Performance Analysis of Space-Time Block Coded Cooperative Wireless Transmission in Rayleigh Fading Channels

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Abstract: **This paper studies theoretically the bit error rate (BER) performance of cooperative transmission using space-time block code (STBC) in a fully distributed manner. Specifically, we first propose a STBC-based cooperative signaling structure to make the cooperation of three single-antenna terminals possible. Then, we derive the closed-form BER expressions for both cooperation and noncooperation schemes under flat Rayleigh fading channel plus additive white Gaussian noise (AWGN). The validity of these expressions is verified by Monte-Carlo simulations. A variety of numerical and simulation results reveal that the cooperative transmission achieves higher diversity gain and better performance than the direct transmission for the same total transmit power.**

Index Terms: **Additive white Gaussian noise (AWGN), cooperative transmission, Rayleigh fading, space-time block code (STBC).**

I. INTRODUCTION

Signal fading due to multi-path propagation is a serious problem in wireless communications. Using a diversified signal in which information related to the same data appears in multiple time instances, frequencies, or antennas that are independently faded can reduce considerably this effect of the channel [1]. Among well-known diversity techniques, the spatial diversity has received a great deal of attention in recent years because of the feasibility of deploying multiple antennas at both transmitter and receiver [2]. However, when wireless mobiles may not be able to support multiple antennas due to size and power limitation, or other constraints [3], the spatial diversity is not exploited. To overcome this restriction, a new technique, called cooperative transmission, was born which allows single-antenna terminals to gain some benefits of transmit diversity. The main idea is that in a multiuser network, two or more users share their information and transmit jointly as a virtual antenna array. This enables them to obtain higher diversity than they could have individually. The way the users share information is by tuning into each other's transmitted signals and by processing information that they overhear. Since the inter-user channel is noisy and faded, this overheard information is not perfect. Hence, one has to carefully study the possible signaling strategies that can exploit the benefits of cooperative communications at most. There are three basic cooperative signaling methods [3] where amplify-and-forward strategy is the simplest and applicable in many wireless networks such as wireless sensor network, mobile communications network, ad-hoc network, relay network, etc. Therefore, it is also exploited in this paper.

The space-time block coding can achieve the full transmit diversity specified by the number of the co-located transmit antennas while allowing a very simple maximum-likelihood (ML) decoding algorithm, based only on linear processing of the received signals [4]. As a result, it is considered as a powerful coding technique to mitigate fading in wireless channels and improve robustness to interference.

In this paper, we propose an appropriate signaling structure to exploit space-time block code (STBC) in a fully distributed fashion. The key idea is to collect the antennas belonging to different users (each user has only one antenna) to create a virtual antenna array and then to build signal sequences sent out on different antennas that are similar to those transmitted on co-located antennas. We call STBCs in this case, "fully distributed" STBCs. The term "fully distributed STBC" means that its structure of the signals transmitted from the single-antennas of users and detection technique at the receiver are identical to those of the conventional STBC for co-located antenna array. Therefore, the proposed cooperative transmission scheme can exploit the ML decoding at the receiver to reduce significantly the implementation complexity. In fact, the concept of space-time block coded cooperative transmission was mentioned in [5] and [6] but only "partially distributed" STBCs. This is because the number of cooperative users is larger than the number of co-located antennas designed for the STBC. Specifically, [5] and [6] use Alamouti code [7] to construct the cooperative transmission scheme for two sensors [5] or two relays [6] in assisting the data transmission for a source node to the destination. Moreover, [5] only mentioned the signal processing from two intermediate nodes to the remote receiver without paying any attention to error transmission caused by the link between the sensor and the intermediate nodes which is faded and noisy in real and generic wireless networks. Such a case was solved by [6] in part but it did not show explicitly the way of detecting the data at the receiver and a closed-form bit error rate (BER) expression. Consequently, the derivation of closed-form BER expression for "fully distributed" STBCs in this paper is considered as the first work that takes into consideration the effects of all channels through which the original data can reach the destination, and exposes a tighter cooperation among the source terminal and two relays. Additionally, we investigate all possible cases of channels between two users to see whether the cooperation is beneficial or not. The numerical results show that the cooperative transmission obtains higher diversity and better performance than noncooperative counterpart for the same total transmit power.

The rest of the paper is organized as follows. Section II presents a cooperative signaling structure as well as the closed-

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				Relay 1			
Source				$-z_{1,2}$	$z_{1,1}$	$-z_{1,4}$	$z_{1,3}$
x_1	$-x_2$	$-x_3$	$-x_4$	Relay 2			
				$-z_{2,3}$	$z_{2,4}$	$z_{2,1}$	$-z_{2,2}$
Time slot 1				Time slot 2			

Fig. 1. Proposed cooperative signaling structure.

form expression derivation for the error probability. Then, the numerical and simulation results that compare the performance of the proposed cooperative transmission with direct transmission are exposed in Section III. Finally, the paper is closed in Section IV with a conclusion.

II. PROPOSED COOPERATIVE SIGNALING STRUCTURE

Consider a cooperative transmission in a generic wireless network where the information is transmitted from a source mobile S to a destination mobile D with the assistance of two relay terminals. All terminals equipped with single-antenna transceivers and sharing the same frequency band are under investigation. In addition, each terminal cannot transmit and receive signal at the same time to mitigate the implementation complexity since the considerable attenuation over the wireless channel and insufficient electrical isolation between the transmit and receive circuitry make a terminal's transmitted signal dominate the signals of other terminals at its receiver input. In order to obtain this, time division multiplexing (TDM) is used for channel access and the signal format of each entity is shown in the Fig. 1.

For simplicity of exposition, we use complex baseband-equivalent models to express all the signals. In addition, the BPSK modulation technique is investigated since it is necessary for the small-size devices requiring the simple signal processing and high performance, for example, wireless sensors.

