Power Control in Uplink and Downlink CDMA Systems with Multiple Flow Types

Yun Li and Anthony Ephremides

Abstract: **We consider a power controlled code division multiple access (CDMA) system with multiple flow types. At each of the N nodes, there are F flow types with different signal-to-interference-and-noise-ratio (SINR) requirements. To keep the complexity of the transmitter low, we assume that each node uses the same power level for all its flows. The single flow case has been fully solved and is well-understood. We concentrate on the multiple flow case, and use a novel and different approach. For the uplink problem with $N = 2$ and F arbitrary, the necessary and sufficient conditions to have a solution are found and proved. For the general $N > 1$ uplink problem, we provide a necessary condition for the problem to have a solution and an iterative algorithm to find the optimum solution. For the downlink case with $F > 1$ some properties of the optimal sequences are obtained.**

Index Terms: **Code division multiple access (CDMA), multiuser detection, power control, user capacity.**

I. INTRODUCTION

Power control is used to balance the received powers of the users of a code division multiple access (CDMA) system, so that no single user can create excessive interference that can destroy the quality of the communication of other users. At the same time, it is desirable to use power levels as low as possible to save energy, provided the quality of service (QoS) objective defined by required signal-to-interference-and-noise-ratio (SINR) requirements is satisfied. In previous works [1]–[3], the optimum power vector was found by inversion of a non-negative matrix related to the channel gains and sequence crosscorrelation property. Subsequently, in [4] iterative power control algorithms were developed for many different power control problems, and it was proved that if the interference function satisfies certain conditions, these iterative power control algorithms converge to the optimum power vector. Later in [5], it was proved that the iterative power control algorithms converge at a geometric rate.

In recent years, power control and signature sequence allocation were studied as tools to maximize the user capacity of CDMA systems [6]–[8]. In [6], the joint optimal signature sequence and power allocation was studied to maximize the network user capacity for a synchronous CDMA system with a MMSE multiuser receiver. The optimal solution also minimizes the sum of the allocated powers, and simplifies the MMSE receiver to a matched filter. In [7], the optimal sequence allocation and the user capacity of asynchronous CDMA systems were derived. Other works that studied sequence selection and power control to satisfy the SINR requirements of the users can be found in [9] and [10].

All the studies mentioned above assume only single flow type at each node. In wireless networks of the future, each node will be multiplexing a variety of flows, each with its own QoS requirement. One example is in a cellular network, where a user (cellular phone) may have data, video, and audio to transmit to other users. These three flow types clearly have different QoS requirements. The QoS requirement is often expressed as the required bit error rate (BER) of the received signal, which, in turn, is a function of the SINR of the received signal. Here, we consider a synchronous CDMA system with a base station and N nodes, all equipped with matched filter receivers. At each node, there are F flow types with SINR requirements $0 < \beta_1 \leq \beta_2 \leq \cdots \leq \beta_F$. Each flow type is assigned a code with processing gain L, and they transmit simultaneously to the base station.

If at each node, the transmitter is able to assign different power levels to different flows, then the uplink power control problem is actually a single flow case with NF virtual users. However, since usually there is only one single power amplifier at each node (especially at the mobile node), it is natural and simpler to assume that there is only single power level available for all the flow types at a node. To use different power levels for each flow will necessitate a significantly more complex design, namely one in which the power levels of the multiplexed flow types are adjusted by appropriate weights or one in which the same goal is achieved via baseband processing. To avoid the additional complexity, it is useful to investigate the solution and performance of the optimal power allocation problem under the constraint of single and common power level for all flows. Therefore, we assume that each node has only one transmitter, i.e., only single power level is available for all F flow types. So when a node sets its power, it has to satisfy all of the SINR requirements of its flow types. One objective of this paper is to evaluate the performance degradation that results from this simple and inexpensive transmitter structure.

For the uplink, the first question is, for fixed codes, to determine the conditions for the power control problem to have a solution (i.e., for the SINR requirements to be satisfied). For the special case of $N = 2$, we have found the sufficient and necessary conditions for the power control problem to have a solution.

Manuscript received August 6, 2005; approved for publication by Fotini-Niovi Pavlidou, Division II Editor, April 4, 2006.

Yun Li is with the Department of Electrical and Computer Engineering, Colorado State University, Fort Collins, Colorado, USA, email: liyun@engr.colostate.edu.

Anthony Ephremides is with the Department of Electrical and Computer Engineering, University of Maryland, College Park, Maryland, USA, email: etony@umd.edu.

Prepared through collaborative participation in the Communications and Networks Consortium sponsored by the U. S. Army Research Laboratory under the Collaborative Technology Alliance Program, Cooperative Agreement DAAD19-01-2-0011. The U. S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation thereon. This research has been supported also by NSF Grant No. ANI0205330 and ONR Grant No. N000140110185.

1229-2370/06/$10.00 © 2006 KICS

For the general problem ($N > 2$), we only have necessary conditions. The next step is to solve the optimum (minimum) power vector to satisfy all the SINR requirements. We do not have an analytical solution to the optimal power vector, but we provide an iterative algorithm to find it. After that, the natural question is to search for the optimal sequences so that the total power of all users is minimized, when the optimum power vectors are used. Because of the lack of an analytical solution, we are only able to give the solution for optimal sequences in the special case of single flow.

In the downlink, the base station transmits to N nodes simultaneously using NF different codes. Its power level P can be adjusted to satisfy the SINR requirements of the users. Since the base station only has one power level, it is relatively easy to obtain the minimum power assignment of the base station to satisfy the SINR requirements for fixed codes. Then the next question is to search the optimal sequences to minimize that power assignment. We do not have an exact solution to the optimal sequences; however, we obtain some properties that the optimal sequences satisfy.

The organization of this paper is as follows. In Section II, we formulate the problem for the uplink and study the special cases for $F = 1$ and $N = 2$, and the general case of $F > 1$, $N > 1$. In Section III, we solve the power assignment of the base station and study the optimal sequences for the $F = 1$ special case, the $N = 1$ special case, and the general $F > 1$, $N > 1$ case. In Section IV, we compare the performance achieved by the optimal solution in the proposed constrained problem with the performance of the same system if each flow type can have its own power level, so as to determine the price we pay to achieve reduced transmitter complexity. Finally, we summarize our work in Section V.

II. UPLINK CASE

In the uplink CDMA system, the signal received in one symbol interval at the base station [11] for our multiple flow type case can be written in the following vector form.

$$\boldsymbol{y} = \sum_{i=1}^{N} \sqrt{P_i}\left(\sum_{f=1}^{F} b_{if}\boldsymbol{s}_{if}\right) + \boldsymbol{w}.$$

Here, P_i is the received power for each of the F flow types of node i, which is the product of the transmission power of user i and the attenuation factor from user i to the base station, b_{if} and $\boldsymbol{s}_{if}$ are the transmitted bits and the signature sequence of flow type f at node i, and $\boldsymbol{w}$ is the additive white Gaussian noise (AWGN) vector with covariance matrix $\sigma^2\boldsymbol{I}$.

