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A Fractional Model Reduction for T-S Fuzzy Systems with State Delay

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Abstract

This paper deals with a fractional model reduction for T-S fuzzy systems with time varying delayed states. A contractive coprime factorization of time delayed T-S fuzzy systems is defined and obtained by solving linear matrix inequalities. Using generalized controllability and observability gramians of the contractive coprime factor, a balanced state space realization of the system is derived. The reduced model will be obtained by truncating states in the balanced realization and an upper bound of model approximation error is also presented. In order to demonstrate efficacy of the suggested method, a numerical example is performed.

Key words : balanced model reduction, coprime factorization, T-S fuzzy system, linear matrix inequality.

1. Introduction

For linear finite dimensional systems with high orders, optimal control techniques such as LQG and H_∞ control theory, usually produce controllers with the same state dimension as the model. Accordingly the problem of model reduction is of significant practical importance in control system design and has been a focus of a wide variety of studies for decades (see [1-6] and the references, therein).

The stability analysis and control of time delayed systems are problems of practical and theoretical interest since many types of processes such as steel making process and chemical process can be modeled as dynamic systems with time delay. In the last decade, the linear matrix inequality (LMI) based controller design method for delayed systems has been developed remarkably [7-9]. A drawback of the LMI based controller synthesis is that computational requirements increase rapidly as the state dimension increases. Therefore the state dimension must be kept as low as possible. In the last decade, several research works related to approximation of linear systems with uncertain parameters have been performed.[5,10,11] More recently, a balanced model reduction method for quadratically stable T-S fuzzy system with time varying delay (FSTVD) is suggested in [12].

But model approximation is mainly focused to quadratically stable systems. Moreover not much works for model reduction of unstable nonlinear systems with state delay have been proposed as far as we know. This motivates our study for a fractional model reduction of FSTVD. In this paper we introduce a contractive coprime factorization of FSTVD and study an approximation technique based on balanced truncation of the contractive coprime factor. We obtain an approximation model by truncating a part of the state variables of the system's coprime factor. This implies that we are trying to keep the coprime factor of the approximation system closed to the coprime factor of the original time delayed system.

We begin by defining a FSTVD and introduce preliminary definitions for model reduction in section 2. A contractive right (left) coprime factorization is introduced in section 3. Model reduction in the coprime factor of the FSTVD is studied in Section 4. Section 5 gives a numerical example to validate the results developed in the previous sections.

In this paper, the notation is fairly standard. R^n denotes n dimensional real vector spaces and $R^{n\times m}$ means the set of $n\times m$ dimensional real matrices. M^T stands for the transpose of M. 0 and I_n denote the zero matrix and the $n\times n$ dimensional identity matrix respectively. In a symmetric block matrix, * in the (i,j) block means the transpose of the submatrix in the (j,i) block. $M<(\le)0$ means that M is negative definite (semi-definite) matrix. Γ_* and Γ_*^{-1} with a subscript * denote the system and the inverse system respectively. Finally $\|\Gamma_P\|_\infty$ denotes the H_∞ norm of the system Γ_P.

2. Preliminary

We consider the following class of FSTVD.

Plant Rule i ($i=1,\cdots,r$):

IF $\rho_1(t)$ is M_{i1} and $\cdots$ and $\rho_g(t)$ is M_{ig}.

THEN

$$\begin{aligned}\dot{x}(t)&=A_ix(t)+A_{di}x(t-d(t))+B_iu(t),\\ y(t)&=C_ix(t),\end{aligned}\qquad(1)$$