In the cooperation process, each relay doesn't perform hard detection on the signal of its source as in [8] but rather, it simply amplifies the received signal under a given transmit power constraint and forwards the resultant signal to the destination. This not only reduces the processing time and power consumption at each relay but also avoids the wrong decisions that can adversely affect the overall performance at the destination.

The cooperation process proceeds as follows. During its own time slot (represented as time slot 1), the source terminal generates four BPSK-modulated symbols x_1, x_2, x_3, x_4 and broadcasts them according to the structure x_1, $-x_2$, $-x_3$, $-x_4$ which will be received by the destination and two relays. Therefore, the sequence of the signals received at the destination and two relays has the common forms as follows

$$y_{Si,1} = \varepsilon_0 \alpha_{Si} x_1 + n_{Si,1} \quad (1)$$

$$y_{Si,2} = -\varepsilon_0 \alpha_{Si} x_2 + n_{Si,2} \quad (2)$$

$$y_{Si,3} = -\varepsilon_0 \alpha_{Si} x_3 + n_{Si,3} \quad (3)$$

$$y_{Si,4} = -\varepsilon_0 \alpha_{Si} x_4 + n_{Si,4} \quad (4)$$

where

- $y_{Si,j}$ is a signal received at the terminal i from the source S during the j-th symbol duration ($i = 1, 2, D$ mean relay 1, relay 2, and the destination D, respectively) in the first time slot; $j = 1, 2, 3, 4$.
- α_{Si} is a fading realization associated with each link from the source S to the target i which is assumed to be an independent zero-mean complex Gaussian random variable (ZMCGRV) with variance λ_{Si}^2.
- $n_{Si,j}$ is a zero-mean additive noise sample of variance σ_{Si}^2 at terminal i in the j-th symbol interval.
- $\varepsilon_0 = \sqrt{P_S}$ is an amplification factor at the source where P_S is average source power.

In (1)–(4) and those following, we assume that the channels among users (inter-user channels) and between the users and the destination (user-destination channels) are independent of each other. Moreover, all channels experience frequency flat fading and are quasi-static, i.e., they are constant during 4-symbol period and change independently to the next.

In the second time slot, the relays are to simply amplify the signals received from the source and forward them simultaneously to the destination. These amplified signals obey the format in Fig. 1. Specifically, they are given by

$$z_{i,j} = \varepsilon_i \frac{\alpha_{Si}^*}{|\alpha_{Si}|} y_{Si,j}. \quad (5)$$

Here, $i = 1, 2$ represent relays 1 and 2, correspondingly; the coefficient $\alpha_{Si}^*/|\alpha_{Si}|$ is used to correct the phase distortion caused by the link between S and i (it is implicitly assumed that the channel state information is estimated perfectly at the receivers and this information is unknown at the transmit sides); $(\cdot)^*$ is the complex conjugate operator; ε_i is the scaling factor at relay i which is chosen as

$$\varepsilon_i = \sqrt{\frac{P_i}{E[|y_{Si,j}|^2]}} = \sqrt{\frac{P_i}{P_S \lambda_{Si}^2 + \sigma_{Si}^2}} \quad (6)$$

where P_i denotes the average power of the relay i and $E[\cdot]$ represents the expectation operator. Selection of ε_i as in (6) ensures that an average output power is maintained [9].

The signal processing at terminal D must be delayed until the relays have transmitted signal sequences $z_{i,j}$. In order to avoid inter-symbol interference at the destination terminal, we assume that the time delay between the two propagation paths containing a relay is negligible. After collecting all signals from the relays, the D is to simply add the relays' received signals synchronously together with those from the source which are delayed a time-slot duration on the symbol-by-symbol basis. For further simplification, we drop the time indices. Then, the signal sequence of 4 consecutive symbols received at the D are given explicitly by

$$r_1 = \underbrace{(-\alpha_{1D} z_{1,2} - \alpha_{2D} z_{2,3} + n_{D,1})}_{\text{from relays 1 and 2 in the 2nd time slot}} + \underbrace{(\varepsilon_0 \alpha_{SD} x_1 + n_{SD,1})}_{\text{from the source in the 1st time slot}}$$

$$\begin{aligned}
&= \left(\begin{array}{l} -\alpha_{1D}\left[\varepsilon_1 \frac{\alpha_{S1}^*}{|\alpha_{S1}|}(-\alpha_{S1}\varepsilon_0 x_2 + n_{S1,2})\right] \\ -\alpha_{2D}\left[\varepsilon_2 \frac{\alpha_{S2}^*}{|\alpha_{S2}|}(-\alpha_{S2}\varepsilon_0 x_3 + n_{S2,3})\right] \\ +n_{D,1} \end{array} \right) \\
&\quad +(\varepsilon_0 \alpha_{SD} x_1 + n_{SD,1}) \\
&= \varepsilon_0 \alpha_{SD} x_1 + \varepsilon_0 \varepsilon_1 \alpha_{1D} |\alpha_{S1}| x_2 \\
&\quad +\varepsilon_0 \varepsilon_2 \alpha_{2D} |\alpha_{S2}| x_3 - \alpha_{1D}\varepsilon_1 \frac{\alpha_{S1}^*}{|\alpha_{S1}|} n_{S1,2} \\
&\quad -\alpha_{2D}\varepsilon_2 \frac{\alpha_{S2}^*}{|\alpha_{S2}|} n_{S2,3} + n_{SD,1} + n_{D,1} \qquad (7)
\end{aligned}$$