The SINR requirements of flow type f at node i can be written as

$$SINR_{i,f} = \frac{P_i}{\sigma^2 + \sum_{(j,g)\neq(i,f)} P_j\rho_{if,jg}^2} \geq \beta_f \qquad f = 1,\cdots,F,\ \ i = 1,\cdots,N. \tag{1}$$

The quantity $\rho_{if,jg}$ denotes the crosscorrelation between flow type f at node i and flow type g at node j, that is, $\rho_{if,jg} = \boldsymbol{s}_{jg}^\top\boldsymbol{s}_{if}$. Define the total squared crosscorrelation between flow type f at node i and all flow types at node j as $\alpha_{ij}^f = \sum_{g=1}^{F}\rho_{if,jg}^2$, then the interference term can be rewritten as

$$\sum_{(j,g)\neq(i,f)} P_j\rho_{if,jg}^2 = \sum_{j=1}^{N}\sum_{g=1}^{F} P_j\rho_{if,jg}^2 - P_i\rho_{if,if}^2 = \sum_{j=1}^{N} P_j\alpha_{ij}^f - P_i$$

and the SINR requirements become

$$P_i \geq \beta_f\sigma^2 + \beta_f\sum_{j=1}^{N} P_j\alpha_{ij}^f - \beta_f P_i \qquad f = 1,\cdots,F,\ \ i = 1,\cdots,N.$$

Or in matrix form

$$\boldsymbol{P} \geq \beta_f\boldsymbol{A}^{(f)}\boldsymbol{P} + \beta_f\sigma^2\mathbf{1},\ \ f = 1,\cdots,F \tag{2}$$

with

$$\boldsymbol{A}^{(f)} = \begin{bmatrix} \alpha_{11}^f - 1 & \alpha_{12}^f & \cdots & \alpha_{1N}^f \\ \alpha_{21}^f & \alpha_{22}^f - 1 & \cdots & \alpha_{2N}^f \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{N1}^f & \alpha_{N2}^f & \cdots & \alpha_{NN}^f - 1 \end{bmatrix}$$

$$\boldsymbol{P} = \begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix}^\top$$

and

$$\mathbf{1} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^\top.$$

A. Special Case of $F = 1$

Now the SINR requirements can be written as

$$\boldsymbol{P} \geq \beta\boldsymbol{A}\boldsymbol{P} + \beta\sigma^2\mathbf{1}$$

while the symmetric non-negative matrix $\boldsymbol{A}$ has entries $a_{ij} = \rho_{ij}^2$ for $i \neq j$, and $a_{ij} = 0$ for $i = j$. For fixed sequences, this is the typical power control problem. Our first two questions, the conditions for the power control problem to have a solution and the optimal power vector, were answered in [1]–[3].

If $\beta < 1/\rho_A$, then a solution exists. Here, ρ_A is the largest eigenvalue of matrix $\boldsymbol{A}$ (Perron-Frobenius eigenvalue [12]). Therefore, the feasible β satisfies $\beta < 1/\rho_A$ and the optimum power vector is given by

$$\boldsymbol{P}^* = \sigma^2\beta(\boldsymbol{I} - \beta\boldsymbol{A})^{-1}\mathbf{1}.$$

We then use these results to find the optimal sequences to minimize the total power $\mathbf{1}^\top\boldsymbol{P}^*$, assuming that the optimal power vector is always used for whatever sequences. The minimization by the sequence assignment now reduces to the problem of $\min_{\boldsymbol{S}}\left(\mathbf{1}^\top(\boldsymbol{I} - \beta\boldsymbol{A})^{-1}\mathbf{1}\right)$. Here, $\boldsymbol{S} = [\boldsymbol{s}_1\ \boldsymbol{s}_2 \cdots \boldsymbol{s}_N]$ is the matrix that consists of N column sequence vectors.

Using the properties of matrix $(\boldsymbol{I} - \beta\boldsymbol{A})$, we have (Appendix A)

$$\left(\mathbf{1}^{\top}(\boldsymbol{I}-\beta\boldsymbol{A})\mathbf{1}\right)\cdot\left(\mathbf{1}^{\top}(\boldsymbol{I}-\beta\boldsymbol{A})^{-1}\mathbf{1}\right) \geq N^2. \quad (3)$$

The equality is achieved when $\sum_{j=1}^{N}\rho_{ij}^2 - 1 = (1-\gamma)/\beta,\ i = 1,\cdots,N$, is true for some constant γ.

For $N = 2$, the Welch bound of the total squared correlation is given by [13] and [14] $\sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{ij}^2 \geq N^2/L$, and the sequences that achieve the bound is called Welch bound equality (WBE) sequences. Notice that the WBE sequences, which obey $\boldsymbol{S}\boldsymbol{S}^T = (N/L)\boldsymbol{I}$, and $\sum_{j=1}^{N}\rho_{ij}^2 = N/L,\ \ i = 1,\cdots,N$, achieve equality in (3) with $\gamma = 1+\beta-\beta N/L$. From the Welch bound, we also have

$$\begin{aligned}\mathbf{1}(\boldsymbol{I}-\beta\boldsymbol{A})\mathbf{1} &= N - \beta\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{ij}^2 - N\right)\\ &\leq N\left(1+\beta-\beta N/L\right). \quad (4)\end{aligned}$$

Using (3) and (4) in succession yields the following minimum achieved by WBE sequences

$$P_{\text{total}} = \beta\sigma^2\mathbf{1}(\boldsymbol{I}-\beta\boldsymbol{A})^{-1}\mathbf{1} \geq N\sigma^2/(1+1/\beta-N/L).$$

For $N \leq L$, orthogonal sequences satisfy (3) with $\gamma = 1$, and it achieves the lowest total crosscorrelation, i.e., $\sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{ij}^2 \geq N$. Thus we have $P_{\text{total}} = N\beta\sigma^2$.

Proposition 1: For $N > L$, the optimal sequences that minimize the total power in an uplink power-controlled CDMA system with SINR requirement β are the WBE sequences and the corresponding optimum power vector is given by $\boldsymbol{P} = \sigma^2/(1+1/\beta-N/L)\mathbf{1}$. For $N \leq L$, the optimal sequences are orthogonal with optimum power vector $\boldsymbol{P} = \beta\sigma^2\mathbf{1}$.

Since the power allocation for $N > L$ case should be positive, we have $N/L < 1 + 1/\beta$. Therefore $1 + 1/\beta$ is the maximum number of users per degree of freedom the system can hold, provided the SINR requirement β is satisfied. It is the so-called user capacity for the uplink CDMA system with matched filter receiver.

The above conclusions about the optimal sequences, the optimum power vector, and the user capacity of the uplink CDMA system with a matched filter were first given in [6]. We include this special case discussion here, although the solution is known, to illustrate our different methodology. Specifically, in [6], the power and sequence assignment were jointly optimized to maximize the user capacity for a synchronous CDMA system with linear MMSE multiuser receiver. Then the optimal sequences were found to be the WBE sequences, which also minimize the total power. Moreover, the MMSE receiver for the optimal sequences and optimum power assignment was found to be a matched filter. Here, we first optimize the power vector for any fixed sequences; then we optimize the sequence assignment for a CDMA cell (which uses the optimum power vector) to minimize the total power.

B. Special Case of $N = 2$

We now study the case of two nodes because it is the simplest case that reveals the different character of our problem. Up to now, we have assumed that there are F flow types, and they are common to all nodes. In this subsection, we remove this assumption and look at the more general case when the numbers of flow types at each node are different. We assume that there are F_1 flow types with $0 < \beta_{11} \leq \beta_{12} \leq \cdots \leq \beta_{1F_1}$ at node 1; and there are F_2 flow types with $0 < \beta_{21} \leq \beta_{22} \leq \cdots \leq \beta_{2F_2}$ at node 2. The conclusion in this subsection applies to the special case with F common flow types if we replace F_1 and F_2 with F, and replace β_{1f} and β_{2f} with β_f.