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where r is the number of fuzzy rules. $\rho_j(t)$ and M_{ij} ($j=1,\cdots,g$) are the premise variables and the fuzzy set respectively. $x(t)\in R^n$ is the state vector, $u(t)\in R^m$ is the input, $y(t)\in R^p$ is the output variable and A_i, $A_{di},\cdots,C_i$ are real matrices with compatible dimensions. We define $A_{di}=F_iG$, where $F_i\in R^{n\times v}$, $G\in R^{v\times n}$. Note that F_i and G are not necessarily full rank matrices. The easiest choice might be $F_i=A_{di}, G=I_n$. We also assume that time varying delay $d(t)$ satisfies

$$0\le d(t)<\infty,\left|\dot{d}(t)\right|\le\beta. \tag{2}$$

Let $\mu_i(\rho(t)), i=1,\cdots,r,$ be the normalized membership function of the inferred fuzzy set $h_i(\rho(t))$,

$$\mu_i(\rho(t))=\frac{h_i(\rho(t))}{\sum_{i=1}^{r}h_i(\rho(t))}, \tag{3}$$

where,

$$h_i(\rho(t))=\prod_{j=1}^{g}M_{ij}(\rho_j(t)),\ \rho(t)=\begin{bmatrix}\rho_1(t) & \rho_2(t) & \cdots & \rho_g(t)\end{bmatrix}^T. \tag{4}$$

In this paper, we assume for all $t\ge 0$,

$$h_i(\rho(t))\ge 0\ (i=1,2,\cdots,r),\ \sum_{i=1}^{r}h_i(\rho(t))>0. \tag{5}$$

Then, we obtain

$$\mu_i(\rho(t))\ge 0\ (i=1,2,\cdots,r),\ \sum_{i=1}^{r}\mu_i(\rho(t))=1. \tag{6}$$

For simplicity, we define

$$\mu_i=\mu_i(\rho(t)),\ (i=1,2,\cdots,r),\ \mu^T=\begin{bmatrix}\mu_1 & \cdots & \mu_r\end{bmatrix}. \tag{7}$$

As shown in fig.1, the fuzzy system (1) can be written as follows:

$$\begin{aligned}\dot{x}(t)&=A(\mu)x(t)+F(\mu)w(t)+B(\mu)u(t)\\&=\sum_{i=1}^{r}\mu_i(A_ix(t)+F_iw(t)+B_iu(t)),\\y(t)&=C(\mu)x(t)=\sum_{i=1}^{r}\mu_iC_ix(t),\\z(t)&=Gx(t),\\w(t)&=\Theta(t)z(t)=Gx(t-d(t)),\end{aligned} \tag{8}$$

where, $\Theta(t)$ is a delay operator. $w(t)$ and $z(t)$ are variables introduced to express the time delay. Thus $w(t)$ and $z(t)$ are an output and input of the delay operator respectively.

In a packed matrix notation, we express the fuzzy system (8) as follows:

$$\Gamma_P=\left[\begin{array}{c|c|c}A(\mu) & F(\mu) & B(\mu)\\ \hline G & 0 & 0\\ \hline C(\mu) & 0 & 0\end{array}\right]. \tag{9}$$

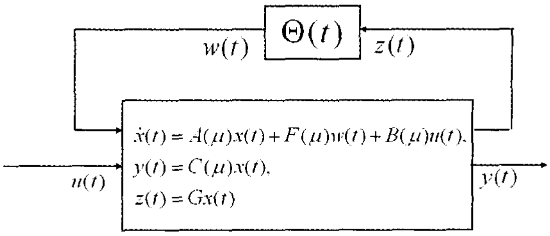

fig. 1 The fuzzy system with time delay

Now we state some definitions for our development.