$$\begin{aligned}
r_2 &= (\alpha_{1D} z_{1,1} + \alpha_{2D} z_{2,4} + n_{D,2}) \\
&\quad +(-\varepsilon_0 \alpha_{SD} x_2 + n_{SD,2}) \\
&= \left(\begin{array}{l} \alpha_{1D}\left[\varepsilon_1 \frac{\alpha_{S1}^*}{|\alpha_{S1}|}(\alpha_{S1}\varepsilon_0 x_1 + n_{S1,1})\right] \\ +\alpha_{2D}\left[\varepsilon_2 \frac{\alpha_{S2}^*}{|\alpha_{S2}|}(-\alpha_{S2}\varepsilon_0 x_4 + n_{S2,4})\right] \\ +n_{D,2} \end{array} \right) \\
&\quad +(-\varepsilon_0 \alpha_{SD} x_2 + n_{SD,2}) \\
&= -\varepsilon_0 \alpha_{SD} x_2 + \varepsilon_0 \varepsilon_1 \alpha_{1D} |\alpha_{S1}| x_1 \\
&\quad -\varepsilon_0 \varepsilon_2 \alpha_{2D} |\alpha_{S2}| x_4 + \alpha_{1D}\varepsilon_1 \frac{\alpha_{S1}^*}{|\alpha_{S1}|} n_{S1,1} \\
&\quad +\alpha_{2D}\varepsilon_2 \frac{\alpha_{S2}^*}{|\alpha_{S2}|} n_{S2,4} + n_{SD,2} + n_{D,2} \qquad (8)
\end{aligned}$$

$$\begin{aligned}
r_3 &= (-\alpha_{1D} z_{1,4} + \alpha_{2D} z_{2,1} + n_{D,3}) \\
&\quad +(-\varepsilon_0 \alpha_{SD} x_3 + n_{SD,3}) \\
&= \left(\begin{array}{l} -\alpha_{1D}\left[\varepsilon_1 \frac{\alpha_{S1}^*}{|\alpha_{S1}|}(-\alpha_{S1}\varepsilon_0 x_4 + n_{S1,4})\right] \\ +\alpha_{2D}\left[\varepsilon_2 \frac{\alpha_{S2}^*}{|\alpha_{S2}|}(\alpha_{S2}\varepsilon_0 x_1 + n_{S2,1})\right] \\ +n_{D,3} \end{array} \right) \\
&\quad +(-\varepsilon_0 \alpha_{SD} x_3 + n_{SD,3}) \\
&= -\varepsilon_0 \alpha_{SD} x_3 + \varepsilon_0 \varepsilon_1 \alpha_{1D} |\alpha_{S1}| x_4 \\
&\quad +\varepsilon_0 \varepsilon_2 \alpha_{2D} |\alpha_{S2}| x_1 - \alpha_{1D}\varepsilon_1 \frac{\alpha_{S1}^*}{|\alpha_{S1}|} n_{S1,4} \\
&\quad +\alpha_{2D}\varepsilon_2 \frac{\alpha_{S2}^*}{|\alpha_{S2}|} n_{S2,1} + n_{SD,3} + n_{D,3} \qquad (9)
\end{aligned}$$

$$\begin{aligned}
r_4 &= (\alpha_{1D} z_{1,3} - \alpha_{2D} z_{2,2} + n_{D,4}) \\
&\quad +(-\varepsilon_0 \alpha_{SD} x_4 + n_{SD,4}) \\
&= \left(\begin{array}{l} \alpha_{1D}\left[\varepsilon_1 \frac{\alpha_{S1}^*}{|\alpha_{S1}|}(-\alpha_{S1}\varepsilon_0 x_3 + n_{S1,3})\right] \\ -\alpha_{2D}\left[\varepsilon_2 \frac{\alpha_{S2}^*}{|\alpha_{S2}|}(-\alpha_{S2}\varepsilon_0 x_2 + n_{S2,2})\right] \\ +n_{D,4} \end{array} \right) \\
&\quad +(-\varepsilon_0 \alpha_{SD} x_4 + n_{SD,4}) \\
&= -\varepsilon_0 \alpha_{SD} x_4 - \varepsilon_0 \varepsilon_1 \alpha_{1D} |\alpha_{S1}| x_3 \\
&\quad +\varepsilon_0 \varepsilon_2 \alpha_{2D} |\alpha_{S2}| x_2 + \alpha_{1D}\varepsilon_1 \frac{\alpha_{S1}^*}{|\alpha_{S1}|} n_{S1,3} \\
&\quad -\alpha_{2D}\varepsilon_2 \frac{\alpha_{S2}^*}{|\alpha_{S2}|} n_{S2,2} + n_{SD,4} + n_{D,4} \qquad (10)
\end{aligned}$$

where α_{mD} are path gains of the channels between the relay m and the destination D; $m = 1, 2$.

In the above expressions, $n_{D,j}$ is an additive noise sample at the destination in j-th symbol duration of time slot 2. The quantities $n_{D,j}$ are modeled as ZMCGRVs with variance σ_D^2. Also, due to the assumption of slow and flat Rayleigh fading, α_{mD} are independent ZMCGRVs with variances λ_{mD}^2.

