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Interactive Fuzzy Linear Programming with Two-Phase Approach

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Abstract

This paper is for applying interactive fuzzy linear programming for the problem of product mix planning, which is one of the aggregate planning problem. We developed a modified algorithm, which has two-phase approach for interactive fuzzy linear programming to get a better solution. Adding two-phase method, we expect to obtain not only the highest membership degree, but also a better utilization of each constrained resource

Key words : interactive fuzzy linear programming, product mix planning, two-phase approach, aggregate planning problem.

1. Introduction

Operations Research (OR) is one of the most famous problem solver and has been used to solve in miscellaneous fields, such as manufacturing, scheduling, transportation, etc.

Lai and Hwang [1] mention that traditional OR (Operations research) approach is not acceptable for the practical decision making problems, because of the nature of fuzziness. The interactive concept provided for decision maker to recognize good solutions. So that he can design a high-productivity system rather than optimize a given system.

OR has been a powerful tool that we have used to solve a various problem until now, but it is doubtful that we can trust the global optimal, which is given by OR approach.

One of the reason that we can not trust the solution that is obtained from OR solver is the fuzziness of data. Since Bellman and Zadeh [2] proposed a fuzzy set theory, we can attack the problem with fuzzy data.

Werners [3] presents the methods of solving linear programming model with crisp or fuzzy constraints and crisp or fuzzy goals. With this method, we can support to make a robust decision for the decision maker with vague informations and imprecise requirements in the real problems.

Zeleny [4] mentioned that to solve the problem more efficiently, we need to move our intention from "optimizing a given system", to "designing an optimal system". Product mix planning problem is referenced in Hopp and Spearman [5] example.

User dependent, problem-oriented and fuzzy set fuzzy set theory improve the flexibility and robustness of linear programming technique. The interactive linear programming provide an efficient and systematic approach, and then shows abundant numerical output using fuzzy IF-THEN rule. And this output help decision maker to design a high productivity system in a given condition [6]-[9].

To obtain the highest membership degree in the objective and better utilization of resources, two-phase method was introduced by Lee and Li [10], and Guu and Wu [11, 12].

Popular and basic linear programming problems with fuzzy constraints.

$$\begin{aligned} &\max \ z = cx \\ &s.t. \ (Ax)_i \le b_i + \theta p_i, \ for\ all\ i \\ &\qquad \theta \in [0,1] \\ &\qquad x_i \ge 0 \end{aligned} \qquad (a)$$

Membership function of ith fuzzy constraints are showed below.

$$\mu_i(x) = \begin{cases} 1 & if\ (Ax)_i < b_i \\ 1 - \dfrac{(Ax)_i - b_i}{p_i} & if\ b_i \le (Ax)_i \le b_i + p_i, \\ 0 & if\ (Ax)_i > b_i + p_i; \end{cases} \quad \text{(b)}$$

The remainder of this paper is organized as follows: Chapter 2 modeling product mix planning problem. Chapter 3 presents modified algorithm procedure of interactive two-phase fuzzy linear programming. In Chapter 4, apply interactive two-phase linear programming to a product mix planning problem. Chapter 5 presents conclusion and future research based on numerical data that we obtained from fuzzy set theory algorithm on the previous chapter. Lastly, references are listed.

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2. Product Mix Planning

One of the realistic aggregate planning is product mix planning. Product mix planning problem can be used to adjust the product of mixing according to the available capacity. In this chapter, linear programming of product mix planning will be presented.

2-1. Problem Description

This is the simple and basic problem of product mix planning. The formulation is modified and added to suitable for applying interactive fuzzy linear programming. We assuming that a planning horizon is only one period even if it is not a realistic in general. Planning horizon is not a big issue in the paper, since we will focus on developing a better algorithm to solve a general linear programming. Four different products, which is called product 1, 2, 3 and 4, produced in the factory. There will be a $7,000 per week fixed cost for labor and capital and 4,800 minutes, five days per week and eight hours per day, of time is available for each workstations. All of these data are identical from week to week; therefore, we do not carry any inventory. We will restrict our period to a single week.