Definition 1(Bounded Real Lemma) : If there exist $X=X^T>0$, $S=S^T>0$ satisfying LMI (10), $\|\Gamma_P\|_\infty\le\gamma$ is achieved in FSTVD (9).

$$\begin{bmatrix}A(\mu)^TX+XA(\mu)+\gamma^{-2}C(\mu)^TC(\mu)+G^TSG & * & *\\ F(\mu)^TX & -(1-\beta)S & *\\ B(\mu)^TX & 0 & -I_m\end{bmatrix}\le 0. \tag{10}$$

Definition 2(Generalized Gramian) : Suppose that Γ_P in (9) is quadratically stable. When there exist $Q=Q^T>0$ and $R=R^T>0$ satisfying LMI (11) for all μ, we say that Q is a generalized observability gramian of the system Γ_P.

$$L_o:=\begin{bmatrix}A(\mu)^TQ+QA(\mu)+C(\mu)^TC(\mu) & * & *\\ F(\mu)^TQ & -(1-\beta)R & *\\ RG & 0 & -R\end{bmatrix}<0. \tag{11}$$

If there exist $P=P^T>0$ and $S=S^T>0$ such that LMI (12) holds for all μ, P is a generalized controllability gramian of the system Γ_P.

$$L_c:=\begin{bmatrix}PA(\mu)^T+A(\mu)P+B(\mu)B(\mu)^T & * & *\\ SF(\mu)^T & -(1-\beta)S & *\\ GP & 0 & -S\end{bmatrix}<0. \tag{12}$$

Remark 1: Generalized controllability and observability gramians defined in definition 2 are not unique contrary to the linear time invariant case.

Now we briefly state a balanced model reduction scheme using generalized controllability and observability gramians. Suppose that the system Γ_P in (9) is quadratically stable. Then $Q=Q^T>0$, $R=R^T>0$, $P=P^T>0$ and $S=S^T>0$ can be computed from LMI's (11) and (12). Let nonsingular matrices T and U be such that

$$\begin{aligned}T^{-1}PT^{-T}=T^TQT=\Sigma=diag(\sigma_1,\sigma_2,\cdots,\sigma_n),\\ \sigma_1\ge\sigma_2\ge\cdots\ge\sigma_n,\end{aligned} \tag{13}$$

$$U^{-1}SU^{-T}=U^TRU=\Pi. \tag{14}$$

Using T and U defined in (13) and (14), the change of coordinates in Γ_P gives

$$\Gamma_P = \left[\begin{array}{c|c|c} T^{-1}A(\mu)T & T^{-1}F(\mu)U & T^{-1}B(\mu) \\ \hline U^{-1}GT & 0 & 0 \\ \hline C(\mu)T & 0 & 0 \end{array}\right] =: \left[\begin{array}{cc|c|c} A_{b11}(\mu) & A_{b12}(\mu) & F_{b1}(\mu) & B_{b1}(\mu) \\ A_{b21}(\mu) & A_{b22}(\mu) & F_{b2}(\mu) & B_{b2}(\mu) \\ \hline G_{b1} & G_{b2} & 0 & 0 \\ \hline C_{b1}(\mu) & C_{b2}(\mu) & 0 & 0 \end{array}\right], \quad (15)$$

where $A_{b11}(\mu) \in R^{v \times v}$, $A_{b22}(\mu) \in R^{(n-v)\times(n-v)}$ and the other matrices are compatibly partitioned.

We obtain a v dimensional reduced order system of Γ_P as follows:

$$\hat{\Gamma}_P = \left[\begin{array}{c|c|c} A_{b11}(\mu) & F_{b1}(\mu) & B_{b1}(\mu) \\ \hline G_{b1} & 0 & 0 \\ \hline C_{b1}(\mu) & 0 & 0 \end{array}\right]. \quad (16)$$

It was proved that the approximation error is bounded by $\left\|\Gamma_P - \hat{\Gamma}_P\right\|_\infty \le 2\sum_{i=v+1}^{n} \sigma_i$.[12]

3. Contractive Coprime Factorization

In this section, we extend some of the results on coprime factorization of linear time invariant system to the FSTVD. For model reduction, we focus on the contractive coprime factorization which is analogous to the normalized coprime factorization of linear time invariant systems.

