論 文
7-1-5

Large Amplitude Oscillations in a Hanging Cable and Suspension Bridge: Some New Connections with Nonlinear Analysis
(케이블과 현수교 다리에서 일어나는 진폭이 큰 진동에 대한 연구)

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Abstract

The motions of suspension bridge as well as hanging cable are governed by nonlinear partial differential equations. Nonlinearity gives rise to a large amplitude oscillation. We use finite difference methods to compute periodic solutions to the torsional partial differential equations. We use the one-noded forcing term and a slight perturbation in the forcing term.

요약

케이블 뿐 아니라 현수교의 운동은 비선형 미분방정식에 지배된다. 미분방정식은 비선형성 때문에 진폭이 큰 해가 존재한다. 유한차분법을 이용하여 비선형 방정식의 주기근을 구한다. 일노드의 힘과 힘의 약간 변형된 형태를 이용하여 방정식의 해를 구한다.

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1. Introduction

If the science of mechanics has a classic movie, it might be the old film of the collapse of the Tacoma Narrows suspension bridge in 1940. Some people have seen that the collapse of the bridge followed the dramatic large-scale oscillation.

For over sixty years, scientists have attempted to explain the cause of the dramatic and destructive torsional oscillations of the Tacoma Narrows suspension bridge. We argue that nonlinear partial differential equations govern the motion of suspension bridges and that the nonlinearity gives rise to a large amplitude oscillation. Papers [2,4,5,6,7] provide a theoretical and numerical evidence for the vertical, torsional, and traveling wave motion of suspension bridge.

Earlier researcher considered a horizontal cross section of the center span of a suspension bridge and ordinary differential equation models for the torsional and coupled torsional and coupled torsional-vertical motions of the cross section.

We show partial differential equation models for the torsional and coupled vertical-torsional motions of the center span in section 2. The forced sine-Gordon equation on a bounded domain governs the torsional motion.

We use finite difference methods to compute periodic solutions to the torsional partial differential equations. As in [5], we demonstrate that under small external forcing, the center span may oscillate periodically with small or large amplitude, depending only on its initial displacement and velocity.

2. Description of the Model

We will treat the center span of the bridge as a beam of length L and width $2l$ suspended by cables. To model the motion of a horizontal cross section of the beam, we treat it as a rod of length $2l$ and mass m suspended by cables. Let $y(t)$ denote the downward distance of the center of gravity of the rod from the unloaded state and let $\Theta(t)$ denote the angle of the rod from horizontal at time t.

We will assume that the cables do not resist compression, but resist extension according to Hooke's law with spring constant K; i.e., the force exerted by the cable is proportional to the extension in the cable with constant K. The extension in the right hand cable is $(y-l\sin\Theta)$, and hence the force exerted by the right hand cable is

$-K(y-l\sin\Theta)^{+}$ where $u^{+}=\max\{u,0\}$.

i.e.,

$$\begin{cases} -K(y-l\sin\Theta) & y-l\sin\Theta\geq 0 \\ 0 & y-l\sin\Theta<0 \end{cases}.$$

Similarly, the extension in the left hand cable is $(y+l\sin\Theta)$ and the force exerted by the left hand cable is $-K(y+l\sin\Theta)^{+}$.

We have the same model as in [9].

$$\begin{aligned} &\theta_{tt}-\epsilon\theta_{xx}+\delta\theta_{t}=-2.4\cos\theta\sin\theta+\lambda f(x,t) \\ &\theta(0,t)=\theta(1,t)=0 \\ &\theta(x,0)=\xi(x),\theta_{t}(x,0)=\eta(x) \end{aligned}$$

Here, we choose $\varepsilon=\delta=0.01$ and external forcing of the form

$$f(x,t)=\sin(\mu t)$$
$$f(x,t)=\sin(\mu t)\sin(2\pi x)$$
or
$$f(x,t)=\sin(\mu t)\sin(\pi x).$$

3. Numerical Results

The experiments I do here are the continuation of [9]. This paper uses the different external forcing from [9]. We change nodal structure and observe the solution and use the one-noded forcing with different μ and λ. The solutions were observed after periods 390 through 400 of the forcing term in order to attempt to avoid transient behaviors.

To determine the physical constants $K, m, l, L, \delta_1, \delta_2, \varepsilon_1, \varepsilon_2$, and the external forcing terms $\bar{f}(t)$ and $f(x,t)$, we depend on [1],[5], and [10]. We choose $L=1000, l=6, m=2500, \delta_1=\delta_2=0.01, K=1000, \epsilon_1=0.01$, and $\varepsilon_2=0.0001$.