2-2. Model Development

Linear programming is formulated based on input data in Table 1. Profit of product 1 is $55 ($90-$45), product 2 is $70, product 3 is $50 and product 4 is $20. Formulation (1) shows that the objective of problem is maximize the profit of four products. Constraint from (2) to (5) states that there is an upper bound on weekly sales. Constraints from (6) to (9) represent that for each workstation has an available time of 4,800 minutes. Formulation (2)-(9) contains capacity constraints for the workstations, but there are many other resources that we have to consider such as people, raw materials and transport devices.

Table 1. Input Data for Modified Single-Period Product Mix Planning Problem

Restriction	Product			
	1	2	3	4
Selling price	90	110	150	40
Raw material cost	45	40	100	20
Maximum weekly sales	80	50	150	50
Minutes per unit on workstation A	15	10	18	7
Minutes per unit on workstation B	15	35	20	10
Minutes per unit on workstation C	15	5	35	10
Minutes per unit on workstation D	25	14	40	5

Formulation (10) refers about time constraint for the inspector. An inspector has to check products and it require 1, 2, 1.5 and 0.5 hours separate per unit to inspect. And an inspector has 190 hours available time per month. Inspector time is a bottleneck and should be removed if this constraint is adhered to the optimal solution.

Lastly, constraint (11) indicates that decision variables are greater than zero.

$$\max\ z(x) = 45x_1 + 70x_2 + 50x_3 + 20x_4 - 7000 \quad (1)$$

$$s.t.$$

$$g_1(x) = x_1 \le 80 \quad (2)$$

$$g_2(x) = x_2 \le 50 \quad (3)$$

$$g_3(x) = x_3 \le 150 \quad (4)$$

$$g_4(x) = x_4 \le 50 \quad (5)$$

$$g_5(x) = 15x_1 + 10x_2 + 18x_3 + 7x_4 \le 4800 \quad (6)$$

$$g_6(x) = 15x_1 + 35x_2 + 20x_3 + 10x_4 \le 4800 \quad (7)$$

$$g_7(x) = 15x_1 + 5x_2 + 35x_3 + 10x_4 \le 4800 \quad (8)$$

$$g_8(x) = 25x_1 + 14x_2 + 40x_3 + 5x_4 \le 4800 \quad (9)$$

$$g_9(x) = x_1 + 2x_2 + 1.5x_3 + 0.5x_4 \le 190 \quad (10)$$

$$x_i \ge 0,\ for\ i = 1, 2, 3, 4 \quad (11)$$

3. Modified Algorithm Procedure of Interactive Two-Phase Fuzzy Linear Programming

Step 0: Solve a traditional linear programming, equation (12). The unique global optimal solution will be obtained. Decision maker make a decision whether go or stop. If satisfied then stop, if not then change the value of available resource b_i and then solve linear programming again. If available resources are changeable and want to apply fuzzy constraints then go to step 1

Traditional LP problem

$$\max\ cx$$

$$s.t.\ (Ax)_i \le b_i,\ for\ all\ i$$

$$x_i \ge 0 \quad (12)$$

Step 1: Solve parametric linear programming problem, equation (13). The global optimal solution is a function of θ. For each θ, optimal solution is obtained and presented to decision maker then decision maker can choose best solution among them. At here, we could identify the Z_0 and Z_1. Z_0 is the optimal solution when θ is 0 and Z_1 is the optimal solution when θ is 1. If satisfied, then stop. If wants to change the available resource b_i, then change the value of b_i and go to step 0 again. If wants to use fuzzy constraint for objective function then go to step 2.

Parametric programming problem

$$\begin{aligned} &\max\ cx \\ &s.t.\ (Ax)_i \le b_i + \theta p_i,\ for\ all\ i \\ &\theta \in [0,1] \\ &x_i \ge 0 \end{aligned} \tag{13}$$

θ is a parameter, a fraction of maximum tolerance p_i.

Step 2: After analyzing the solution of step 1, decision maker is asked for subjective goal b_0 and tolerance p_0. By this way, fuzzy value will be applied both objective function and constraints. If b_0 is given then go to step 4, if not go to step 3 to obtain b_0.

Step 3: Solve the equation (14). If b_0 is conceived then assign the goal b_0 and go to step 4.

$$\begin{aligned} &\min\ \theta \\ &s.t.\ c^T x \ge Z_1 - \theta(Z_1 - Z_0), \\ &(Ax)_i \le b_i + \theta p_i,\ for\ all\ i, \\ &\theta \in [0,1] \\ &x_i \ge 0 \end{aligned} \tag{14}$$

θ is a fraction of $(Z_1\text{-}Z_0)$,which is obtained in step 1.