We can rewrite the received signals r_j in more compact forms as

$$\begin{aligned}
r_1 &= h_1 x_1 + h_2 x_2 + h_3 x_3 + n_1 && (11)\\
r_2 &= -h_1 x_2 + h_2 x_1 - h_3 x_4 + n_2 && (12)\\
r_3 &= -h_1 x_3 + h_2 x_4 + h_3 x_1 + n_3 && (13)\\
r_4 &= -h_1 x_4 - h_2 x_3 + h_3 x_2 + n_4 && (14)
\end{aligned}$$

by letting

$$\begin{aligned}
h_1 &= \varepsilon_0 \alpha_{SD} && (15)\\
h_2 &= \varepsilon_0 \varepsilon_1 \alpha_{1D} |\alpha_{S1}| && (16)\\
h_3 &= \varepsilon_0 \varepsilon_2 \alpha_{2D} |\alpha_{S2}| && (17)\\
n_1 &= -\alpha_{1D}\varepsilon_1 \frac{\alpha_{S1}^*}{|\alpha_{S1}|} n_{S1,2} - \alpha_{2D}\varepsilon_2 \frac{\alpha_{S2}^*}{|\alpha_{S2}|} n_{S2,3} \\
&\quad + n_{SD,1} + n_{D,1} && (18)\\
n_2 &= \alpha_{1D}\varepsilon_1 \frac{\alpha_{S1}^*}{|\alpha_{S1}|} n_{S1,1} + \alpha_{2D}\varepsilon_2 \frac{\alpha_{S2}^*}{|\alpha_{S2}|} n_{S2,4} \\
&\quad + n_{SD,2} + n_{D,2} && (19)\\
n_3 &= -\alpha_{1D}\varepsilon_1 \frac{\alpha_{S1}^*}{|\alpha_{S1}|} n_{S1,4} + \alpha_{2D}\varepsilon_2 \frac{\alpha_{S2}^*}{|\alpha_{S2}|} n_{S2,1} \\
&\quad + n_{SD,3} + n_{D,3} && (20)\\
n_4 &= \alpha_{1D}\varepsilon_1 \frac{\alpha_{S1}^*}{|\alpha_{S1}|} n_{S1,3} - \alpha_{2D}\varepsilon_2 \frac{\alpha_{S2}^*}{|\alpha_{S2}|} n_{S2,2} \\
&\quad + n_{SD,4} + n_{D,4}. && (21)
\end{aligned}$$

Due to the fact that all additive noise r.v.'s are mutually independent of each other, conditional on the channel attenuations, n_j $(j = 1, 2, 3, 4)$ are also independent ZMCGRVs with the same variance

$$\begin{aligned}
\sigma_n^2 &= |\alpha_{1D}|^2 \varepsilon_1^2 \sigma_{S1}^2 + |\alpha_{2D}|^2 \varepsilon_2^2 \sigma_{S2}^2 + \sigma_{SD}^2 + \sigma_D^2 \\
&= a \varepsilon_1^2 \sigma_{S1}^2 + b \varepsilon_2^2 \sigma_{S2}^2 + \sigma_{SD}^2 + \sigma_D^2 \qquad (22)
\end{aligned}$$

where $a = |\alpha_{1D}|^2$ and $b = |\alpha_{2D}|^2$ are exponentially distributed r.v.'s with mean values λ_{1D}^2, λ_{2D}^2; that is, $f_a(a) = \lambda_a e^{-\lambda_a a}$, $f_b(b) = \lambda_b e^{-\lambda_b b}$ in which $a,\ b \geq 0$ and $\lambda_a = 1/\lambda_{1D}^2$, $\lambda_b = 1/\lambda_{2D}^2$ are pdf's of r.v.'s a and b, respectively.

Equations (11)–(14) are actually equivalent to the analytical expressions of the conventional rate 3/4 STBC for the co-located 3-antenna array with transmission matrix given by

$$\begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \end{bmatrix}.$$

As a consequence, the maximum likelihood decoding can be applied and results in the decision statistics for the transmitted signals x_i as [4]

$$\overline{x}_i = \sum_{j \in \xi(i)} sig_j(i) r_j h_{\phi_j(i)}^* \qquad (23)$$

where $i = 1, 2, 3, 4; \xi(i)$ is the set of columns of the transmission matrix in which x_i appears; $\phi_j(i)$ represents the row position of x_i in the j-th column and the sign of x_i in the j-th column is denoted by $sig_j(i)$. Specifically, we have

$$\begin{aligned}
\overline{x}_1 &= r_1 h_1^* + r_2 h_2^* + r_3 h_3^* \\
&= \left(|h_1|^2 + |h_2|^2 + |h_3|^2\right) x_1 + h_1^* n_1 + h_2^* n_2 + h_3^* n_3 \\
&= \lambda x_1 + N_1 \qquad (24)\\
\overline{x}_2 &= r_1 h_2^* - r_2 h_1^* + r_4 h_3^* \\
&= \left(|h_1|^2 + |h_2|^2 + |h_3|^2\right) x_2 + h_2^* n_1 - h_1^* n_2 + h_3^* n_4 \\
&= \lambda x_2 + N_2 \qquad (25)\\
\overline{x}_3 &= r_1 h_3^* - r_3 h_1^* - r_4 h_2^* \\
&= \left(|h_1|^2 + |h_2|^2 + |h_3|^2\right) x_3 + h_3^* n_1 - h_1^* n_3 - h_2^* n_4 \\
&= \lambda x_3 + N_3 \qquad (26)\\
\overline{x}_4 &= -r_2 h_3^* + r_3 h_2^* - r_4 h_1^* \\
&= \left(|h_1|^2 + |h_2|^2 + |h_3|^2\right) x_4 - h_3^* n_2 + h_2^* n_3 - h_1^* n_4 \\
&= \lambda x_4 + N_4. \qquad (27)
\end{aligned}$$

Here,

$$\begin{aligned}
N_1 &= h_1^* n_1 + h_2^* n_2 + h_3^* n_3 \\
N_2 &= h_2^* n_1 - h_1^* n_2 + h_3^* n_4 \\
N_3 &= h_3^* n_1 - h_1^* n_3 - h_2^* n_4 \\
N_4 &= -h_3^* n_2 + h_2^* n_3 - h_1^* n_4 \\
\lambda &= |h_1|^2 + |h_2|^2 + |h_3|^2 \\
&= u + v + k \\
u &= \varepsilon_0^2 |\alpha_{SD}|^2 \qquad (28)\\
v &= \varepsilon_0^2 \varepsilon_1^2 |\alpha_{1D}|^2 |\alpha_{S1}|^2 = \varepsilon_0^2 \varepsilon_1^2 a |\alpha_{S1}|^2 \qquad (29)\\
k &= \varepsilon_0^2 \varepsilon_2^2 |\alpha_{2D}|^2 |\alpha_{S2}|^2 = \varepsilon_0^2 \varepsilon_2^2 b |\alpha_{S2}|^2. \qquad (30)
\end{aligned}$$

Equations (24)–(27) show that the proposed cooperative transmission protocol can provide exactly the same performance as the 3-level receive maximum ratio combining.