Now the SINR requirements for the flow types at nodes 1 and 2 are as follows:

$$P_1 \geq \beta_{1f}\sigma^2 + \beta_{1f}P_1(\alpha_{11}^f - 1) + \beta_{1f}P_2\alpha_{12}^f$$

with

$$\alpha_{11}^f = \sum_{g=1}^{F_1}\rho_{1f,1g}^2,\ \alpha_{12}^f = \sum_{g=1}^{F_2}\rho_{1f,2g}^2,\ f = 1,\cdots,F_1$$

and

$$P_2 \geq \beta_{2f}\sigma^2 + \beta_{2f}P_2(\alpha_{22}^f - 1) + \beta_{2f}P_1\alpha_{21}^f$$

with

$$\alpha_{21}^f = \sum_{g=1}^{F_1}\rho_{2f,1g}^2,\ \alpha_{22}^f = \sum_{g=1}^{F_2}\rho_{2f,2g}^2,\ f = 1,\cdots,F_2.$$

Because of the non-negativity of α_{ij}^f, if a positive solution (P_1, P_2) exists, then we must have

$$P_1(1 + 1/\beta_{1f} - \alpha_{11}^f) \geq \sigma^2 + P_2\alpha_{12}^f > 0,\ f = 1,\cdots,F_1$$

and

$$P_2(1 + 1/\beta_{2f} - \alpha_{22}^f) \geq \sigma^2 + P_1\alpha_{21}^f > 0,\ f = 1,\cdots,F_2.$$

Therefore, we have the following necessary conditions in order for the problem to have a solution

$$\alpha_{11}^f < 1 + 1/\beta_f,\ f = 1,\cdots,F_1 \quad (5)$$

$$\alpha_{22}^f < 1 + 1/\beta_f,\ f = 1,\cdots,F_2. \quad (6)$$

Let us rewrite the SINR requirements as

$$P_1 \geq a_f + b_f P_2, f = 1,\cdots,F_1$$

where

$$a_f = \frac{\sigma^2}{1 + 1/\beta_{1f} - \alpha_{11}^f},\ b_f = \frac{\alpha_{12}^f}{1 + 1/\beta_{1f} - \alpha_{11}^f}$$

and

$$P_2 \ \geq c_f + d_f P_1, f = 1,\cdots,F_2$$

where

$$c_f = \frac{\sigma^2}{1 + 1/\beta_{2f} - \alpha_{22}^f},\ d_f = \frac{\alpha_{21}^f}{1 + 1/\beta_{2f} - \alpha_{22}^f}.$$

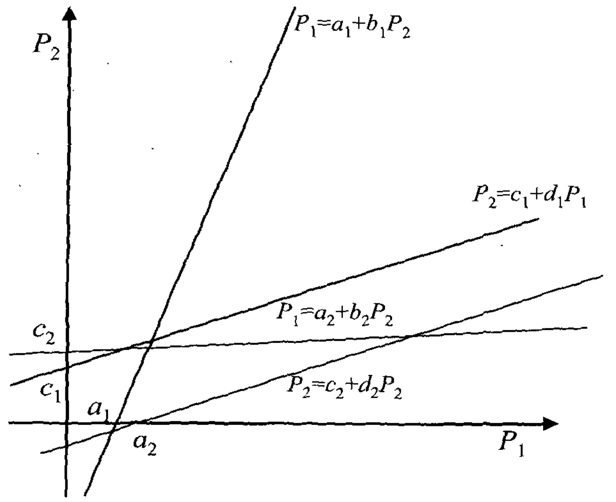


Fig. 1. When $1/b_2 \leq d_1$ is true, feasible area of flow type 1 is always above the feasible area of flow type 2, hence the two solution areas do not overlap.

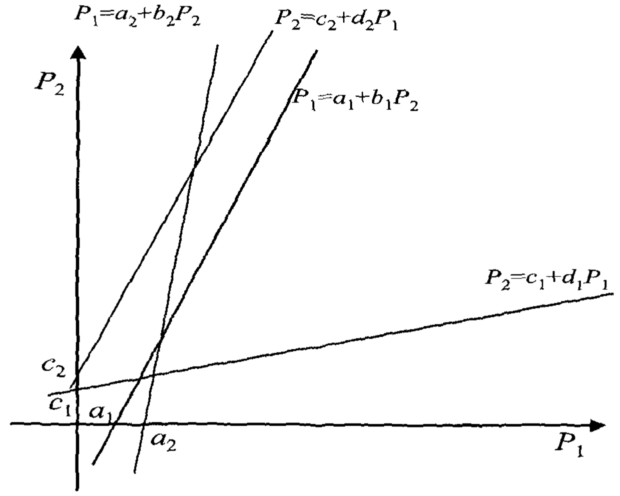


Fig. 2. When $1/b_1 \leq d_2$ is true, feasible area of flow type 1 is always under the feasible area of flow type 2, and there is no overlap of these 2 areas.

If (5) and (6) are satisfied, these coefficients defined above are either positive or non-negative. On the (P_1, P_2) plane, the straight lines $P_2 = c_f + d_f P_1$ and $P_1 = a_f + b_f P_2$ have non-negative or positive slopes and intersections. Furthermore, the SINR requirements can also be written as

$$P_1 \geq \max_{f=1,\cdots,F_1} (a_f + b_f P_2) \tag{7}$$

$$P_2 \geq \max_{f=1,\cdots,F_2} (c_f + d_f P_1). \tag{8}$$

The region defined by (7) is the infinite area to the right of lines $P_1 = a_f + b_f P_2, f = 1,\cdots,F_1$. Its behavior when P_1 and P_2 are large enough is determined by the minimum slope $\min_{f=1,\cdots,F_1}(1/b_f)$. The area defined by (8) is the infinite area above lines $P_2 = c_f + d_f P_1, f = 1,\cdots,F_2$. Its behavior when P_1 and P_2 are large enough is determined by the largest slope $\max_{f=1,\cdots,F_2} d_f$. If a solution exists, the power vectors that satisfy the SINR requirements are in the overlapped area of (7) and (8). Therefore, the existence of a solution can be completely determined by these coefficients, namely the values of $\max_{f=1,\cdots,F_2} d_f$ and $\min_{f=1,\cdots,F_1}(1/b_f)$.

Proposition 2: In the $N = 2$ uplink power control problem with SINR requirements $0 < \beta_{11} \leq \beta_{12} \leq \cdots \leq \beta_{1F_1}$ at node 1 and $0 < \beta_{21} \leq \beta_{22} \leq \cdots \leq \beta_{2F_2}$ at node 2, a solution exists if and only if

$$\alpha_{11}^f < 1 + 1/\beta_f, \quad f = 1,\cdots,F_1$$

$$\alpha_{22}^f < 1 + 1/\beta_f, \quad f = 1,\cdots,F_2$$

and

$$\max_{f=1,\cdots,F_1} b_f \cdot \max_{f=1,\cdots,F_2} d_f < 1. \tag{9}$$

Proof: The solution set A can be described as

$$A = \left\{ (P_1, P_2) > 0 : \begin{array}{l} P_1 \geq a_f + b_f P_2, \; f = 1,\cdots,F_1, \\ P_2 \geq c_f + d_f P_1, \; f = 1,\cdots,F_2. \end{array} \right\}.$$

Necessity: Suppose (5) and (6) are not satisfied, then there is no solution. Suppose (5) and (6) are satisfied, but (9) is not satisfied, then there exist at least one pair of h and g ($h \neq g$) such that $0 < 1/b_h \leq d_g$. From the non-negative property of the coefficients, for any $P_1 > 0$, we have $c_g + d_g P_1 > -a_h/b_h + P_1/b_h$. Then

$$\begin{aligned} &\{(P_1, P_2) > 0 : \; P_2 \geq c_g + d_g P_1, \text{ and } P_1 \geq a_h + b_h P_2\} \\ &= \{(P_1, P_2) > 0 : \; c_g + d_g P_1 \leq P_2 \leq -a_h/b_h + P_1/b_h\} \\ &= \emptyset. \end{aligned}$$

Since solution set A is a subset of this set, the solution set is also empty, i.e., no solution exists.