Definition 3 : Let Γ_P be a system given in (9). Γ_P admits a quadratically stable proper right coprime(left coprime) factorization if there exist quadratically stable systems Γ_X, Γ_Y, Γ_N, Γ_M ($\Gamma_{\tilde{X}}, \Gamma_{\tilde{Y}}, \Gamma_{\tilde{N}}, \Gamma_{\tilde{M}}$) such that $\Gamma_P = \Gamma_N \Gamma_M^{-1}$, $\Gamma_X\Gamma_N + \Gamma_Y\Gamma_M = I$ ($\Gamma_P = \Gamma_{\tilde{M}}^{-1}\Gamma_{\tilde{N}}$, $\Gamma_{\tilde{N}}\Gamma_{\tilde{X}} + \Gamma_{\tilde{M}}\Gamma_{\tilde{Y}} = I$). Moreover we say that the pair (Γ_N, Γ_M) (($\Gamma_{\tilde{N}}, \Gamma_{\tilde{M}}$)) is a contractive right coprime(left coprime) factorization if $\left\|\begin{matrix}\Gamma_N \\ \Gamma_M\end{matrix}\right\|_\infty \le 1$ ($\left\|\Gamma_{\tilde{N}} \;\; \Gamma_{\tilde{M}}\right\|_\infty \le 1$).

First, we obtain a coprime factorization of the FSTVD Γ_P.

Lemma 1 : Suppose that $K(\mu)$ and $L(\mu)$ are quadratically stabilizing state feedback and output injection matrix of the system Γ_P. Define

$$\begin{bmatrix} \Gamma_N & \Gamma_{\tilde{Y}} \\ \Gamma_M & \Gamma_{\tilde{X}} \end{bmatrix} := \left[\begin{array}{c|c|cc} A(\mu)+B(\mu)K(\mu) & F(\mu) & B(\mu) & -L(\mu) \\ \hline G & 0 & 0 & 0 \\ \hline C(\mu) & 0 & 0 & I_p \\ K(\mu) & 0 & I_m & 0 \end{array}\right],$$

$$\begin{bmatrix} \Gamma_X & \Gamma_Y \\ \Gamma_{\tilde{M}} & -\Gamma_{\tilde{N}} \end{bmatrix} := \left[\begin{array}{c|c|cc} A(\mu)+L(\mu)C(\mu) & F(\mu) & L(\mu) & -B(\mu) \\ \hline G & 0 & 0 & 0 \\ \hline K(\mu) & 0 & 0 & I_m \\ C(\mu) & 0 & I_p & 0 \end{array}\right]. \quad (17)$$

Then,

$$\begin{bmatrix} \Gamma_X & \Gamma_Y \\ \Gamma_{\tilde{M}} & -\Gamma_{\tilde{N}} \end{bmatrix}\begin{bmatrix} \Gamma_N & \Gamma_{\tilde{Y}} \\ \Gamma_M & \Gamma_{\tilde{X}} \end{bmatrix} = I. \quad (18)$$

Proof : The proof will be omitted due to lack of space.

The next theorem states that a contractive coprime factorization of the FSTVD Γ_P can be obtained using solutions of LMI's.

Theorem 2: Suppose that there exist $Z_1 = Z_1^T > 0$, $R_1 = R_1^T > 0$, $Z_2 = Z_2^T > 0$ and $R_2 = R_2^T > 0$ satisfying matrix inequalities (19) and (20) for all μ.

$$\begin{aligned} &Z_1A(\mu)^T + A(\mu)Z_1 - B(\mu)B(\mu)^T + \frac{1}{1-\beta}F(\mu)R_1F(\mu)^T \\ &+ Z_1G^TR_1^{-1}GZ_1 + Z_1C(\mu)^TC(\mu)Z_1 < 0, \end{aligned} \quad (19)$$

$$\begin{aligned} &A(\mu)^TZ_2 + Z_2A(\mu) - C(\mu)^TC(\mu) + G^TR_2G \\ &+ \frac{1}{1-\beta}Z_2F(\mu)R_2^{-1}F(\mu)^TZ_2 + Z_2B(\mu)B(\mu)^TZ_2 < 0. \end{aligned} \quad (20)$$