Step 4: If decision maker can decided the maximum tolerance p_0, then go to step 5, if not then go to step 6. It should be denoted that $p_0 \in [0, b_0 - Z_0]$.

Step 5: Solve the equation (15), which is known as Zimmermann's model. The unique global optimal solution will be obtained. If decision maker satisfied then stop. If decision maker find a better value of the goal b_0 then go to step 4 with a new value of b_0. If wants to change the value of p_i then go to step 1, after changing the value of p_i.

$$\begin{aligned} &\min\ \theta \\ &s.t.\ c^T x \ge b_0 - \theta p_0, \\ &(Ax)_i \le b_i + \theta p_i,\ for\ all\ i, \\ &\theta \in [0,1] \\ &x_i \ge 0 \end{aligned} \tag{15}$$

where b_0 and p_0 are given. is a fraction of the maximum tolerances.

Step 6: Solve problem with fuzzy objective and fuzzy constraints with the condition of the goal b_0 is given and tolerance p_0 is not given. The solution set is given to the decision maker and then the decision maker will have a good concept of p_0 at this point and go to the step 7.

Step 7: Max-min operator, which is called first phase in two-phase method, will be deployed in the step 7. Obtained solution value of step 7 will be used to get the membership function value that will be used in step 8.

$$\begin{aligned} &\max\ \alpha \\ &s.t.\ c^T x - (Z_1 - Z_0)\alpha \ge Z_0, \\ &c^T \le Z_1, \\ &(Ax)_i + \alpha p_i \le b_i + p_i,\ for\ all\ i, \\ &(Ax)_i \ge b_i \\ &\theta \in [0,1] \\ &x_i \ge 0 \end{aligned} \tag{16}$$

Step 8: Second phase of two-phase method is applied in step 8. It was proved that second phase solution is at least better than the solution of first phase. We will obtain the highest membership degree in the objective and also desire to acquire a better utilization of each constrained resources.

$$\begin{aligned} &\max\ \alpha_i \\ &s.t.\ \alpha_i \ge u_i(x^*), \\ &c^T x - (Z_1 - Z_0)\alpha_0 \ge Z_0, \\ &c^T x \le Z_1, \\ &(Ax)_i + \alpha_i p_i \le b_i + p_i,\ for\ all\ i \\ &(Ax)_i \ge b_i,\ for\ all\ i, \\ &\theta \in [0,1] \\ &x_i \ge 0 \end{aligned} \tag{17}$$

4. Application of Interactive Linear Programming to a Product Mix Planning Problem

This chapter Implement a modified interactive fuzzy linear programming algorithm to product mix planning problem.

Step 0: Solve the original product mix planning problem in chapter 2-2, which is formulated as linear programming using simplex method. The global optimal solution is found. Objective value of $Z^* = 575$ and $x^* = (80, 42.5, 0, 50)$. The actual amount of units produced in the workstation is 1975 for workstation 1, 3187.5 for workstation 2, 1912.5 for workstation 3, and 2845 for workstation 4. And the precise units required for each arc is x*. modified constraints (18) is as follows:

And solve formulation (18) again, the result is the same as previous, however; unnecessary resources are removed. This is the way to design an optimal system instead of optimizing a given system. If the decision maker satisfied with the result then stop, If the decision maker wants to implement the fuzzy constraints then go to step 1.

Step 1: For the more specific analysis, we can consider the tolerance for some constraints. We can give some tolerance for constraints from (2) to (5), because weekly sales can be increased the amount of product to maximize the profit. When we considering some tolerance on each workstation work time, there is some constraint change on constraint from (6) to (9).

$$
\begin{aligned}
&\max\ z = 45x_1 + 70x_2 + 50x_3 + 20x_4 - 7000 \\
&s.t. \\
&x_1 \le 80 \\
&x_2 \le 42.5 \\
&x_3 \le 0 \\
&x_4 \le 50 \\
&15x_1 + 10x_2 + 18x_3 + 7x_4 \le 1975 \\
&15x_1 + 35x_2 + 20x_3 + 10x_4 \le 3187.5 \\
&15x_1 + 5x_2 + 35x_3 + 10x_4 \le 1912.5 \\
&25x_1 + 14x_2 + 40x_3 + 5x_4 \le 2845 \\
&x_1 + 2x_2 + 1.5x_3 + 0.5x_4 \le 190 \\
&x_i \ge 0,\ for\ i\ =\ 1, 2, 3, 4 \qquad (18)
\end{aligned}
$$

In the formulation (10), we will give some tolerance for the inspection time. The problem is changed to the linear programming with fuzzy constraint and crisp objective function and modified formulation is shown in the formulation (19).