Sufficiency: Suppose (5), (6), and (9) are all satisfied. From

$$\max_{f=1,\cdots,F_1} a_f + P_2 \max_{f=1,\cdots,F_1} b_f \geq a_f + b_f P_2, \; f = 1,\cdots,F_1$$

$$\max_{f=1,\cdots,F_2} c_f + P_1 \max_{f=1,\cdots,F_2} d_f \geq c_f + d_f P_1, \; f = 1,\cdots,F_2$$

we have

$$\begin{aligned} A &\supseteq \left\{ (P_1, P_2) > 0 : \begin{array}{l} P_1 \geq \max_{f=1,\cdots,F_1} a_f + P_2 \max_{f=1,\cdots,F_1} b_f, \\ P_2 \geq \max_{f=1,\cdots,F_2} c_f + P_1 \max_{f=1,\cdots,F_2} d_f \end{array} \right\} \\ &= \left\{ (P_1, P_2) > 0 : \begin{array}{l} \max_{f=1,\cdots,F_2} c_f + P_1 \max_{f=1,\cdots,F_2} d_f < P_2 \\ < \dfrac{(P_1 - \max_{f=1,\cdots,F_1} a_f)}{\max_{f=1,\cdots,F_1} b_f} \end{array} \right\} \\ &\neq \emptyset. \end{aligned}$$

The last step is justified because

$$\max_{f=1,\cdots,F_2} c_f + P_1 \max_{f=1,\cdots,F_2} d_f < \frac{P_1 - \max_{f=1,\cdots,F_1} a_f}{\max_{f=1,\cdots,F_1} b_f}$$

is guaranteed when P_1 is large enough, specifically, when

$$P_1 > \frac{\max_{f=1,\cdots,F_2} c_f + \max_{f=1,\cdots,F_1} a_f / \max_{f=1,\cdots,F_1} b_f}{1/\max_{f=1,\cdots,F_1} b_f - \max_{f=1,\cdots,F_2} d_f} > 0.$$

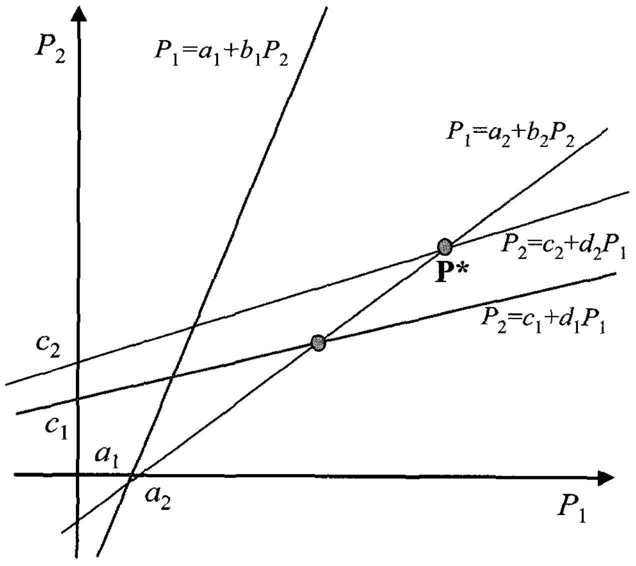


Fig. 3. The case $d_1 < 1/b_2 \leq 1/b_1$(the larger slope of the solution area of flow type 2 is between the smaller and larger slopes of the solution area of flow type 1.

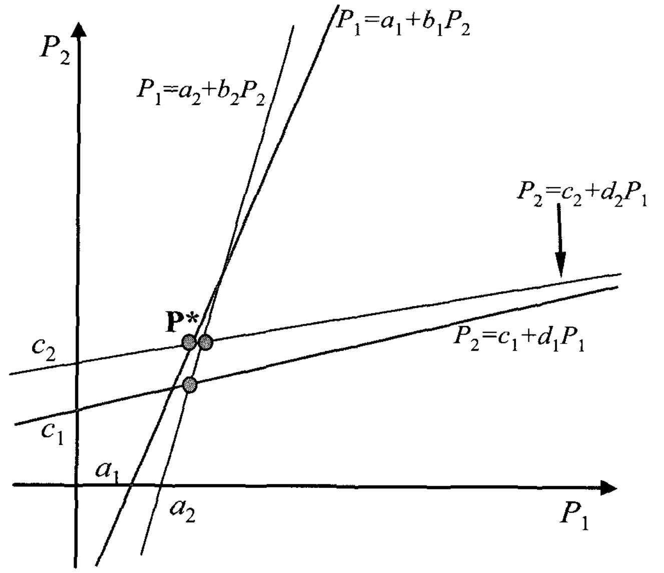


Fig. 4. The case $d_2 < 1/b_1 < 1/b_2$ (the larger slope of area 1 is between the smaller and larger slopes of area 2).

The last part "> 0" follows from (9) and the non-negative properties of the coefficients. □

To illustrate the idea, we give examples of the power control problem with $N = 2$ and $F = 2$, in Figs. 1–4. In the figures, the feasible area of flow type 1 in 2-D (P_1, P_2) space is the shaded area, and the feasible area for flow type 2 is the slanted area. In order for flow type 1 to be feasible, we need to have $1/b_1 > d_1$. Similarly, we require $1/b_2 > d_2$ for flow type 2. Whether the two feasible areas of flow types 1 and 2 overlap depends on the value of the coefficients defined above. Figs. 1 and 2 illustrate why $1/b_2 > d_1$ and $1/b_1 > d_2$ are necessary conditions for both flow types to have solutions simultaneously. When these are satisfied, there are two possible cases shown in Figs. 3 and 4.

The optimum power vector $\boldsymbol{P}^*$ is obtained at the intersection of curves $P_1 = \max_{f=1,\cdots,F_1}(a_f + b_f P_2)$ and $P_2 = \max_{f=1,\cdots,F_2}(c_f + d_f P_1)$. An iterative algorithm can be used to find the minimum power solution in this fixed-point problem. Namely

$$P_1^{(i+1)} = \max_{f=1,\cdots,F_1} (a_f + b_f P_2^{(i)}) \tag{10}$$

$$P_2^{(i+1)} = \max_{f=1,\cdots,F_2} (c_f + d_f P_1^{(i)}). \tag{11}$$

The proof of the convergence is discussed later in Section II-C.

C. General Case of $N > 1$

For the general case of $N > 1$ with F common flow types at each node, the SINR requirements (2) can be written as $\boldsymbol{P} \geq \boldsymbol{I}(\boldsymbol{P})$ if we define the interference function $\boldsymbol{I}(\boldsymbol{P})$ as

$$I_i(\boldsymbol{P}) = \max_{f=1,\cdots,F} \left\{ \beta_f (\boldsymbol{A}^{(f)}\boldsymbol{P})_i + \beta_f \sigma^2 \right\}, \; i = 1, \cdots, N. \tag{12}$$

If a solution exists, the problem is to find the minimum $\boldsymbol{P}$, such that $\boldsymbol{P} \geq \boldsymbol{I}(\boldsymbol{P})$ is satisfied. The iterative algorithm we used in (10) and (11) can be written as $\boldsymbol{P}^{(i+1)} = \boldsymbol{I}(\boldsymbol{P}^{(i)})$.

We use the conclusions from [4] to prove the convergence of the algorithm. Reference [4] defined an interference function $\boldsymbol{I}(\boldsymbol{P})$ to be standard if for all $\boldsymbol{P} > 0 \cdot \boldsymbol{1}$, it satisfied positivity (i.e., $\boldsymbol{I}(\boldsymbol{P}) > 0 \cdot \boldsymbol{1}$), monotonicity (i.e., if $\boldsymbol{P} > \boldsymbol{P}'$, then $\boldsymbol{I}(\boldsymbol{P}) \geq \boldsymbol{I}(\boldsymbol{P}')$), and scalability (i.e., for all $\alpha > 1$, $\alpha \boldsymbol{I}(\boldsymbol{P}) > \boldsymbol{I}(\alpha \boldsymbol{P})$).

It was shown that if there is a fixed point, then it is unique and it is the optimum power vector (component-wise minimum). Furthermore, if $\boldsymbol{P} \geq \boldsymbol{I}(\boldsymbol{P})$ has a solution, then, for any initial power vector $\boldsymbol{P}$, the power control algorithm with standard $\boldsymbol{I}(\boldsymbol{P})$ converges to the optimum power vector $\boldsymbol{P}^*$. Since the proof of convergence was given in [4], we only need to verify here that the interference function defined above is standard. This can be verified in a straightforward way.