Then the pair (Γ_N, Γ_M) ($(\Gamma_{\tilde{N}}, \Gamma_{\tilde{M}})$) is a contractive right coprime(left coprime) factorization of Γ_P, where

$$\begin{bmatrix} \Gamma_N \\ \Gamma_M \end{bmatrix} := \left[\begin{array}{c|c|c} A(\mu)+B(\mu)K(\mu) & F(\mu) & B(\mu) \\ \hline G & 0 & 0 \\ \hline C(\mu) & 0 & 0 \\ K(\mu) & 0 & I_m \end{array}\right], \quad (21)$$

$$K(\mu) = -B(\mu)^T Z_1^{-1},$$

$$\begin{bmatrix} \Gamma_{\tilde{N}} & \Gamma_{\tilde{M}} \end{bmatrix} := \left[\begin{array}{c|c|cc} A(\mu)+L(\mu)C(\mu) & F(\mu) & B(\mu) & L(\mu) \\ \hline G & 0 & 0 & 0 \\ \hline C(\mu) & 0 & 0 & I_p \end{array}\right], \quad (22)$$

$$L(\mu) = -Z_2^{-1}C(\mu)^T.$$

Proof : We restrict ourselves to the contractive right coprime case. The proof for the left coprime case can be performed similarly.

With the definition $K(\mu)$ in (21) and by defining $P_1 = Z_1^{-1}$ and $S_1 = R_1^{-1}$, matrix inequality (19) is equivalent to the existence of $P_1 = P_1^T > 0$ and $S_1 = S_1^T > 0$ satisfying

$$(A(\mu)+B(\mu)K(\mu))^T P_1 + P_1(A(\mu)+B(\mu)K(\mu)) + C(\mu)^T C(\mu) + K(\mu)^T K(\mu) + G^T S_1 G + \frac{1}{1-\beta} P_1 F(\mu) S_1^{-1} F(\mu)^T P_1 < 0. \tag{23}$$

Thus, $K(\mu)$ is a quadratically stabilizing state feedback gain matrix. It is easy to show $\Gamma_P = \Gamma_N \Gamma_M^{-1}$, so will be omitted. For a quadratically stabilizing output injection matrix $L(\mu)$, define quadratically stable FSTVD Γ_X and Γ_Y as follows:

$$[\Gamma_X \quad \Gamma_Y] = \left[\begin{array}{c|c|cc} A(\mu)+L(\mu)C(\mu) & F(\mu) & L(\mu) & -B(\mu) \\ \hline G & 0 & 0 & 0 \\ \hline K(\mu) & 0 & 0 & I_m \end{array}\right]. \tag{24}$$

Then it is obvious that $\Gamma_X \Gamma_N + \Gamma_Y \Gamma_M = I$ from Lemma 1. Now it suffices to prove $\|\Gamma_R\|_\infty \le 1$, where

$$\Gamma_R = \begin{bmatrix} \Gamma_N \\ \Gamma_M \end{bmatrix} = \left[\begin{array}{c|c|c} A(\mu)+B(\mu)K(\mu) & F(\mu) & B(\mu) \\ \hline G & 0 & 0 \\ \hline C(\mu) & 0 & 0 \\ K(\mu) & 0 & I_m \end{array}\right]. \tag{25}$$

From definition 1 and using Schur complement, $\|\Gamma_R\|_\infty \le 1$ is equivalent to the existence of $X = X^T > 0$ and $S = S^T > 0$ satisfying

$$\begin{bmatrix} L_{11}(\mu) & * \\ B(\mu)^T X & -I_m \end{bmatrix} \le 0, \tag{26}$$

where

$$L_{11}(\mu) = (A(\mu)+B(\mu)K(\mu))^T X + X(A(\mu)+B(\mu)K(\mu)) + C(\mu)^T C(\mu) + K(\mu)^T K(\mu) + G^T S G + \frac{1}{1-\beta} X F(\mu) S^{-1} F(\mu)^T X.$$

We know that $X = P_1$, $S = S_1$ and $K(\mu) = -B(\mu)^T P_1$ solve matrix inequality (26). This completes the proof.

