가로 등방성 복합재료의 초음파에 관한 연구

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The Wave Propagation in Transversely Isotropic Composite Laminates

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ABSTRACT

In transversely isotropic composite laminates, the velocities, the particle directions and the amplitudes of reflected and transmitted waves were obtained using the equation of motion, the constitutive equation, and the displacement equation expressed by wave number and frequency. Eigenvalue problem involving a velocity was solved by Snell's law. Finally, the results were confirmed by T300 Carbon fiber/5208 Epoxy materials. This approach could be applied to the detection of flaws in transversely isotropic composite laminates by the water immersion C-scan procedure.

초 록

가로 등방성 복합재료에서 반사되거나 굴절된 파동의 속도와 입자방향, 그리고 진폭을 운동방정식과 구성방정식 그리고 파동수와 진동수로 표현된 변위식을 사용하여 구하였다. Snell 법칙을 사용하여 Eigenvalue 문제를 풀어 파동속도를 구하였으며 그 결과는 T300 Carbon fiber/5208 Epoxy 재료 성질을 이용하여 검증하였다. 이러한 분석은 수분 침수 C-scan을 이용하여 가로등방성 복합재료의 결점을 찾아내는데 응용될 수 있다.

Key Words: Wave Propagation, transversely isotropic, laminates, reflected, refracted, transmitted, wave number, Snell's law, immersion, C-scan

1. Introduction

Immersion C-scan procedure has become the method of identifying gross composite defects by an analysis of the reflected wave amplitudes. This procedure works well in many cases. However, one is concerned with more subtle defects which are difficult to identify with conventional data analysis procedures (e.g., porosity, local variation in fiber orientation, segregation of reinforcing

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fibers, etc.). Since the defects will principally affect the local moduli, ultrasonic velocity measurements are quite useful in analysing these types of problems. Further, since fiber reinforced composites are anisotropic materials, one would ideally like to examine directional dependence on the properties by measuring all pertinent elastic moduli. Previous investigators have performed the ultrasonic tests required to completely characterize all nine elastic moduli for orthotropic materials [1-2]. However, this procedure is rarely used in practice for several important reasons. These tests were all conducted using contact transducers at normal incidence. This approach, while useful for measuring at a given point, is unsuitable for the scanning of large parts due to the difficulty of maintaining shear coupling as the transducer is scanned. An alternative approach is to use immersion transducer with mode conversion to generate the required waves additional to the longitudinal wave. Unfortunately, the mode conversion approach to the generation of waves in an anisotropic media is significantly more complicated than the isotropic case [3]. However, a considerable degree of simplification can be achieved by restricting to the special case of a unidirectional transversely isotropic composite materials. The principal objective of this work was to develop a simplified method for analyzing reflection-refraction phenomena in transversely isotropic materials for arbitrary angles of incidence. Finally, a method was determined which was rapid, accurate and sufficiently compact to be implemented on a laboratory microcomputer so that it would be useful for the detection of flaws. The results were confirmed by T300 Carbon fiber/5208 Epoxy materials.

2. Theory

In an anisotropic media, the equation of motion is described as follows.

$$\sigma_{ij,j} + \rho b_i = \rho \ddot{u_i} \qquad (1)$$

where, σ=stress, u=displacement
ρ=density, b=body force.

The constitutive equation is

$$\sigma_{ij} = c_{ijkl}\epsilon_{kl} \qquad (2)$$

where, c=stiffness matrix
ϵ=strain.

And the strain and the displacement relatinship is

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad (3)$$

Substituting equations (2) and (3) into equation (1) without body force gives the following equation.

$$\rho \ddot{u_i} = c_{ijkl}u_{k,ij} \qquad (4)$$

For the cases to be studied, the wave vector for the incident wave lies in a plane either parallel or perpendicular to the fiber reinforcement. The incident wave may be represented as

$$u_i = A_0\alpha_i e^{i(kl_jx_j - \omega t)} \qquad (5)$$

where, k=wave number
l=wave normal vector
ω=fraquency
A_0=amplitude
α=polarization vector

Substituting eq. (5) into eq. (4) gives as

$$-\rho\omega^2 u_i = c_{ijkl}(-k^2 l_l l_j)u_k \tag{6}$$

Rearranging eq. (6) gives the governing equation as

$$(\underline{\lambda} - \rho v^2 \underline{I})\underline{\alpha} = 0 \tag{7}$$

where, I=identity matrix
v=velocity
$\lambda_{ik} = c_{ijkl} l_j l_l$.