Proposition 3: If a solution to $\boldsymbol{P} \geq \boldsymbol{I}(\boldsymbol{P})$ exists, the algorithm $\boldsymbol{P}^{(i+1)} = \boldsymbol{I}(\boldsymbol{P}^{(i)})$ with the interference function defined in (12) converges to the optimum power vector $\boldsymbol{P}^*$. (Appendix B).

This iterative power control converges very fast to the minimum power vector. As shown in [5], the standard power control algorithms converge at geometric rate.

Now, we discuss the existence of the solution for the general case of $N > 1$. Similarly to the $N = 2$ case, we have the following necessary condition from the positivity of the power vector.

Proposition 4: If the uplink power control problem for $N > 1$ with SINR requirements $0 < \beta_1 \leq \beta_2 \leq \cdots \leq \beta_F$ has a solution, then

$$\alpha_{ii}^f < 1 + 1/\beta_f, \; f = 1, \cdots, F, \; i = 1, \cdots, N.$$

Let us assume that these necessary conditions are satisfied. Then we wish to determine under what condition a solution exists. In the N-dimensional space $(P_1, P_2, \cdots, P_N)$, the requirements are not as clearly seen as in the 2-dimensional space

(P_1, P_2). We have not obtained a complete answer to the question of the existence of the solution. However, we can relate the $N > 2$ case with the $N = 2$ case and obtain some insights in the form of necessary or sufficient conditions.

For a necessary condition, we construct a simplified, easier $N = 2$ problem from the original problem by assuming that all other nodes have zero SINR requirement except nodes i and j. Then only 2 inequality sets for P_i and P_j are left in $\boldsymbol{P} \geq \boldsymbol{I}(\boldsymbol{P})$. If $\boldsymbol{P}$ is a solution of the original problem, then $\boldsymbol{Z}(i,j) \cdot \boldsymbol{P}$ ($\boldsymbol{Z}$ is an $N \times N$ zero matrix, except $Z_{ii} = Z_{jj} = 1$) is a solution of this simplified $N = 2$ problem. Therefore, one necessary condition for the original $N > 2$ problem to have a solution is that any simplified $N = 2$ problem, as described above, satisfies $\max_{f=1,\cdots,F} b_f \cdot \max_{f=1,\cdots,F} d_f < 1$.

For a sufficient condition, we construct a more difficult $N = 2$ problem by adding more constraints. Let us partition the N nodes into two distinct sets, one with N_1 nodes $i_1, i_2, \cdots, i_{N_1}$, and the other with N_2 nodes $j_1, j_2, \cdots, j_{N_2}$, $(N = N_1 + N_2)$. Let the nodes in set 1 have the same power P_1 and the nodes in set 2 have the same power P_2. Set 1 has a total of $N_1 F$ flow types and set 2 has a total of $N_2 F$ flow types. Then the flow types in set 1 and set 2 can be re-indexed, and the α factor and coefficients a, b, c, and d can be defined as in the $N = 2$ case. If the constraint $N = 2$ problem has a solution $[P_1\ P_2]^\top$, then the power vector whose $i_1, i_2, \cdots, i_{N1}$ elements are P_1 and whose $j_1, j_2, \cdots, j_{N2}$ elements are P_2 is a solution for the original problem. Therefore, one sufficient condition for the original problem is that any constrained $N = 2$ problem satisfies $\max_{f=1,\cdots,F} b_f \cdot \max_{f=1,\cdots,F} d_f < 1$.

For the general case of $N > 1$ with different flow types, the SINR requirements in (1) should be modified to the following form

$$P_i \geq \beta_{if}\sigma^2 + \beta_{if} P_i(\alpha_{ii}^f - 1) + \beta_{if} \sum_{j \neq i} P_j \alpha_{ij}^f$$
$$i = 1, \cdots, N, \quad f = 1, \cdots, F_i.$$

The interference is separated into two terms, interference from flow types in the same node, and the interference from other nodes. Proposition 3 holds if the interference function is defined by

$$\boldsymbol{I}_i(\boldsymbol{P}) = \max_{f=1,\cdots,F_i} \left\{ \beta_{if} \left(\sigma^2 + P_i(\alpha_{ii}^f - 1) + \sum_{j \neq i} P_j \alpha_{ij}^f \right) \right\}$$
$$\alpha_{ij}^f = \sum_{g=1}^{F_j} \rho_{if,jg}^2, \quad i = 1, \cdots, N. \tag{13}$$

III. DOWNLINK CASE

In the downlink case, the base station transmits to all the flow types at all the nodes simultaneously. Its power level P can be adjusted to satisfy the SINR requirements of all the flow types at all the nodes. We assume that the attenuation factors from the base station to the nodes are the same.

The SINR requirement of flow type f at node i is given by

$$SINR_{i,f} = \frac{P}{\sigma^2 + \sum_{(j,g) \neq (i,f)} P \cdot \rho_{if,jg}^2} \geq \beta_f$$
$$f = 1, \cdots, F, \quad i = 1, \cdots, N.$$

Define the total squared crosscorrelation between flow type f at node i and all flow types at all nodes as $\alpha_{if} = \sum_{j=1}^N \sum_{g=1}^F \rho_{if,jg}^2$. Then the SINR requirements become

$$\sigma^2/P \leq 1 + 1/\beta_f - \alpha_{if}, \ f = 1, \cdots, F, \ i = 1, \cdots, N.$$

Since the power level P is positive, the following conditions must be satisfied in order for the problem to have a solution

$$\max_i \alpha_{if} < 1 + 1/\beta_f, \ f = 1, \cdots, F. \tag{14}$$

Then the minimum power is given by

$$P^* = \frac{\sigma^2}{\min_f(1 + 1/\beta_f - \max_i \alpha_{if})}. \tag{15}$$

Now, we wish to minimize P^* by selecting appropriate sequences; that is

$$\max_S \left(\min_f (1 + 1/\beta_f - \max_i \alpha_{if}) \right) \tag{16}$$
$$\alpha_{if} = \sum_{j=1}^N \sum_{g=1}^F (\boldsymbol{s}_{if}^\top \cdot \boldsymbol{s}_{jg})^2.$$

Here, $\boldsymbol{S} = [\boldsymbol{s}_{11}, \cdots, \boldsymbol{s}_{1F}, \cdots, \boldsymbol{s}_{N1}, \cdots, \boldsymbol{s}_{NF}]$ is the matrix that consists of NF column sequence vectors. The optimal sequences to minimize the power are now derived from a max-min problem. We will first discuss this problem for the special case of $F = 1$, then for the special case of $N = 1$, and finally for the general case of $N > 1$ and $F > 1$.

A. Special Case of $F = 1$

For $F = 1$, problem (16) becomes $\min_S \max_i \sum_{j=1}^N \rho_{ij}^2$. For $N \leq L$, obviously $\max_i \sum_{j=1}^N \rho_{ij}^2 \geq 1$ holds, and the minimum power is obtained by orthogonal sequences. For $N > L$, we have

$$\max_i \sum_{j=1}^N \rho_{ij}^2 \geq \frac{1}{N} \sum_{i=1}^N \sum_{j=1}^N \rho_{ij}^2 \geq \frac{1}{N} \cdot \frac{N^2}{L} = \frac{N}{L}.$$

The first inequality holds because the maximum value is always greater than or equal to the average, and the second is a consequence of the Welch bound of total squared crosscorrelation [13], [14]. The WBE sequences achieve both equalities.

Proposition 5: For $N > L$, the optimal sequences that minimize the power in a downlink CDMA system with SINR requirement β are the WBE sequences and the minimum power assignment is $P^* = \sigma^2/(1 + 1/\beta - N/L)$. For $N \leq L$, the optimal sequences for this problem are the orthogonal sequences with $P^* = \sigma^2\beta$.

For the $N > L$ case, $0 < P^* < \infty$ implies that $N/L < 1 + 1/\beta$. This is the maximum number of users per degree of freedom the system can have, provided the SINR requirements are satisfied. It is equal to the user capacity for the downlink CDMA system that uses matched filter receivers. Notice that this result is the same as in the uplink case.