Remark 2: Z_1 and R_1 satisfying the matrix inequality (19) can be computed by solving following LMI (27). Z_2 and R_2 satisfying the matrix inequality (20) can also be computed similarly.

$$\begin{aligned} &\Phi_{ii} < 0, \text{ for all } i = 1,\cdots,r, \\ &\Phi_{ij} + \Phi_{ji} < 0, \text{ for all } i = 1,\cdots,r,\ j = i+1,\cdots,r, \end{aligned} \tag{27}$$

where,

$$\Phi_{ij} = \begin{bmatrix} Z_1 A_i^T + A_i Z_1 - B_i B_j^T & * & * & * \\ R_1 F_i^T & -(1-\beta)R_1 & * & * \\ G Z_1 & 0 & -R_1 & * \\ C_i Z_1 & 0 & 0 & -I_p \end{bmatrix}.$$

4. Fractional Balanced Model Reduction

Up to now, we defined a contractive coprime factorization. In this section, we present a balanced realization and a model reduction scheme in the contractive coprime factor of FSTVD Γ_P.

Generalized controllability gramian P and observability gramian Q of the right coprime factor Γ_R can be obtained as solutions satisfying LMI's (28) and (29).

$$L_c := \begin{bmatrix} L_{c1} & * & * \\ SF(\mu)^T & -(1-\beta)S & * \\ GP & 0 & -S \end{bmatrix} < 0, \tag{28}$$

where,

$$L_{c1} = P(A(\mu)+B(\mu)K(\mu))^T + (A(\mu)+B(\mu)K(\mu))P + B(\mu)B(\mu)^T.$$

$$L_o := \begin{bmatrix} L_{o1} & * & * \\ F(\mu)^T Q & -(1-\beta)R & * \\ RG & 0 & -R \end{bmatrix} < 0, \tag{29}$$

where,

$$L_{o1} = (A(\mu)+B(\mu)K(\mu))^T Q + Q(A(\mu)+B(\mu)K(\mu)) + C(\mu)^T C(\mu) + K(\mu)^T K(\mu).$$

The following lemma states that P,Q,R,S satisfying LMI's (28) and (29) can also be obtained from solutions of matrix inequalities (19) and (20).

Lemma 3 : Let Z_1, R_1, Z_2, R_2 be solutions of (19) and (20). Then, $Q = Z_1^{-1}$, $R = R_1^{-1}$, $P = (Z_1^{-1} + Z_2)^{-1}$ and $S = (R_1^{-1} + R_2)^{-1}$ solve LMI's (28) and (29).

Proof : The proof will be omitted due to lack of space.

As described in (7) and (8), we compute T and U using P,Q,R,S obtained from lemma 3. With transformation matrices T and U, define a balanced realization of the system Γ_R as

$$\Gamma_R = \left[\begin{array}{c|c|c} T^{-1}(A(\mu)+B(\mu)K(\mu))T & T^{-1}F(\mu)U & T^{-1}B(\mu) \\ \hline U^{-1}GT & 0 & 0 \\ \hline C(\mu)T & 0 & 0 \\ K(\mu)T & 0 & I_m \end{array}\right] =: \left[\begin{array}{cc|c|c} A_{b11}(\mu)+B_{b1}(\mu)K_{b1}(\mu) & A_{b12}(\mu)+B_{b1}(\mu)K_{b2}(\mu) & F_{b1}(\mu) & B_{b1}(\mu) \\ A_{b21}(\mu)+B_{b2}(\mu)K_{b1}(\mu) & A_{b22}(\mu)+B_{b2}(\mu)K_{b2}(\mu) & F_{b2}(\mu) & B_{b2}(\mu) \\ \hline G_{b1} & G_{b2} & 0 & 0 \\ \hline C_{b1}(\mu) & C_{b2}(\mu) & 0 & 0 \\ K_{b1}(\mu) & K_{b2}(\mu) & 0 & I_m \end{array}\right], \tag{30}$$