3. Application to Orthotropic Medium

The wave normal for the refracted waves in the orthotropic media is

$$\underline{l} = \begin{pmatrix} \cos\theta^i \\ 0 \\ \sin\theta^i \end{pmatrix}$$

Once the wave normal vector is found, $\underline{\lambda}$ can be evaluated as

$$\underline{\lambda} = \begin{pmatrix} c_{11}\cos^2\theta^i + c_{55}\sin^2\theta^i & 0 & c_{13}\sin\theta^i\cos\theta^i + c_{55}\sin\theta^i\cos\theta^i \\ 0 & c_{66}\cos^2\theta^i + c_{44}\sin^2\theta^i & 0 \\ c_{13}\sin\theta^i\cos\theta^i + c_{55}\sin\theta^i\cos\theta^i & 0 & c_{55}\cos^2\theta^i + c_{33}\sin^2\theta^i \end{pmatrix} \tag{8}$$

In solving the eigenvalue problem, the most difficult problem is that the directional cosines of the refracted wave can not be determined from Snell's law because of the directional dependence of the wave velocities.

Defining the slowness vector as

$$\underline{m} = \frac{1}{\omega}\underline{k} = \frac{1}{v}\underline{l}$$

where, k=wave vector=$|k|\,l$

$k = |\underline{k}|$=wave number

$\underline{l}$ =wave normal.

The slowness surface represents the locus of the endpoints of the slownes vectors. For as anisotropic media, there are three distinct sheets of arbitrary shape. The shape of slowness surface is an important factor in determining the nature of reflected waves and refracted waves. The problem under consideration consists of a plane longitudinal wave in water incident upon the boundary of a unidirectional composite panel. For the cases to be studied, the wave vector for the incident wave lies in a plane either parallel or perpendicular to the fiber reinforcement.

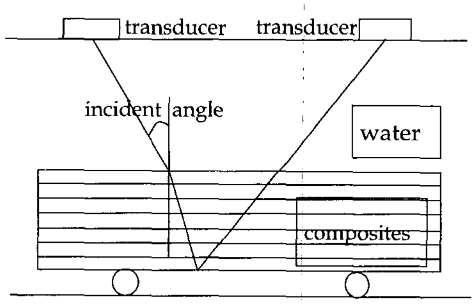

Fig. 1 Experimental arrangement of the oblique wave in the water

With this geometry, the incident wave may be represented as

$$\tilde{u}_{in} = \tilde{A} e^{i\omega^{in}(m_k^{in} x_k - t)}$$

Similarly, the reflected longitudinal wave can be represented as

$$\tilde{u}_{re} = \tilde{A}_{re} e^{i\omega^{re}(m_k^{re} x_k - t)}$$

For the transmitted waves, we have

$$\tilde{u}^i = \tilde{A}_i e^{i\omega^i (m^i_k x_k - t)}$$

where, the superscript i is used to differentiate between the transmitted work. In order to satisfy the boundary conditions at the interface, the frequencies of all waves must be equal, i.e

$$\omega = \omega^{in} = \omega^{re} = \omega^i$$

and

$$m^{in}_k x_k = m^{re}_k x_k = m^i_k x_k \tag{9}$$

which is equivalent to

$$a_i = \epsilon_{ijk} m^{in}_j \nu_k = \epsilon_{ijk} m^{re}_j \nu_k = \epsilon_{ijk} m^i_j \nu_k$$

where,

$\tilde{\nu}$ = normal to interface

$\tilde{a}$ = constant vector quantity.