Table 2 will be presented to the decision maker. If the decision maker satisfied with the one of these answers then decision maker would make a decision and stop here. If the dicision maker wants to change the resources b_i, then change b_i, p_i and go back to the step 0. Otherwise, proceed to the step 2 to give fuzzy constraint to objective function.

$$
\begin{aligned}
&\max\ z = 45x_1 + 70x_2 + 50x_3 + 20x_4 - 7000 \\
&s.t. \\
&x_1 \le 80 + 15\alpha \\
&x_2 \le 42.5 + 10\alpha \\
&x_3 \le 0 + 5\alpha \\
&x_4 \le 50 + 5\alpha \\
&15x_1 + 10x_2 + 18x_3 + 7x_4 \le 1975 + 200\alpha \\
&15x_1 + 35x_2 + 20x_3 + 10x_4 \le 3187.5 + 300\alpha \\
&15x_1 + 5x_2 + 35x_3 + 10x_4 \le 1912.5 + 200\alpha \\
&25x_1 + 14x_2 + 40x_3 + 5x_4 \le 2845 + 250\alpha \\
&x_1 + 2x_2 + 1.5x_3 + 0.5x_4 \le 190 + 15\alpha \\
&x_i \ge 0,\ for\ i\ =\ 1, 2, 3, 4 \\
&\alpha \ge 0 \\
&\alpha \le 1 \qquad (19)
\end{aligned}
$$

Step 2: After analyzing the table 2, Decision maker have to choose the subjective goal b_0 and tolerance p_0. If he can choose b_0, then go to step 4, otherwise go to step 3 to get the value of b_0

Table 2. Parametric Programming Problem Solutions

	Z*	x*				Actual Used Resources			
						Workstation			
		1	2	3	4	A	B	C	D
0.0	575	80	42.5	0	50	1975	3187.5	1912.5	2845
0.1	636.89	80.74	42.68	0	50.8	1993.5	3212.9	1932.5	2870.02
0.2	698.78	81.48	42.86	0	51.6	2012	3238.3	1952.5	2895.04
0.3	760.68	82.22	43.04	0	52.4	2030.5	3263.7	1972.5	2920.06
0.4	822.57	82.95	43.22	0	53.21	2048.92	3289.05	1992.45	2944.88
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.
0.9	1132.03	86.65	44.12	0	57.22	2141.49	3416.15	2092.55	3070.03
1.0	1193.92	97.39	44.3	0	58.02	2159.99	3441.55	2112.55	3095.05

In step 4, Werner's approach help decision maker decide the value of b_0.

Step 3: Solve the formulation (20) to obtain b_0. The solution for the formulation (20) is x^*=(83.69, 43.4, 0, 54.01), Z^*=884.459 and α is 0.5. The value of Z is improved if compared with the results of step 0, and also the value of x^* is quite different. If decision maker can assign the goal b_0, then go to the next step 4. If he wants to change the value of b_i, then go back to the step 0.

$$
\begin{aligned}
&\min\ \alpha \\
&s.t. \\
&45x_1 + 70x_2 + 50x_3 + 20x_4 - 7000 \geq 1193.919 \\
&-618.919\alpha \\
&x_1 \leq 80 + 15\alpha \\
&x_2 \leq 42.5 + 15\alpha \\
&x_3 \leq 0 + 15\alpha \\
&x_4 \leq 50 + 15\alpha \\
&15x_1 + 10x_2 + 18x_3 + 7x_4 \leq 1975 + 200\alpha \\
&15x_1 + 35x_2 + 20x_3 + 10x_4 \leq 3187.5 + 300\alpha \\
&15x_1 + 5x_2 + 35x_3 + 10x_4 \leq 1912.5 + 200\alpha \\
&25x_1 + 14x_2 + 40x_3 + 5x_4 \leq 2845 + 250\alpha \\
&x_1 + 2x_2 + 1.5x_3 + 0.5x_4 \leq 190 + 15\alpha \\
&x_i \geq 0,\ for\ i = 1, 2, 3, 4 \\
&\alpha \geq 0 \\
&\alpha \leq 1 \qquad (20)
\end{aligned}
$$