B. Special Case of $N = 1$

For $N = 1$, both the downlink and the uplink are simple point-to-point links. We have $\alpha_f = \sum_g \rho_{fg}^2$, and the problem (16) becomes $\max_S \min_f (1 + 1/\beta_f - \alpha_f)$.

If $F \leq L$, we have $\alpha_f \geq 1$, with orthogonal sequences achieve the equality. Hence

$$\min_f (1 + 1/\beta_f - \alpha_f) \leq \min_f (1/\beta_f) = \beta_F$$

and the minimum power is given by $P^* = \beta_F \sigma^2$.

If $F > L$, it is difficult to get an exact analytic solution. Here, we find some properties that the optimal sequences must satisfy. If F codes are available with $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_F$, then the question is how to assign these F codes to the F flow types so that $\min_f (1 + 1/\beta_f - \alpha_{i(f)})$ is maximized. Let notation $i(f)$ represent the code that is assigned to flow type f. We find that the code with the least correlation (i.e., the code with minimum α) should be assigned to the most demanding flow type (i.e., the flow type with maximum β) and the code with the highest correlation (i.e., the code with maximum α) to the least demanding flow type (i.e., the flow type with minimum β). This assignment is always better than or at least as good as other assignments. Therefore, when we search for the optimal sequences, we can limit the search to the code sets that satisfy $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_F$ without missing the optimum power.

Proposition 6: There is at least one optimal set of sequences which maximize $\min_f (1 + 1/\beta_f - \alpha_f)$ $(0 < \beta_1 \leq \beta_2 \leq \cdots \leq \beta_F)$ and that has the following property (Appendix C)

$$\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_F.$$

Using the definition of α_i, we can transform the condition for α into conditions that ρ_{ij} must satisfy when F is small. For $F = 3$, this property implies $\rho_{12}^2 \geq \rho_{13}^2 \geq \rho_{23}^2$. For $F = 4$, it implies $\rho_{12}^2 - \rho_{34}^2 \geq \rho_{13}^2 - \rho_{24}^2 \geq |\rho_{23}^2 - \rho_{14}^2| \geq 0$. For $F \geq 5$, we were not able to obtain similar simple analytic conditions.

C. General Case of $N > 1$ and $F > 1$

For the general case of $N > 1$ and $F > 1$, similarly to Proposition 6, we have the following property of the optimal sequences.

Proposition 7: *There is at least one optimal set of sequences* which maximize $\min_f (1 + 1/\beta_f - \max_i \alpha_{if})$ $(0 < \beta_1 \leq \beta_2 \leq \cdots \leq \beta_F)$ and that have the following property (Appendix C)

$$\max_i \alpha_{i1} \geq \max_i \alpha_{i2} \geq \cdots \geq \max_i \alpha_{iF}.$$

From this conclusion, we know that at least one optimal set of sequences will assign the N sequences with largest value of α to the flow type 1 (any permutation is fine within these N sequences), the next N sequences with next largest value of α to flow type 2, and so on.

IV. EFFECT OF THE SINGLE POWER LEVEL CONSTRAINT

In this paper, we assume that all flow types at each node have the same power level. This constraint simplifies the transmitter structure; on the other hand, it may degrade the system performance. In this section, we examine the performance degradation by comparing the performance of the system with this same power level constraint (constrained problem) to that of the system without this constraint (unconstrained problem).

The unconstrained problem is a synchronous CDMA system with one base station and NF nodes. The NF separate power levels satisfies

$$P_{if} \geq \beta_f \sigma^2 + \beta_f \sum_{j=1}^{N} \sum_{g=1}^{F} (P_{jg} \rho_{if,jg}^2) - \beta_f P_{if}$$
$$f = 1, \cdots, F, \quad i = 1, \cdots, N.$$

For the uplink if power vector $\boldsymbol{P} = [P_1, P_2, \cdots, P_N]^T$ is a solution of the constrained problem, then the power vector $\boldsymbol{P} \otimes \mathbf{1}_{1\times F}$ is a solution of the unconstrained problem. (Notation $\boldsymbol{P} \otimes \mathbf{1}_{1\times F}$ is the Kronecker product, which denotes the vector whose i-th element is replaced by $P_i \cdot [1, \cdots, 1]_{1\times F}^T$). Therefore, if solutions exist for the constrained problem, then solutions must also exist for the unconstrained problem. The contrary is not true.

If solutions exist to both problems, assume that the optimum power vectors of the constrained and unconstrained uplink problem are $\boldsymbol{P}^*$ and $\tilde{\boldsymbol{P}}^*$, respectively. Since $\boldsymbol{P}^* \otimes \mathbf{1}_{1\times F}$ is a solution of the unconstrained problem, from the property of the optimum power vector, we have $\boldsymbol{P}^* \otimes \mathbf{1}_{1\times F} \geq \tilde{\boldsymbol{P}}^*$. Therefore, minimum total power of the constrained problem is always larger than or equal to that of the unconstrained problem.

To see how large the performance difference is between the constraint and unconstrained system, we give one analytical example and one numerical example below.

Example of $N = 1$ and $F = 2$: One user has two flow types *with SINR requirements $\beta_1 \leq \beta_2$, and the cross-correlation between the two sequences is ρ. For the constrained problem, the* sufficient and necessary conditions to have a solution can be obtained from (14)

$$\beta_1 < \frac{1}{\rho^2} \text{ and } \beta_2 < \frac{1}{\rho^2} \tag{17}$$

and the minimum total power can be obtained from (15)

$$P_{\text{total}} = \frac{2\sigma^2 \beta_2}{1 - \rho^2 \beta_2}.$$

For the unconstrained problem, the condition to have a solution can be obtained from the Perron-Frobenius eigenvalue

$$\beta_1 \beta_2 < \frac{1}{\rho^4} \tag{18}$$

and the minimum total power is given by

$$\tilde{P}_{\text{total}} = \frac{\sigma^2}{1 - \rho^4 \beta_1 \beta_2} (\beta_1 + \beta_2 + \rho^2 \beta_1^2 + \rho^2 \beta_2^2).$$

Table 1. Comparison of the power control problem with and without the constraint of single power level ($N = 3,\ F = 2,\ L = 20$).

Number of times in 100 simulations that	$\beta_1 = 1.0$ $\beta_2 = 1.5$	$\beta_1 = 2.0$ $\beta_2 = 3.0$
Power ratio if in $[1.0, 1.2)$	11	4
Power ratio if in $[1.2, 1.4)$	69	23
Power ratio if in $[1.4, 2.0)$	16	23
Power ratio if in $[2.0, 10)$	2	21
Power ratio if in $[10, \infty)$	2	12
Only unconstrained problem has solutions	0	13
Neither problem has solutions	0	4

Comparing (17) and (18), it is obvious that (18) defines a larger region for (β_1, β_2), i.e., it is easier for the unconstrained problem to have a solution.

From $\beta_1 \le \beta_2$, we always have $P_{\text{total}} \ge \tilde{P}_{\text{total}}$, i.e., the constrained problem needs larger total power.

Numerical example: Let $N = 3$, $F = 2$, and $L = 20$. We generate NF random sequences from random variable V_{ij} (+1 or −1 with half probability) as $s_i = [V_{i1}, V_{i2}, \cdots, V_{iL}]$, $i = 1, \cdots, NF$. Then we solve iteratively the constrained and unconstrained power control problem. Each time we run the program, a new set of sequences is generated, and the corresponding power control problems are solved and the results for both problems are compared. We define the power ratio as the ratio of the total power needed for the constrained problem to that for the unconstrained problem.

Basically, the performance difference between the constrained and unconstrained problem depends on the SINR requirements and the degree of sequence correlation. For SINR requirements $\beta_1 = 1.0$ and $\beta_2 = 1.5$, we run the program 100 times. Both problems have solutions every time, and the power ratio is between 1.0 and 1.4 most of the time (in about 80% of the cases). Please refer to Table 1 for the details. For stricter SINR requirements, e.g., $\beta_1 = 2.0$ and $\beta_2 = 3.0$, we also run the program 100 times. There are 4 times for which no solution exist for either of the two problems. There are 13 times that only the unconstrained problem has a solution. For the remaining 83 times, both problems have a solution. This again confirms that it is easier for the unconstrained problem to have a solution. When both of the problems have a solution, the power ratio varies more widely, with an average value close to 2.