In the case that

$$\tilde{\nu} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

and,

$$\tilde{m}^{in} = \frac{1}{v_w} \begin{pmatrix} \cos\theta^{in} \\ 0 \\ \sin\theta^{in} \end{pmatrix}$$

$$\tilde{m}^{re} = \frac{1}{v_w} \begin{pmatrix} -\cos\theta^{re} \\ 0 \\ \sin\theta^{re} \end{pmatrix}$$

$$\tilde{m}^{i} = \frac{1}{v_w} \begin{pmatrix} \cos\theta^{i} \\ 0 \\ \sin\theta^{i} \end{pmatrix}$$

where, v_w is the velocity in the water and the negative sign in the slowness vector of the reflected wave indicates that it is propagating away from the interface, then we have the Snell's law as

$$\frac{\sin\theta^{in}}{v_w} = \frac{\sin\theta^{re}}{v_w} = \frac{\sin\theta^i}{v_i} \tag{10}$$

4. Eigenvalue Problem

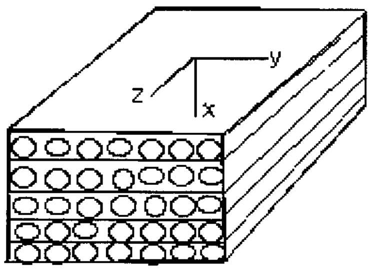

Fig. 2 Coordinate system in unidirectional reinforced

We can rewrite the Snell's law as

$$\sin\theta^i = \frac{v_i}{v_w} sin\theta^{in} = kv_i$$

for each mode, then eq. (8) becomes the Eigenvalue problem for v_i. Since the pure mode shear wave could not be excited in the experimental arrangement, we restrict our attention to the characteristic equation of remaining two waves as

$$\begin{pmatrix}[c_{11}(1-(kv_i)^2)+c_{55}(kv_i)^2-\rho v_i^2]\\ [c_{55}(1-(kv_i)^2)+c_{33}(kv_i)^2-\rho v_i^2]\end{pmatrix}$$
$$-(c_{13}+c_{55})^2(kv_i)^2(1-(kv_i)^2)=0$$

This is a simple equation for v_i^2 which may be solved numerically for the two real roots.

5. Amplitude considerations

The particle displacements for the incident wave are represented as

$$\tilde{u}_{in}=A_0\begin{pmatrix}\cos\theta^{in}\\ 0\\ \sin\theta^{in}\end{pmatrix}e^{ik^L}(\cos\theta^{in}x+\sin\theta^{in}z-\omega t)$$

Similarly, the particle displacements for the reflected wave are

$$\tilde{u}_{re}=A_r\begin{pmatrix}\cos\theta^{re}\\ 0\\ \sin\theta^{re}\end{pmatrix}e^{ik^L}(-\cos\theta^{re}x+\sin\theta^{re}z-\omega t)$$

For wave propagation in the composite, similar expressions for the generated quasilongitudinal and quasitransverse waves are

$$\tilde{u}_{QL}=A_{QL}\begin{pmatrix}\alpha_1^{QL}\\ 0\\ \alpha_3^{QL}\end{pmatrix}e^{ik_{QL}}(\cos\theta^{QL}x+\sin\theta^{QL}z-\omega t)$$

$$\tilde{u}_{QT}=A_{QT}\begin{pmatrix}\alpha_1^{QT}\\ 0\\ \alpha_3^{QT}\end{pmatrix}e^{ik_{QT}}(\cos\theta^{QT}x+\sin\theta^{QT}z-\omega t)$$

Here, the eigenvectors $\tilde{\alpha}^{QL}$ and $\tilde{\alpha}^{QT}$ are perpendicular to one another, but in general $\tilde{\alpha}^{QL}*\tilde{l}^{QL}\neq 1$ and $\tilde{\alpha}^{QT}*\tilde{l}^{QT}\neq 0$. Therefore, they are called as quasilongitudinal and quasitransverse waves. Three boundary conditions at the fluid solid interface are required to calculate the reflection and transmission coefficients at the interface.