Step 4: Let us assume that b_0 = 884.46 at α=0.5, p_0=300, since, $p_0 \in [0, b_0 - Z_0]$ and proceed to step 5. If can not decide p_0 and want to see more analysis then go to step 6.

Step 5: Solve Zimmermann's model, formulation (21), with given b_0 and p_0. Solution for step 5 is Z^* is 783.43, x^* is (82.49, 43.11, 0, 52.7) when α = 0.337. Formulation (21) is almost the same with the formulation (20), but the membership function has changed.

Step 6: Table 3 show that the results of simulation with different value of p_0. There are five different value of p_0 and it applied in formulation (21). As noted before, p_0 should be between 0 and 309.46. The result will be presented to the decision maker and p_0 should be decided in step 6. Now we have all of the information, which obtained using fuzzy set theory and it is given to decision maker.

Step 7: Step 7 and 8 is the last procedure, which we can make sure that there is no room to improve. If there is any possibility that we can improve the solution, then step7 and 8 could find a better solution for the degree of each membership function for each objective and utilization.

$$
\begin{aligned}
&\min\ \alpha \\
&s.t. \\
&45x_1 + 70x_2 + 50x_3 + 20x_4 - 7000 \geq 884.46 - 300\alpha \\
&x_1 \leq 80 + 15\alpha \\
&x_2 \leq 42.5 + 10\alpha \\
&x_3 \leq 0 + 5\alpha \\
&x_4 \leq 50 + 30\alpha \\
&15x_1 + 10x_2 + 18x_3 + 7x_4 \leq 1975 + 200\alpha \\
&15x_1 + 35x_2 + 20x_3 + 10x_4 \leq 3187.5 + 300\alpha \\
&15x_1 + 5x_2 + 35x_3 + 10x_4 \leq 1912.5 + 200\alpha \\
&25x_1 + 14x_2 + 40x_3 + 5x_4 \leq 2845 + 250\alpha \\
&x_1 + 2x_2 + 1.5x_3 + 0.5x_4 \leq 190 + 15\alpha \\
&x_i \geq 0,\ for\ i = 1, 2, 3, 4 \\
&0 \leq \alpha \leq 1 \qquad (21)
\end{aligned}
$$

Table 3. Results of simulation with different value of p_0

	α	Z^*	x^*				Actual Used Resources			
							Workstation			
			1	2	3	4	A	B	C	D
0	0.5	884.46	83.69	43.40	0	54.01	2067.42	3314.45	2012.45	2969.90
80	0.443	849.04	83.27	43.30	0	53.55	2056.90	3300.05	2001.05	2955.70
120	0.419	834.20	83.09	43.25	0	53.36	2052.37	3293.70	1996.20	2949.55
160	0.397	820.89	82.93	43.22	0	53.16	2048.48	3288.55	1991.95	2944.28
309.5	0.333	781.32	82.46	43.10	0	53.67	2036.59	3272.10	1979.10	2928.25

The membership function, formulations (22) to (30), for each nine fuzzy constraints and for the objective function, formulation (31) are as follows.

$$\mu_1(x) = \begin{cases} 1 & if\ g_1(x) < 80, \\ \dfrac{95 - g_1(x)}{15} & if\ 80 \le g_1(x) \le 95, \\ 0 & if\ g_1(x) > 95; \end{cases} \quad (22)$$

$$\mu_2(x) = \begin{cases} 1 & if\ g_2(x) < 42.5, \\ \dfrac{52.5 - g_2(x)}{10} & if\ 42.5 \le g_2(x) \le 52.5, \\ 0 & if\ g_2(x) > 52.5; \end{cases} \quad (23)$$

$$\mu_3(x) = \begin{cases} 1 & if\ g_3(x) < 0, \\ \dfrac{5 - g_3(x)}{5} & if\ 0 \le g_3(x) \le 5, \\ 0 & if\ g_3(x) > 5; \end{cases} \quad (24)$$