V. SUMMARY

We studied the problem of a power-controlled CDMA system with N nodes and F flow types with the constraint that each node uses the same power level for all flows that it multiplexes. We revisited the single flow case, and rederived the same results as in [6] about the optimum sequence and user capacity but with a different method. For the uplink problem with $N = 2$, we found and proved the necessary and sufficient conditions to have a solution. For the general $N > 1$ uplink problem, we provided an iterative algorithm to find the optimal solution and derived conditions that the optimum sequence must satisfy. For the downlink case with $F > 1$, some properties of the optimal sequences were also derived. Finally, the single power level constraint, which simplifies the transmitter structure, causes a performance degradation that was assessed in some examples.

APPENDICES

A. Proof of (3)

Matrix $\boldsymbol{A}$ is real and symmetric, so it can be diagonalized to $\boldsymbol{A} = \boldsymbol{U}\Lambda_A\boldsymbol{U}^\top$. Here, $\boldsymbol{U}$ is a unitary matrix (i.e., $\boldsymbol{U}\boldsymbol{U}^\top = \boldsymbol{U}^\top\boldsymbol{U} = \boldsymbol{I}$), and Λ_A is a diagonal matrix, whose diagonal elements are equal to the real eigenvalues of $\boldsymbol{A}$. Define $\boldsymbol{G} = \boldsymbol{I} - \beta\boldsymbol{A}$. Then, $\boldsymbol{G}$ can be diagonalized to $\boldsymbol{G} = \boldsymbol{U}\Lambda\boldsymbol{U}^\top$, with $\Lambda = \boldsymbol{I} - \beta\Lambda_A$. From the feasibility assumption on β, (i.e., $\beta < 1/\rho_A$), the diagonal elements of Λ (eigenvalues of $\boldsymbol{G}$) are all positive. So $\boldsymbol{G}$ and $\boldsymbol{G}^{-1}$ are positive definite, and they can be written as $\boldsymbol{G} = \boldsymbol{U}\Lambda^{\frac{1}{2}}\Lambda^{\frac{1}{2}}\boldsymbol{U}^\top$ and $\boldsymbol{G}^{-1} = \boldsymbol{U}\Lambda^{-\frac{1}{2}}\Lambda^{-\frac{1}{2}}\boldsymbol{U}^\top$. Then, by the Cauchy-Schwartz inequality, we have

$$
\begin{aligned}
&(\mathbf{1}^\top\boldsymbol{G}\mathbf{1})(\mathbf{1}^\top\boldsymbol{G}^{-1}\mathbf{1}) \\
&= \left[(\Lambda^{\frac{1}{2}}\boldsymbol{U}^\top\mathbf{1})^\top(\Lambda^{\frac{1}{2}}\boldsymbol{U}^\top\mathbf{1})\right]\cdot\left[(\Lambda^{-\frac{1}{2}}\boldsymbol{U}^\top\mathbf{1})^\top(\Lambda^{-\frac{1}{2}}\boldsymbol{U}^\top\mathbf{1})\right] \\
&\ge \left|(\Lambda^{\frac{1}{2}}\boldsymbol{U}^\top\mathbf{1})^\top\cdot(\Lambda^{-\frac{1}{2}}\boldsymbol{U}^\top\mathbf{1})\right|^2 \\
&= \left|\mathbf{1}^\top\mathbf{1}\right|^2 \\
&= N^2.
\end{aligned}
$$

The condition to have equality is $\Lambda^{\frac{1}{2}}\boldsymbol{U}^\top\mathbf{1} = \gamma\Lambda^{-\frac{1}{2}}\boldsymbol{U}^\top\mathbf{1}$, for some constant γ. This means that $\boldsymbol{G} = \gamma\mathbf{1}$ and $\boldsymbol{A}\mathbf{1} = [(1-\gamma)/\beta]\mathbf{1}$; i.e., the row summation of matrix $\boldsymbol{A}$ is a constant. Then

$$
\left(\sum_{j=1}^{N}\rho_{ij}^2\right) - 1 = \frac{1-\gamma}{\beta},\ i = 1, \cdots, N, \ \text{ for some constant } \gamma.
$$

B. Proof of Proposition 3

The positivity property follows directly from the nonnegativeness of matrix $\boldsymbol{A}^{(f)}$.

For the monotonicity property, assume $\boldsymbol{P} \ge \boldsymbol{P}'$, then we have

$$
\begin{aligned}
\boldsymbol{I}(\boldsymbol{P}) &= \max_f\left\{\beta_f\boldsymbol{A}^{(f)}\cdot\boldsymbol{P} + \beta_f\sigma^2\mathbf{1}\right\} \\
&\ge \max_f\left\{\beta_f\boldsymbol{A}^{(f)}\cdot\boldsymbol{P}' + \beta_f\sigma^2\mathbf{1}\right\} \\
&= \boldsymbol{I}(\boldsymbol{P}').
\end{aligned}
$$

For scalability, assume $\alpha > 1$, then

$$
\begin{aligned}
\alpha\boldsymbol{I}(\boldsymbol{P}) &= \max_f\left\{\alpha\beta_f\boldsymbol{A}^{(f)}\cdot\boldsymbol{P} + \alpha\beta_f\sigma^2\mathbf{1}\right\} \\
&> \max_f\left\{\beta_f\boldsymbol{A}^{(f)}\cdot(\alpha\boldsymbol{P}) + \beta_f\sigma^2\mathbf{1}\right\} \\
&= \boldsymbol{I}(\alpha\boldsymbol{P}).
\end{aligned}
$$

Therefore, $\boldsymbol{I}(\boldsymbol{P})$ defined in (12) is standard. Using the main theorem from [4], the iterative algorithm $\boldsymbol{P}^{(i+1)} = \boldsymbol{I}(\boldsymbol{P}^{(i)})$ converges to the optimum power vector $\boldsymbol{P}^*$.

The proof is similar to the $\boldsymbol{I}(\boldsymbol{P})$ defined in (13).

C. Proof of Propositions 5, 6, and 7

We use notation $(i, j, \cdots)$ to imply the assignment that code i is assigned to flow type β_1, code j is assigned to flow type β_2, and so on. We want to select the assignment to maximize $f(i,j,\cdots) = \min(1/\beta_1 - \alpha_i, 1/\beta_2 - \alpha_j, \cdots)$. We will use the following lemma.

Lemma: if $0 < \beta_i \leq \beta_j$ and $\alpha_i \geq \alpha_j$, then

$$\min(1/\beta_i - \alpha_i, 1/\beta_j - \alpha_j) \geq \min(1/\beta_i - \alpha_j, 1/\beta_j - \alpha_i).$$

(Proved immediately by using $1/\beta_i - \alpha_i \geq 1/\beta_j - \alpha_i$, $1/\beta_j - \alpha_j \geq 1/\beta_j - \alpha_i$, and $1/\beta_i - \alpha_j \geq 1/\beta_j - \alpha_i$).

We start from $F = 2$. Suppose there are 2 codes with $\alpha_1 \geq \alpha_2$ and 2 flow types with $0 < \beta_1 \leq \beta_2$. We want to maximize $f(i,j) = \min(1/\beta_1 - \alpha_i, 1/\beta_2 - \alpha_j)$. Using the lemma, we have

$$\min(1/\beta_1 - \alpha_1, 1/\beta_2 - \alpha_2) \geq \min(1/\beta_1 - \alpha_2, 1/\beta_2 - \alpha_1).$$

Therefore, for $F = 2$, assigning codes to flow types (indexed in an increasing order of β) in the order of decreasing crosscorrelation α is better than, or equal to, any other assignment.