As mentioned in the introduction section, this paper concentrates on analyzing the BER performance of "fully distributed" STBC. Therefore, in the second time-slot (sTS), the source terminal does not transmit data. As a result, it seems that there is a loss of data rate for the cooperative transmission compared to the noncooperative case. However, for some wireless networks such as relay networks, wireless sensor networks, ad-hoc networks, etc, where there exist many idle users, taking advantage of these idle users as relays to obtain the diversity gain and improve the performance represents the reasonable utilization of the system resource. Moreover, in such networks, the total consumed power of all cooperative users, rather than data rate, is usually a crucial factor because of large bandwidth resources and the power limitation of users. Consequently, we are interested in the consumed power when comparing the proposed cooperation with noncooperation much more than data rate as commonly assumed in [10]–[19]. Furthermore, it is obvious that the performance will be increased if the source node uses the sTS. For example, in the sTS, the source node retransmits the same signal sequence as in the first time slot (fTS). Then, three signal sequences from the source and two relays in the sTS are combined as above. Afterward, the resulted signals in (24)–(27) can be used in maximum ratio combining with the signals in the fTS. At this time, we expect the diversity level of 4. However, such operation is not desired since STBC is not fully distributed.

It is straightforward to infer that N_j $(j = 1, 2, 3, 4)$ are also independent ZMCGRVs, given the channel realizations, with the same variance

$$\sigma_N^2 = \lambda \sigma_n^2. \qquad (31)$$

By observing (24)–(27), we find that x_j are attenuated and corrupted by the same fading and noisy level, their error probability is equal. As a result, BER of x_1 is sufficient to evaluate the performance of the system. For BPSK transmission, the recovered bit of x_1 is given by

$$\widehat{x}_i = sign\left(Re\{\overline{x}_1\}\right) \qquad (32)$$

where $sign(\cdot)$ is a signum function and $Re\{\cdot\}$ is the real part of a complex number.

Then, the error probability of x_1 conditional on channel realizations is easily found as

$$P_e = Q\left(\sqrt{2\gamma}\right) \qquad (33)$$

where $\gamma = \lambda^2/\sigma_N^2 = \lambda/\sigma_n^2$ can be interpreted as the signal-to-noise ratio at the output of Gaussian channel.

To find the average error probability, we must know the pdf of γ. Expressing γ explicitly results in the following formula

$$\gamma = \frac{\lambda}{\sigma_n^2} = \frac{u + v + k}{\sigma_n^2}. \qquad (34)$$

Since σ_n^2 is a function of only two r.v.'s a and b, we can find the pdf of r.v. $(u + v + k)$ given a and b. From (28)–(30), we realize that u, v, and k have exponential distribution with mean values $\varepsilon_0^2 \lambda_{SD}^2$, $\varepsilon_0^2 \varepsilon_1^2 a \lambda_{S1}^2$, $\varepsilon_0^2 \varepsilon_2^2 b \lambda_{S2}^2$, respectively. Their pdf's are of the following forms

$$\begin{aligned}
f_u(u) &= \lambda_u e^{-\lambda_u u} \\
f_v(v) &= \lambda_v e^{-\lambda_v v} \\
f_k(k) &= \lambda_k e^{-\lambda_k k}
\end{aligned}$$

where $\lambda_u = 1/\left(\varepsilon_0^2 \lambda_{SD}^2\right)$, $\lambda_v = 1/\left(\varepsilon_0^2 \varepsilon_1^2 a \lambda_{S1}^2\right)$, $\lambda_k = 1/\left(\varepsilon_0^2 \varepsilon_2^2 b \lambda_{S2}^2\right)$, and $u, v, k \geq 0$.

The pdf of $w = u + v$ is computed by using convolution theorem

$$\begin{aligned}
f_{u+v}(w) &= \int_0^w \lambda_u e^{-\lambda_u x_1} \lambda_v e^{-\lambda_v (w - x_1)} dx_1 \\
&= \frac{\lambda_u \lambda_v}{\lambda_u - \lambda_v}\left(e^{-\lambda_v w} - e^{-\lambda_u w}\right).
\end{aligned}$$

Then, repeating that process, we can obtain the pdf of $\lambda = w + k$ as

$$\begin{aligned}
f_\lambda(\lambda) &= f_{u+v}(\lambda) \circ f_k(\lambda) \\
&= \begin{pmatrix} \frac{\lambda_u}{\lambda_u - \lambda_v} \lambda_v e^{-\lambda_v \lambda} \\ -\frac{\lambda_v}{\lambda_u - \lambda_v} \lambda_u e^{-\lambda_u \lambda} \end{pmatrix} \circ \left(\lambda_k e^{-\lambda_k \lambda}\right) \\
&= \frac{\lambda_u}{\lambda_u - \lambda_v} \frac{\lambda_v \lambda_k}{\lambda_v - \lambda_k}\left(e^{-\lambda_k \lambda} - e^{-\lambda_v \lambda}\right)
\end{aligned}$$

$$-\frac{\lambda_v}{\lambda_u-\lambda_v}\frac{\lambda_u\lambda_k}{\lambda_u-\lambda_k}\left(e^{-\lambda_k\lambda}-e^{-\lambda_u\lambda}\right)$$
$$=\frac{\lambda_u\lambda_v\lambda_k\begin{bmatrix} e^{-\lambda_k\lambda}(\lambda_u-\lambda_v)- \\ e^{-\lambda_v\lambda}(\lambda_u-\lambda_k)+ \\ e^{-\lambda_u\lambda}(\lambda_v-\lambda_k)\end{bmatrix}}{(\lambda_u-\lambda_v)(\lambda_v-\lambda_k)(\lambda_u-\lambda_k)} \qquad (35)$$

where $\circ$ is the convolution operator.