$$\mu_4(x) = \begin{cases} 1 & if\ g_4(x) < 50, \\ \dfrac{80 - g_4(x)}{30} & if\ 50 \le g_4(x) \le 80, \\ 0 & if\ g_4(x) > 80; \end{cases} \quad (25)$$

$$\mu_5(x) = \begin{cases} 1 & if\ g_5(x) < 1975, \\ \dfrac{2175 - g_5(x)}{200} & if\ 1975 \le g_5(x) \le 2175, \\ 0 & if\ g_5(x) > 2175; \end{cases} \quad (26)$$

$$\mu_6(x) = \begin{cases} 1 & if\ g_6(x) < 3187.5, \\ \dfrac{3487.5 - g_6(x)}{300} & if 3187.5 \le g_6(x) \le 3487.5, \\ 0 & if\ g_6(x) > 3487.5; \end{cases} \quad (27)$$

$$\mu_7(x) = \begin{cases} 1 & if\ g_7(x) < 1912.5, \\ \dfrac{2212.5 - g_7(x)}{300} & if\ 1912.5 \le g_7(x) \le 2212.5, \\ 0 & if\ g_7(x) > 2212.5; \end{cases} \quad (28)$$

$$\mu_8(x) = \begin{cases} 1 & if\ g_8(x) < 2845, \\ \dfrac{3095 - g_8(x)}{250} & if\ 2845 \le g_8(x) \le 3095, \\ 0 & if\ g_8(x) > 3095; \end{cases} \quad (29)$$

$$\mu_9(x) = \begin{cases} 1 & if\ g_9(x) < 190, \\ \dfrac{200 - g_9(x)}{10} & if\ 190 \le g_9(x) \le 200, \\ 0 & if\ g_9(x) > 200; \end{cases} \quad (30)$$

$$\mu_0(x) = \begin{cases} 1 & if\ z(x) < 575, \\ \dfrac{z(x) - 575}{618.92} & if\ 575 \le z(x) \le 618.92, \\ 0 & if\ z(x) > 618.92; \end{cases} \quad (31)$$

Max-min operator formulation is shown in formulation (32).

After solving formulation (32), solution Z^* is 835.55, x^* is (85.79, 42.5, 0, 50) when $\alpha^* = 0.42$. We can get the value of $\mu_i(x^*)$ using membership function. $\mu_0(x^*) = \mu_8(x^*) = \mu_9(x^*) = 0.42$, $\mu_1(x^*) = 0.614$, $\mu_2(x^*) = \mu_3(x^*) = \mu_4(x^*) = 1$, $\mu_5(x^*) = 0.566$, $\mu_6(x^*) = \mu_7(x^*) = 0.71$. These values will be used in step 8.

Step 8: Second phase formulation (33) is made using the value, which is obtained at step 7. The optimal solution for a_i = (0.42, 0.62, 1, 1, 1, 0.57, 0.71, 1, 0.42, 0.42), x^*=(85.78, 42.5, 0, 50) and Z^*=835.55. Unfortunately, there is no significant improvement is made. It means that the solution that we achieved in the previous step is good enough. If there is a room to improve then different value of a_i is attained. For example, if a_6 = 1 instead of 0.57, then 1975 minutes used on workstation A, while at step 7, 2061 minutes required on workstation A.

5. Conclusions and Future Research

When we solve the traditional linear programming problem, we expect only one answer for the question.

$$\begin{aligned}
&\max\ \alpha \\
&s.t. \\
&45x_1 + 70x_2 + 50x_3 + 20x_4 - 7000 - 618.919\alpha \ge 575 \\
&45x_1 + 70x_2 + 50x_3 + 20x_4 - 7000 \le 1193.919 \\
&x_1 + 15\alpha \le 95 \\
&x_1 \ge 80 \\
&x_2 + 10\alpha \le 52.5 \\
&x_2 \ge 42.5 \\
&x_3 + 5\alpha \le 5 \\
&x_3 \ge 0 \\
&x_4 + 30\alpha \le 80 \\
&x_4 \ge 50 \\
&15x_1 + 10x_2 + 18x_3 + 7x_4 + 200\alpha \le 2175 \\
&15x_1 + 10x_2 + 18x_3 + 7x_4 \ge 1975 \\
&15x_1 + 35x_2 + 20x_3 + 10x_4 + 300\alpha \le 3487.5 \\
&15x_1 + 35x_2 + 20x_3 + 10x_4 \ge 3187.5
\end{aligned}$$