3.1 One-noded forcing and initial conditions

Experiment 1. In this experiment, we use $\lambda=0.02, \mu=1.3$. We apply forcing of the form

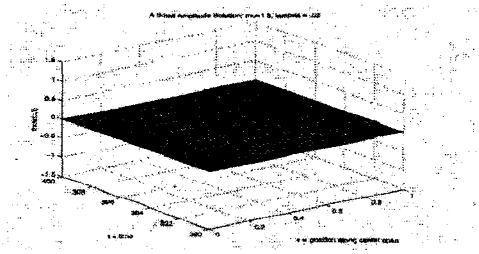
$$\lambda f(x,t)=\lambda\sin\mu t\sin\pi x$$

1a. $\Theta(x,0)=\Theta(x,\Delta t)=1.2\sin\pi x$

Despite the large initial displacement, we see in [Fig. 1] that the bridge has settled down to no-noded, periodic oscillation of small amplitude (approximately 0.0248 radians).

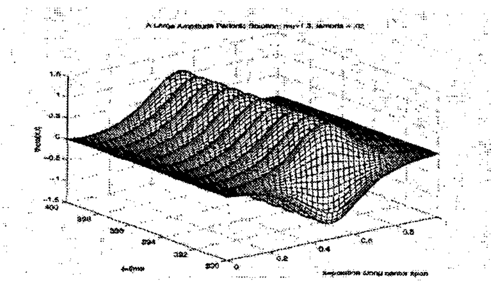
1b. $\Theta(x,0)=\Theta(x,\Delta t)=1.1\sin\pi x$

We decreased the amplitude of the initial displacement only slightly different from 1a, but we see in [Fig. 2] that this small change has a dramatic impact on the motion of the bridge. As in 1a, the bridge has settled down to periodic oscillation. But instead of settling to near equilibrium behavior, as in 1a, the amplitude of the oscillation is approximately 1.171 radians. This is close to the amplitude observed at the Tacoma Narrows suspension bridge on the day of the collapse, [1].



[Fig. 1] Experiment 1a: A Small Amplitude Solution at $\mu=1.3, \lambda=0.02$.

[그림 1] 실험 1a: $\mu=1.3, \lambda=0.02$ 에서 작은 진폭의 근



[Fig. 2] Experiment 1b: A Large Amplitude Periodic Solution at $\mu=1.3, \lambda=0.02$.

[그림 2] 실험 1b: $\mu=1.3, \lambda=0.02$ 에서 큰 진폭의 주기근

3.2 Solutions which change nodal structure

The torsional oscillations which preceded the collapse of the Tacoma Narrows suspension bridge were one-noded. Occasionally, the motion would change to be no-noded and then back to be one-noded. In this experiment, we use forcing of the form

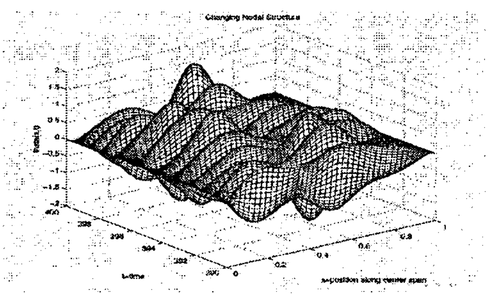
$$\lambda f(x,t) = \lambda \sin \mu t [\sin(2\pi x) + 0.01 \sin(\pi x)]$$

Experiment 2. We use $\lambda = 0.06, \mu = 1.4$. We use the following large initial displacement.

$$\theta(x,t) = \theta(x, \Delta t) = 1.4 \sin 2\pi x$$

We see a complicated motion in [Fig. 3]. The parameters are the same as those of [9]. Different forcing term and displacement give the different result from [9].

[Fig. 4] shows the angular displacement along the length of the bridge at two different points in time; the solid curve describes a one-noded twist while the dashed curve has no node.



[Fig. 3] Experiment 2a: A Small Amplitude Solution at $\mu=1.4$, $\lambda=0.06$.

[그림 3] 실험 2: $\mu=1.4$, $\lambda=0.06$ 에서 큰 진폭의 근

4. Conclusion

All numerical results above were only approximately solved by a finite difference method. From the various numerical experiments, there are rich phenomena associated with the oscillation of the Tacoma Narrows suspension bridge and hanging cable.

In this paper, we noticed the following mathematical phenomena.

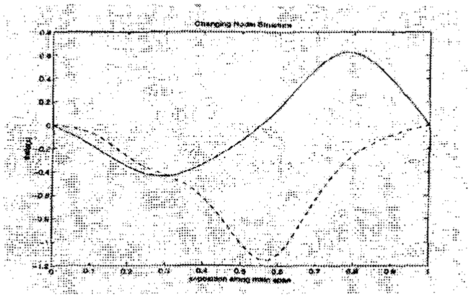
1. The suspension bridge and hanging cable have the large scale oscillation.

2. The suspension bridge and cable would go into large oscillation under the impulse of a single gust, and at other times would remain motionless.

3. The motions would change rapidly from one nodal type to another.

4. The large vertical oscillation could rapidly change to torsional.

Later, this research has to be investigated for higher frequency forcing. Further investigation will include the result for the coupled vertical-torsional motion of the main span of a suspension bridge.



[Fig. 4] Experiment 2: The angular displacement at $\mu=1.4$, $\lambda=0.06$.

[그림 4] 실험 2b: $\mu=1.4$, $\lambda=0.06$ 에서 변위

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