Finally, the pdf of γ given a and b is easily found as

$$f_{\gamma|a,b}(\gamma|a,b)=\frac{\sigma_n^2\lambda_u\lambda_v\lambda_k\begin{bmatrix} e^{-\lambda_k\sigma_n^2\gamma}(\lambda_u-\lambda_v)- \\ e^{-\lambda_v\sigma_n^2\gamma}(\lambda_u-\lambda_k)+ \\ e^{-\lambda_u\sigma_n^2\gamma}(\lambda_v-\lambda_k)\end{bmatrix}}{(\lambda_u-\lambda_v)(\lambda_v-\lambda_k)(\lambda_u-\lambda_k)}. \qquad (36)$$

Now, we can calculate the average error probability as follows

$$P_{eAVG}=\int_0^\infty\int_0^\infty\left[\int_0^\infty P_e f_{\gamma|a,b}(\gamma|a,b)d\gamma\right]f_a(a)f_b(b)dadb \qquad (37)$$

where the integral inside the square bracket can be reduced as

$$\begin{bmatrix} \int_0^\infty Q\left(\sqrt{2\gamma}\right)\sigma_n^2\lambda_k e^{-\lambda_k\sigma_n^2\gamma}\frac{\lambda_u\lambda_v}{(\lambda_v-\lambda_k)(\lambda_u-\lambda_k)}d\gamma- \\ \int_0^\infty Q\left(\sqrt{2\gamma}\right)\sigma_n^2\lambda_v e^{-\lambda_v\sigma_n^2\gamma}\frac{\lambda_u\lambda_k}{(\lambda_u-\lambda_v)(\lambda_v-\lambda_k)}d\gamma+ \\ \int_0^\infty Q\left(\sqrt{2\gamma}\right)\sigma_n^2\lambda_u e^{-\lambda_u\sigma_n^2\gamma}\frac{\lambda_v\lambda_k}{(\lambda_u-\lambda_v)(\lambda_u-\lambda_k)}d\gamma \end{bmatrix}$$
$$=\begin{bmatrix} \frac{\lambda_u\lambda_v}{2(\lambda_v-\lambda_k)(\lambda_u-\lambda_k)}\left[1-\sqrt{\frac{1}{1+\sigma_n^2\lambda_k}}\right]- \\ \frac{\lambda_u\lambda_k}{2(\lambda_u-\lambda_v)(\lambda_v-\lambda_k)}\left[1-\sqrt{\frac{1}{1+\sigma_n^2\lambda_v}}\right]+ \\ \frac{\lambda_v\lambda_k}{2(\lambda_u-\lambda_v)(\lambda_u-\lambda_k)}\left[1-\sqrt{\frac{1}{1+\sigma_n^2\lambda_u}}\right] \end{bmatrix}. \qquad (38)$$

By substituting the term in (38) into (37), we obtain the closed-form BER expression for the proposed cooperative transmission scheme. The integrals in (37) can be approximated as sums [20].

In noncooperation case, the source transmits the data with power P_T and consequently, the signal received at the destination is given by

$$y=\alpha_{SD}\sqrt{P_T}c+g \qquad (39)$$

where c is a BPSK-modulated data symbol; g is a sample of zero-mean complex Gaussian random variable with variance σ_{SD}^2.

The symbol c is recovered by

$$\begin{aligned}\bar{c} &= sign\left(Re\{\alpha_{SD}^*y\}\right)\\ &= sign\left(|\alpha_{SD}|^2\sqrt{P_T}c+\bar{g}\right)\end{aligned}$$

where $\bar{g}=Re\{\alpha_{SD}^*g\}$ is ZMCGRV with variance $\sigma^2=\left(|\alpha_{SD}|^2\sigma_{SD}^2\right)/2$ conditioned on the channel realization. Thus, we can derive the BER of noncooperation case P_{ne} as

$$\begin{aligned}P_{ne} &= \int_{|\alpha_{SD}|^2\sqrt{P_T}}^\infty\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{x^2}{2\sigma^2}}dx\\ &= Q\left(\sqrt{\frac{2P_T|\alpha_{SD}|^2}{\sigma_{SD}^2}}\right).\end{aligned}$$

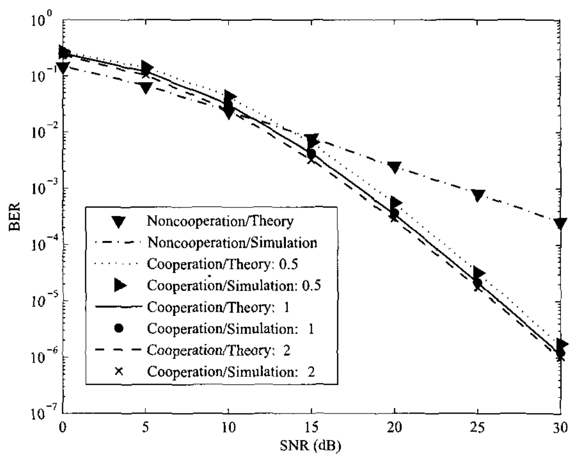

Fig. 2. Effect of inter-user channels on BER performance.