$$15x_1 + 5x_2 + 35x_3 + 10x_4 + 200\alpha \le 2212.5$$
$$15x_1 + 5x_2 + 35x_3 + 10x_4 \ge 1912.5$$
$$25x_1 + 14x_2 + 40x_3 + 5x_4 + 250\alpha \le 3095$$
$$25x_1 + 14x_2 + 40x_3 + 5x_4 \ge 2845$$
$$x_1 + 2x_2 + 1.5x_3 + 0.5x_4 + 10\alpha \le 200$$
$$x_1 + 2x_2 + 1.5x_3 + 0.5x_4 \ge 190$$
$$x_i \ge 0, \textit{ for } i = 1, 2, 3, 4$$
$$0 \le \alpha \le 1 \quad (32)$$

This research shows that by incorporating with fuzzy set theory, decision maker can obtain more specific information about the problem.

This is the way how to design the system instead of just optimize and get only one global optimal solution. Interactive fuzzy linear programming method used to approach the product mix planning problem. With the given result, decision maker can decide how product can be mixed. This is a small problem, it needs to challenge much practical and larger problem.

$$\max\ a_0+a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9$$
$$s.t.$$
$$a_0 \ge 0.42$$
$$a_1 \ge 0.641$$
$$a_2 \ge 1$$
$$a_3 \ge 1$$
$$a_4 \ge 1$$
$$a_5 \ge 0.56575$$
$$a_6 \ge 0.7105$$
$$a_7 \ge 0.7105$$
$$a_8 \ge 0.421$$
$$a_9 \ge 0.421$$
$$45x_1 + 70x_2 + 50x_3 + 20x_4 - 7000 - 618.919a_0 \ge 575$$
$$45x_1 + 70x_2 + 50x_3 + 20x_4 - 7000 \le 1193.919$$
$$x_1 + 15a_1 \le 95$$
$$x_1 \ge 80$$
$$x_2 + 10a_2 \le 52.5$$
$$x_2 \ge 42.5$$
$$x_3 + 5a_3 \le 5$$
$$x_3 \ge 0$$
$$x_4 + 30a_4 \le 80$$
$$x_4 \ge 50$$
$$15x_1 + 10x_2 + 18x_3 + 7x_4 + 200a_5 \le 2175$$
$$15x_1 + 10x_2 + 18x_3 + 7x_4 \ge 1975$$
$$15x_1 + 35x_2 + 20x_3 + 10x_4 + 300a_6 \le 3487.5$$
$$15x_1 + 35x_2 + 20x_3 + 10x_4 \ge 3187.5$$
$$15x_1 + 5x_2 + 35x_3 + 10x_4 + 200a_7 \le 2212.5$$
$$15x_1 + 5x_2 + 35x_3 + 10x_4 \ge 1912.5$$
$$25x_1 + 14x_2 + 40x_3 + 5x_4 + 250a_8 \le 3095$$
$$25x_1 + 14x_2 + 40x_3 + 5x_4 \ge 2845$$
$$x_1 + 2x_2 + 1.5x_3 + 0.5x_4 + 10a_9 \le 200$$
$$x_1 + 2x_2 + 1.5x_3 + 0.5x_4 \ge 190$$
$$x_i \ge 0, \textit{ for } i = 1, 2, 3, 4$$
$$0 \le a_i \le 1, \textit{ for } i = 0,1,2,3,4,5,6,7,8,9 \quad (33)$$

Two-phase method will find a better utilization and degree of membership function for each objective if there is any room to improve. Even if there is no improvement with two-phase method, we can confirm the obtained solution that we have acquired from the previous step is good. In our model, there is no significant improvement even if we used two-phase method, but we showed that two-phase method could have a better utilization by presenting an example. The modified algorithm can be applied any linear programming problem. It is obvious that this algorithm does not fit to integer programming. Applying interactive fuzzy linear programming to integer programming can be one of the future research.

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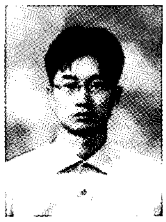

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