Now consider $F = 3$ with 3 codes $\alpha_1 \geq \alpha_2 \geq \alpha_3$ and 3 flow types with SINR requirements $0 < \beta_1 \leq \beta_2 \leq \beta_3$. Consequently, using the conclusion of $F = 2$, we have

$$\begin{aligned} f(1,2,3) &= \min(1/\beta_1 - \alpha_1, 1/\beta_2 - \alpha_2, 1/\beta_3 - \alpha_3) \\ &= \min(1/\beta_1 - \alpha_1, \min(1/\beta_2 - \alpha_2, 1/\beta_3 - \alpha_3)) \\ &\geq \min(1/\beta_1 - \alpha_1, \min(1/\beta_2 - \alpha_3, 1/\beta_3 - \alpha_2)) \\ &= f(1,3,2). \end{aligned}$$

Similarly, we show that $f(1,2,3) \geq f(2,1,3) \geq f(2,3,1)$ and $f(1,3,2) \geq f(3,1,2) \geq f(3,2,1)$.

Therefore, $f(1,2,3)$ is the maximal assignment among all 6 possible ones, i.e., for $F = 3$, assigning codes to flow types (indexed in an increasing order of β) in the order of decreasing crosscorrelation α is better than, or equal to, other possible assignments.

Then assume that for $F = n$ the proposition is true, that is

$$f(1,2,\cdots,n) \geq f(i_1, i_2, \cdots, i_n)$$

is true for any permutation $(i_1, i_2, \cdots, i_n)$.

Using the same procedure as above, we prove that the proposition is true for $F = n + 1$.

Hence, the induction process implies that for any F flow types with $0 < \beta_1 \leq \beta_2 \leq \cdots \leq \beta_F$, there is at least one optimal set of codes that have the property $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_F$.

We can use the same induction process to prove Proposition 7, by changing α_f to $\max_i \alpha_{if}$.

REFERENCES

[1] J. Zander, "Distributed cochannel interference control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, no. 3, pp. 305–311, Aug. 1992.

[2] J. Zander, "Performance of optimum transmitter power control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, no. 1, pp. 57–62, Feb. 1992.

[3] S. V. Hanly, "An algorithm for combined cell-site selection and power control to maximize cellular spread spectrum capacity," *IEEE J. Select. Areas Commun.*, vol. 13, no. 7, pp. 1332–1340, Sept. 1995.

[4] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Select. Areas Commun.*, vol. 13, no. 7, pp. 638–644, Sept. 1995.

[5] C. Y. Huang and R. Yates, "Rate of convergence for minimum power assignment algorithms in cellular radio systems," *Baltzer/ACM Wireless Networks*, vol. 4, no. 3, pp. 223–231, Apr. 1998.

[6] P. Viswanath, V. Anantharam, and D. N. C. Tse, "Optimal sequence, power control, and user capacity of synchronous CDMA systems with linear MMSE multiuser receivers," *IEEE Trans. Inform. Theory*, vol. 45, no. 6, pp. 1968–1983, Sept. 1999.

[7] S. Ulukus and R. D. Yates, "User capacity of asynchronous CDMA systems with matched filter receivers and optimum signature sequences," *IEEE Trans. Inform. Theory*, vol. 50, no. 5, pp. 903–909, May 2004.

[8] S. V. Hanly and D. N. C. Tse, "Power control and capacity of spread-spectrum wireless networks," *Automatica*, vol. 35, no. 12, pp. 1987–2012, Dec. 1999.

[9] H. Y. Kwan and T. M. Lok, "Power control scheme with signature sequence adaptation for DS-CDMA systems," in *Proc. WCNC 2000*, vol. 1, Sept. 2000, pp. 333–337.

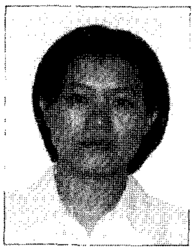
[10] L. Gao and T. F. Wong, "Power control and spreading sequence allocation in a CDMA forward link," *IEEE Trans. Inform. Theory*, vol. 50, no. 1, pp. 105–124, Jan. 2004.

[11] S. Verdu, *Multiuser Detection*, Cambridge Univ. Press, 1998.

[12] E. Seneta, *Non-Negative Matrices and Markov Chains*, Springer Verlag, 1981.

[13] J. L. Massey and T. Mittelholzer, "Welch's bound and sequence sets for code-division multiple-access systems," *Sequences II: Methods in Communication, Security and computer Science*, R. Capocelli, A. De Santis and U. Vaccaro, Eds., New York: Springer-Verlag, 1991.

[14] L. Welch, "Lower bounds on the maximum cross correlation of signals," *IEEE Trans. Inform. Theory*, vol. 20, no. 3, pp. 397–399, May 1974.



Yun Li received the B.S. and M.S. degrees in physics, from Tsinghua University, Beijing, China, in 1993 and 1996, respectively, and the M.S. and Ph.D. degrees in electrical engineering from University of Maryland, College Park, Maryland, in 1998 and 2004, respectively. She is now a postdoctoral fellow at Colorado State University, Fort Collins, Colorado. Her research interests include wireless communications, ad hoc networks, and sensor management.

Anthony Ephremides received his B.S. degree from the National Technical University of Athens (1967), and M.S. (1969) and Ph.D. (1971) degrees from Princeton University, all in Electrical Engineering. He has been at the University of Maryland since 1971, and currently holds a joint appointment as Professor in the Electrical Engineering Department and in the Institute of Systems Research (ISR) of which he is a founding member. He is co-founder of the NASA Center for Commercial Development of Space on Hybrid and Satellite Communications Networks established in 1991 at Maryland as an off-shoot of the ISR. He served as Co-Director of that Center from 1991 to 1994.

He was a visiting professor in 1978 at the National Technical University in Athena's, Greece, and in 1979 at the EECS Department of the University of California, Berkeley, and at INRIA, France. During 1985–1986 he was on leave at MIT and ETH in Zurich, Switzerland. He was the General Chairman of the 1986 IEEE Conference on Decision and Control in Athens, Greece and of the 1991 IEEE International Symposium on Information Theory in Budapest, Hungary. He also organized two workshops on Information theory in 1984 (Hot Springs, VA) and in 1999 (Metsovo, Greece). He was the Technical Program Co-Chair of the IEEE INFOCOM in New York City in 1999 and of the IEEE International Symposium on Information theory in Sorrento, Italy in 2000. He has also been the Director of the Fairchild Scholars and Doctoral Fellows Program, an academic and research partnership program in Satellite Communications between Fairchild Industries and the University of Maryland. He won the IEEE Donald E. Fink Prize Paper Award (1992) and he was the first recipient of the Sigmobile Award of the ACM (Association of Computer Machinery) for contributions to wireless communications in 1997. He has been the President of the Information Theory Society of the IEEE (1987) and has served on its Board of Governors

almost continuously from 1981 until the present. He was elected to the Board of Directors of the IEEE in 1989 and 1990.

Dr. Ephremides has authored or co-authored over 100 technical journal papers and 300 technical conference presentations. He has also contributed chapters to several books and edited numerous special issues of scientific journals. He has also won awards from the Maryland Office of Technology Liaison for the commercialization of products and ideas stemming from his research. He has served on the Editorial Boards of the IEEE Transactions on Automatic Control, IEEE Transactions on Information theory, the Journal of Wireless Networks, and the International Journal of Satellite Communications.

He has been the Dissertation Supervisor of over twenty Ph.D. students who now hold prominent positions in academia, industry, and research labs. He is the founder and President of Pontos, Inc., a Maryland company that provides technical consulting services, since 1980.

Dr. Ephremides' interests are in the areas of communication theory, communication systems and networks, queuing systems, signal processing, and satellite communications. His research has been continuously supported since 1971 by NSF, NASA, ONR, ARL, NRL, NSA, and Industry. In 2003, Dr. Ephremides was awarded the Cynthia Kim Eminent Professorship in Information Technology at the University of Maryland.