Because $x=2P_T|\alpha_{SD}|^2/\sigma_{SD}^2$ is exponentially distributed with mean value of $\chi=2P_T\lambda_{SD}^2/\sigma_{SD}^2$, the average BER in the case of noncooperation has the following form

$$\begin{aligned}P_{neAVG} &= \int_0^\infty P_{ne}f_x(x)dx\\ &= \int_0^\infty Q\left(\sqrt{x}\right)\frac{1}{\chi}e^{-\frac{x}{\chi}}dx\\ &= \frac{1}{2}\left[1-\sqrt{\frac{\chi}{2+\chi}}\right]\\ &= \frac{1}{2}\left[1-\sqrt{\frac{P_T\lambda_{SD}^2/\sigma_{SD}^2}{1+P_T\lambda_{SD}^2/\sigma_{SD}^2}}\right].\end{aligned} \qquad (40)$$

III. NUMERICAL RESULTS

In the presented results below, we set the noise variances equally as $\sigma_{SD}^2=\sigma_{S1}^2=\sigma_{S2}^2=\sigma_D^2=1$. The signal-to-noise ratio on the x-axes of all figures is defined as SNR $=P_T/\sigma_{SD}^2$. Additionally, Monte-Carlo simulations are performed to verify the closed-form BER expression in (37) and to evaluate the BER performance of the proposed cooperation.

For a fair comparison, it is essential that the total consumed energy of the cooperative system does not exceed that of corresponding direct transmission system. This is a strict and conservative constraint; allowing the relays to add additional power can then only increase the attractiveness of the cooperation. Therefore, complying this energy constraint requires $P_T=P_S+P_1+P_2$. Then, we choose $P_S=P_T/2$ and $P_1=P_2=P_T/4$. As a consequence, the consumed power of each cooperative user is considerably reduced in comparison to that of each noncooperative user, thus contributing to solving the problem of power limitation for small-size devices, for example, wireless sensors.

Fig. 2 compares the performance between noncooperation and the proposed collaboration when the quality of user-destination channels is constant $\lambda_{1D}^2=\lambda_{2D}^2=\lambda_{SD}^2=1$ but that of inter-user channels changes $\lambda_{S1}^2=\lambda_{S2}^2=0.5,\ 1,\ 2$. In

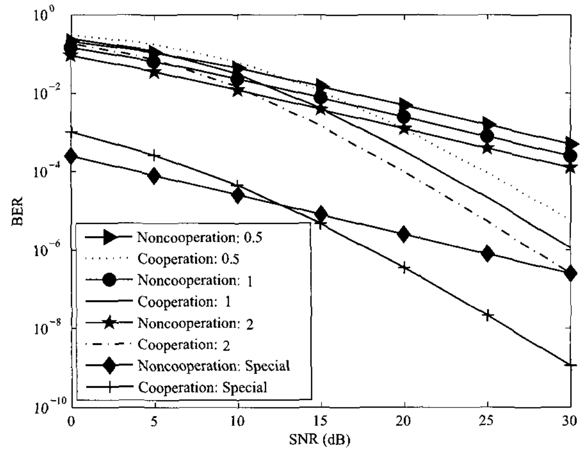

Fig. 3. Effect of user-destination channels on BER performance (the numbers in the legend box denote the values of $\lambda_{1D}^2 = \lambda_{2D}^2 = \lambda_{SD}^2$). For the special case, we choose $\lambda_{1D}^2 = \lambda_{2D}^2 = 1$ and $\lambda_{SD}^2 = 1000$.

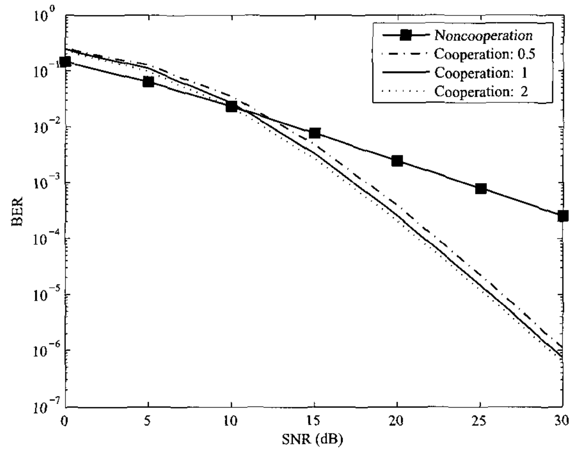

Fig. 4. Effect of asymmetry of the inter-user channels on BER performance.

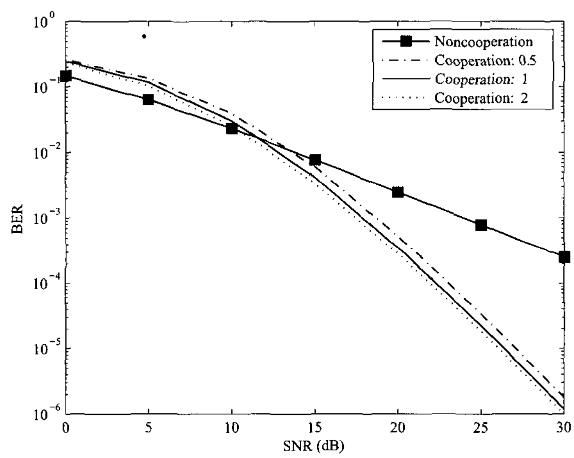

Fig. 5. Effect of asymmetry of the user-destination and inter-user channels on BER performance.