where $A_{b11}(\mu) \in R^{v \times v}$, $A_{b22}(\mu) \in R^{(n-v)\times(n-v)}$ and the other matrices are compatibly partitioned. Let $\hat{\Gamma}_R$ be the reduced order system with v states obtained by truncating Γ_R. $\hat{\Gamma}_R$ can be described as follows:

$$\hat{\Gamma}_R=\begin{bmatrix}\hat{\Gamma}_N\\ \hat{\Gamma}_M\end{bmatrix}=\left[\begin{array}{c:c|c}A_{b11}(\mu)+B_{b1}(\mu)K_{b1}(\mu) & F_{b1}(\mu) & B_{b1}(\mu)\\ \hdashline G_{b1} & 0 & 0\\ \hline C_{b1}(\mu) & 0 & 0\\ K_{b1}(\mu) & 0 & I_m\end{array}\right]. \quad (31)$$

Finally we associate $\hat{\Gamma}_R$ the following FSTVD $\hat{\Gamma}_P$ with v states.

$$\hat{\Gamma}_P=\left[\begin{array}{c:c|c}A_{b11}(\mu) & F_{b1}(\mu) & B_{b1}(\mu)\\ \hdashline G_{b1} & 0 & 0\\ \hline C_{b1}(\mu) & 0 & 0\end{array}\right] =\sum_{i=1}^{r}\mu_i\left[\begin{array}{c:c|c}A_{bi,11} & F_{bi,1} & B_{bi,1}\\ \hdashline G_{b1} & 0 & 0\\ \hline C_{bi,1} & 0 & 0\end{array}\right]. \quad (32)$$

From [12], the following lemma is immediate.

Lemma 4 : The reduced order $\hat{\Gamma}_R$ is quadratically stable and balanced. Moreover, the reduction error is bounded by $\left\|\Gamma_R-\hat{\Gamma}_R\right\|_\infty \le 2\sum_{i=v+1}^{n}\sigma_i$.

Remark 3: Suppose that $\hat{\Gamma}_K$ is a controller stabilizing the reduced order system $\hat{\Gamma}_P$. Thus $(I+\hat{\Gamma}_P\hat{\Gamma}_K)^{-1}\hat{\Gamma}_P\hat{\Gamma}_K$ is stable. By the small gain theorem, $(I+\Gamma_P\hat{\Gamma}_K)^{-1}\Gamma_P\hat{\Gamma}_K$ is also stable if and only if $\left\|\hat{\Gamma}_M^{-1}(I+\hat{\Gamma}_K\hat{\Gamma}_P)^{-1}\begin{bmatrix}\hat{\Gamma}_K & I\end{bmatrix}\right\|_\infty <$ $< 1/\left\|\Gamma_R-\hat{\Gamma}_R\right\|_\infty$ holds.

5. Numerical Example

We now describe a numerical algorithm for fractional model reduction. In order to get a less reduction error bound, it is necessary for $\sigma_{v+1},\cdots,\sigma_n$ to be small. Hence we choose a cost function as $J=trace(PQ)=\sum_{i=1}^{n}\sigma_i^2$. Thus, we will minimize the non-convex cost function subject to convex constraints. This optimization problem is very difficult to solve it. So, we suggest a suboptimal procedure using an iterative method. From the results of lemma 3, the cost function can be written as

$$J=trace(PQ)=trace(I+Z_1^{1/2}Z_2Z_1^{1/2})^{-1} \\ =trace(I+Z_2^{1/2}Z_1Z_2^{1/2})^{-1}. \quad (33)$$

step 1 : Set $i=0$. Initialize $Z_{2,i}$ as a positive definite matrix.
step 2 : Set $i=i+1$.

