress concentration at thinnest place is highest due to less number of fibres. As a consequence, the load per fibre at the place of the thinnest place is higher as compared to the fibres of other links. Therefore, yarn breakage is initiated at the thinnest place of the yarn.

A schematic diagram of the position of fibres in the weakest link of a yarn is presented in Figure 9. Actually, at the place of weakest link, when the generated tension of a fibre is greater than its breaking load, it will break. It has been reported by some researchers that the approximate length of the weakest link lies in between 2-7 mm [17,18]. Now, during the process of yarn failure a slipped fibre in the weakest link must be slipped. But for a gripped fibre having one end at the weakest link must be slipped, since it requires certain distance of a fibre from its end to support the full tension and usually this distance is greater than the length of a weakest link of twisted fibre bundle in the yarn [1,19]. Otherwise, a gripped fibre, which passes through a weakest link, should be broken during yarn rupture.

After breakage of certain number of fibres in the weakest link broken and slipped fibres do not contribute to bear the load directly. Thus the stress on surviving fibres concentrates causing rupture of the gripped fibres to propagate faster. Consequently, ultimate yarn failure takes place with the mixed mode of fibre breakage and slippage.

Modeling of Yarn Tenacity

In order to derive the mathematical equation for spun yarn tenacity as a function of fibre bundle tenacity the following assumptions are made:

- When a yarn is subjected to tensile load, it breaks at the weakest portion, which comprises the minimum number of fibres in the cross-section. The thinnest place having minimum number of fibres has the lowest torsional stiffness so that twist tends to concentrate here, leading to more than optimum twist at this place. Although, higher twist at this place reduces the fibre slippage, but increased fibre obliquity predominates over the effect of reduced fibre slippage leading to lowest strength for thinnest place. Further stress concentration at thinnest place is highest due to less number of fibres. As a consequence, the load per fibre at the place of the thinnest place is higher as compared to the fibres of other links. Therefore, it is logical enough to assume that yarn breakage is initiated at the thinnest place instead of other places in the yarn.
- The number of fibre in the yarn cross-section follows a normal distribution. Thus for 95 % confidence level, the relationship between the average number of fibres in the yarn cross-section and number of fibres at the weakest portion can be expressed as

$$n_h = n_e - 1.96\sigma \tag{21}$$

where n_h: number of fibres at the place of yarn break, n_e: average number of fibres in the yarn cross-section, σ: standard deviation of number of fibres in the yarn cross-section. Hence, from the definition of coefficient of variation, equation 3.21 can be transformed into

$$n_h = n_e(1 - 0.0196V) \tag{22}$$

where V = coefficient of variation of number of fibres in the yarn cross-section.

- The yarn volume remains constant throughout the tensile testing. Hearle [1] has worked out the following relationship for constant volume deformation:

$$\frac{\tan\theta}{\tan\theta'} = (1+\varepsilon)^{3/2} \tag{23}$$

where θ = average helix angle of the fibre in the yarn before break, θ' = average helix angle of fibre at the time of yarn failure, ε = yarn breaking strain.

Translation of Fibre Tenacity into Yarn Tenacity

To start with the derivation of yarn tenacity from the fibre bundle tenacity, the first thing to be taken into account is the length of yarn failure zone. From the yarn elongation at break and the length of yarn failure zone, the actual length of twisted fibre strand over which the breakage of yarn takes place can be estimated from equation (20).

Let F_h be the fibre bundle tenacity measured at a gauge length equal to l_h, which is the actual length of the weakest zone in the yarn before the break. But as the fibres have helical configuration in the yarn, therefore, the contribution of the fibre bundle tenacity into yarn tenacity is $F_h\cos^2\theta'$, since the tenacity of fibre bundle is defined as its breaking load per unit effective linear density. A simple representation

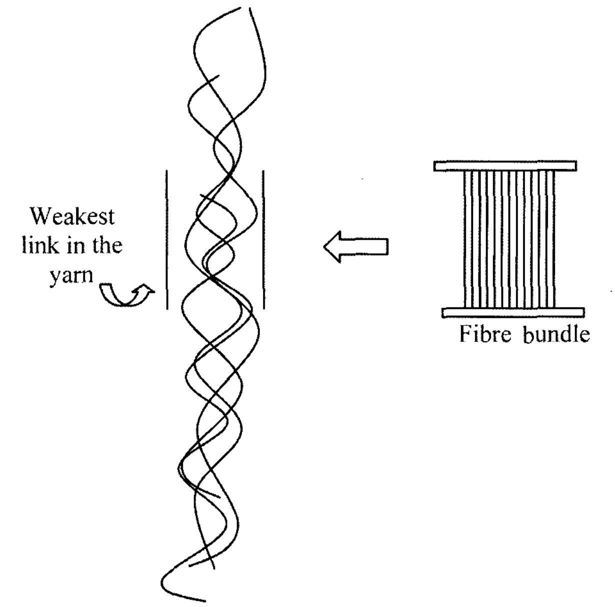

Figure 10. The translation of fibre bundle tenacity to yarn tenacity.

of the translation of fibre bundle tenacity to yarn strength is shown in Figure 10.

As the yarn rupture zones comprise of both broken and slipped fibre, therefore, a proportion of broken fibres to the number of fibres at the place of break has to be multiplied with the term $F_h\cos^2\theta'$ to get the calculated yarn tenacity. Moreover, the yarn tenacity is related to the linear density of yarn which is considerably in excess of that portion of yarn which actually breaks, and thus the resultant experimental tenacity of yarn is underestimated. As the yarn linear density is proportional to the number of fibre in the yarn cross-section, therefore, to compare the experimental yarn tenacity and the theoretical yarn tenacity, the later can be expressed as

$$Q_t\frac{n_e}{n_h} = \frac{\psi}{n_h}F_h\cos^2\theta' \tag{24}$$

where Q_t: theoretical yarn tenacity (cN/tex), ψ is the number of broken fibres at the place of yarn failure.

Now the percentage broken fibres is related with ψ as

$$\psi = \frac{n_h\varphi_b}{100} \tag{25}$$

where φ_b is the percentage broken fibres in the yarn failure zone. By substituting ψ in equation (24), the theoretical yarn tenacity is expressed as

$$Q_t = \frac{n_h}{n_e}\cdot F_h\cdot\frac{\varphi_b}{100}\cdot\cos^2\theta' \tag{26}$$

The model provides insight in understanding the mechanism of failure process of spun yarns from the point of translation of fibre bundle tenacity into yarn tenacity. The equation (26) is based on the assumption that only the broken fibres contribute to the yarn breaking strength. From the above discussion, it is clear that the contribution of slipping fibres on the maximum breaking load of yarn is extensively less as compared to that of gripped fibres. Therefore, the contribution of slipping fibres is not taken into account in deriving the model of yarn tenacity.

Now from the equations (22) and (26), we have

$$Q_t = (1 - 0.0196\cdot V)\cdot F_h\cdot\frac{\varphi_b}{100}\cdot\cos^2\theta' \tag{27}$$

The equation (27) indicates the relationship of yarn CV% and yarn tenacity. It is also evident from equation (27) that a higher value of yarn CV% would reduce the value of theoretical yarn tenacity.

Conclusions

The overall descriptions present a qualitative explanation for the mechanism of staple yarn failure. The mechanism provides a better insight in understanding the failure process of spun yarns. A mathematical expression for the translation of fibre bundle tenacity into the tenacity of single yarns has been proposed. The model provides further insight in understanding the mechanism of failure process of spun yarns.

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