this figure and those following, the numbers corresponding to the scheme in the legend box, for example *Cooperation/Theory*: 0.5, mean the performance of the cooperation strategy with respect to the varying parameters; specifically, $\lambda_{S1}^2 = \lambda_{S2}^2 = 0.5$ in this example. As shown in Fig. 2, the performance of the cooperation scheme slightly depends on the inter-user channels since the cooperative transmission must utilize them to achieve the spatial diversity and since using the intermediate nodes to forward the source signal also causes the noise amplification at those nodes. Nevertheless, under any of their conditions the cooperation always outperforms noncooperation at high SNR (SNR $\geq$ 15 dB) and this benefit increases when the channel quality is better. For instance, at the target of BER 10^{-3} the cooperative transmission acquires SNR gains of about 5 dB, 6 dB, 7 dB according to $\lambda_{S1}^2 = \lambda_{S2}^2 = 0.5,\ 1,\ 2$ relative to the noncooperation. Moreover, due to the steeper BER curves of the cooperation scheme than those of noncooperation, the further performance improvement can be yielded when the power of the source (or equivalently, SNR) increases. These illustrated results assert that using STBC in the distributed diversity way can also obtain all benefits of transmit diversity as for physically co-located antenna array. Furthermore, Fig. 2 also reveals that simulation results agree with theoretical ones (using (37) and (40)) and thus, the analysis is completely exact. As a consequence, only formulas in (37) and (40) are used for evaluating the potentials of the cooperation scheme in enhancing the BER performance in comparison to noncooperation scheme in the sequel.

It is certain that the direct transmission can only obtain low error probability when the channel $S-D$ is good. This is demonstrated in Fig. 3 where the fading variances of the user-destination channels vary $\lambda_{1D}^2 = \lambda_{2D}^2 = \lambda_{SD}^2 = 0.5,\ 1,\ 2$ while the others are unchanged $\lambda_{S1}^2 = \lambda_{S2}^2 = 1$. Compared to the noncooperation, the proposed cooperative transmission yields a better BER performance of about 7 dB for any value of $\lambda_{1D}^2 = \lambda_{2D}^2 = \lambda_{SD}^2$ at BER of 10^{-3} and this improvement is further increased in the quality of user-destination channels. Moreover, the large diversity gain achieved from the cooperation significantly contributes to the performance enhancement as the SNR increases. Fig. 3 also demonstrates a special case where $\lambda_{1D}^2 = \lambda_{2D}^2 = 1$ and very large λ_{SD}^2; specifically, $\lambda_{SD}^2 = 1000$ in this case. It is realized that the diversity gain of the cooperative transmission is constant regardless of the variation of λ_{SD}^2 (BER curves are parallel for different values of λ_{SD}^2). Consequently, in the range of high SNR, the cooperation is always superior to the noncooperation counterpart even though the direct propagation path is very good (very large λ_{SD}^2).

Figs. 2 and 3 only show the BER performance of the cooperative transmission in symmetric scenarios of inter-user channels and user-destination channels. In order to evaluate the effects of channels on the feasibility of fully distributed STBC more totally and to recognize which channel conditions are appropriate for the cooperation, we investigate some possible asymmetric cases of channels between two arbitrary users. The numerical results are depicted in Figs. 4 and 5.

Fig. 4 considers the impact of asymmetric inter-user channels on the performance of the cooperation. We examine this case by fixing the variances of user-destination channels $\lambda_{1D}^2 = \lambda_{2D}^2 = \lambda_{SD}^2 = 1$ and that of *source-relay* 1 channel $\lambda_{S1}^2 = 1$ while changing the quality of *source-relay* 2 channel λ_{S2}^2 from 0.5 to 2. Therefore, the numbers in the legend box denote the values of λ_{S2}^2. This figure shows that the cooperative transmission is very robust to the changes of inter-user channels. Specifically, when the source signal experiences the deep fade four times (λ_{S2}^2 from 2 down to 0.5), the performance degradation is just around 0.5 dB for any values of SNR.

The asymmetry of all possible propagation paths in the cooperative wireless networks is examined in which only λ_{S2}^2 changes from 0.5 to 2 while the others are constant $\lambda_{1D}^2 = 0.5$, $\lambda_{SD}^2 = 1$, $\lambda_{2D}^2 = 2$, $\lambda_{S1}^2 = 1$. The result is depicted in Fig. 5 where the numbers represent the values of λ_{S2}^2. It is recognized that the performance degradation of the cooperation is negligible (less than 0.3 dB for any value of SNR and λ_{S2}^2). Therefore, the cooperation brings a considerable benefit in combating the adverse effects of fading channels.

IV. CONCLUSION

Performance analysis of the conventional rate 3/4 STBC in the cooperative transmission scenario was presented and its closed-form error probability expression was also derived. Under the Rayleigh fading channel plus Gaussian noise, the numerical results demonstrate that the proposed cooperation considerably improves the performance over the noncooperation regardless of the faded noisy inter-user channels in the range of high SNR. In addition, the receiver structure with ML detector can be implemented with negligible hardware complexity. Moreover, this cooperation scheme is straightforwardly extended to the cases of more than two single-antenna relays by creating a new signaling structure and applying the high-level STBCs [21] to reduce further the probability of error. Furthermore, it is not difficult to realize that the proposed scheme can be associated with relays of more than two antennas. Therefore, deploying STBCs in the distributed diversity manner is feasible and is a promising technique for the future wireless networks where there exist idle users to take advantage of system resources efficiently.

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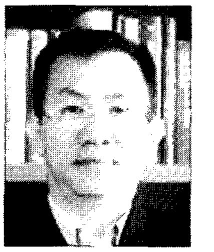

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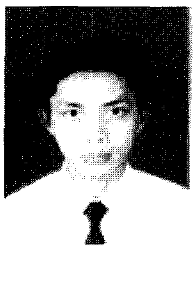

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