1) Minimize $J_i=trace(I+Z_{2,i-1}^{1/2}Z_{1,i}Z_{2,i-1}^{1/2})^{-1}$ subject to (19).
2) Minimize $J_i=trace(I+Z_{1,i}^{1/2}Z_{2,i}Z_{1,i}^{1/2})^{-1}$ subject to (20).

step 3 : If $|J_i-J_{i-1}|$ is less than a small tolerance level stop iteration, otherwise go to step 2.

We consider a FSTVD with 2 rules given by

$$A_1=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&-12&-3&-5\end{bmatrix},\ A_2=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&-14&-5&-6\end{bmatrix},$$
$$A_{d1}=\begin{bmatrix}0&0.1&0&0\\0&0&0.1&0\\0&0&0.1&0\\0&0&0&0.1\end{bmatrix},\ A_{d2}=\begin{bmatrix}0&0.1&0&0\\0&0&0.1&0\\0&0&0.2&0\\0&0&0&0.2\end{bmatrix},$$
$$B_1=B_2=\begin{bmatrix}0\\0\\0\\1\end{bmatrix},\ C_1^T=C_2^T=\begin{bmatrix}1\\0\\0\\0\end{bmatrix},$$

and $|\dot{d}(t)|\le 0.1$. For convenience, we choose $F_1=A_{d1}, F_2=A_{d2}, G=I$. It is easy to show that this fuzzy system is unstable. Accordingly the conventional balanced truncation method can not be applied to this system[12].

Using an initial estimate $Z_{2,0}=I$, we obtain the balanced gramian $\Sigma=diag(1,0.9584,0.9495,0.1961)$. By truncating the last state variable, we obtain the truncated right coprime factor $\hat{\Gamma}_R$ in (31) and the associated FSTVD $\hat{\Gamma}_P$ in (32), where

$$A_{b1,11}=\begin{bmatrix}0.047&37.82&27.51\\-0.002&-0.094&1.710\\0.0&-1.411&-0.057\end{bmatrix},$$
$$A_{b2,11}=\begin{bmatrix}0.047&-1.501&16.53\\-0.002&-0.417&1.619\\0.0&-1.583&-0.105\end{bmatrix}$$

$$F_{b1,1}=\begin{bmatrix}49.96&7.142&5.230&0.0\\0.007&0.321&-0.024&0.0\\-0.017&-0.185&-0.129&0.0\end{bmatrix},$$
$$F_{b2,1}=\begin{bmatrix}48.84&35.14&-1.850&0.0\\-0.004&0.546&-0.085&0.0\\-0.041&-0.117&-0.184&0.0\end{bmatrix},$$

$$B_{b1,1}=B_{b2,1}=\begin{bmatrix}-71.6\\-0.588\\-0.313\end{bmatrix},\ C_{b1,1}^T=C_{b2,1}^T=\begin{bmatrix}-0.001\\0.133\\-0.036\end{bmatrix},$$

$$G_{b1}=\begin{bmatrix}0.0006&0.404&-0.286\\-0.0001&0.454&0.144\\-0.0001&0.011&-0.013\\-0.0000&0.000&-0.0002\end{bmatrix}.$$

From lemma 4, we can expect that an upper bound of the reduction error in the right coprime factor is 0.3922.

6. Conclusion

In this paper, we have studied a fractional model reduction for a FSTVD. A contractive coprime factorization analogous to the normalized coprime factorization of the linear time invariant systems is derived by solving LMI's. Based on the contractive coprime factor, a balanced realization is obtained from generalized controllability and observability gramians which can be computed from solutions of LMI's. However, from a numerical example we observe that the upper bound of reduction error is conservative. The conservativeness may follow from the optimization problem which has the non-convex cost function. But we think that the proposed method may be suitable for model reduction of a class of FSTVD which are not quadratically stable.

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