(1) Continuity of normal displacement

$$u_{water}|_{x=0}=u_{composite}|_{x=0}$$

which leads to an expression of Snell's law for the composite as before as well as the relationship

$$A_0\cos\theta^{in}-A_r\cos\theta^{re}=A_{QL}\alpha_1^{QL}+A_{QT}\alpha_1^{QT}$$

(2) Continuity of normal stress

$$\sigma_{11(water)}|_{x=0}=\sigma_{11(composite)}|_{x=0}$$

which becomes as

$$\lambda k_L[(\cos^2\theta^{in}+\sin^2\theta^{in})A_0+(\cos^2\theta^{re}+\sin^2\theta^{re})A_r$$
$$=c_{11}[A_{QL}\alpha_1^{QL}k_{QL}\cos\theta^{QL}+A_{QT}\alpha_1^{QT}k_{QT}\cos\theta^{QT}]$$
$$+c_{13}[A_{QL}\alpha_3^{QL}k_{QL}\cos\theta^{QL}+A_{QT}\alpha_3^{QT}k_{QT}\cos\theta^{QT}]]$$

(3) Zero transverse stress
Since the fluid can not support a shear stress,

$$\sigma_{13(water)}|_{x=0}=\sigma_{13(composite)}|_{x=0}$$

which becomes as

$$A_{QL}k_{QL}(\alpha_1^{QL}\sin\theta^{QL}+\alpha_3^{QL}\cos\theta^{QL})$$
$$+A_{QT}k_{QT}(\alpha_1^{QT}\sin\theta^{QT}+\alpha_3^{QT}\cos\theta^{QT})=0$$

Those yields three equations in three unknowns which can be solved for the reflection and transmission coefficients $(R=\frac{A_r}{A_0},\ T_{QL}=\frac{A_{QL}}{A_0},\ \ T_{QT}=\frac{A_{QT}}{A_0})$.

6. Results considerations

T300 Carbon fiber/5208 Epoxy composite was studied for wave propagation at oblique angles incidence. The stiffness coefficients as

$$\tilde{c}=\begin{bmatrix}11.2 & 5 & 7.1 & 0 & 0 & 0\\ 5 & 11.2 & 7.1 & 0 & 0 & 0\\ 7.1 & 7.1 & 153 & 0 & 0 & 0\\ 0 & 0 & 0 & 7.1 & 0 & 0\\ 0 & 0 & 0 & 0 & 7.1 & 0\\ 0 & 0 & 0 & 0 & 0 & 3.1\end{bmatrix}*10^9\frac{kg}{m^2}$$

and the density $\rho=1.8\frac{g}{cm^3}$.

The oblique incident beam was projected to a unidirectionally reinforced composites in the reinforcement plane parallel and perpendicular to the fiber axis.

A) x-y plane (isotropic plane)

(1) longitudinal wave slowness radius:

$$R_1=\frac{1}{v_1}=\sqrt{\frac{\rho}{c_{11}}}=0.4mm$$

(2) SV slowness radius:

$$R_{SV}=\frac{1}{v_2}=\sqrt{\frac{\rho}{c_{44}}}=0.5mm$$

(3) SH slowness radius:

$$R_{SH}=\frac{1}{v_3}=\sqrt{\frac{\rho}{c_{66}}}=0.76mm$$

B) x-z plane

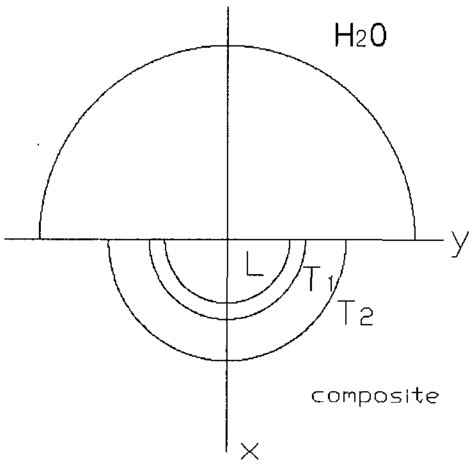

Fig. 3 The slowness surface (transversely isotropic plane)

(1) S-H wave

$$R_{SH}(\theta)=\frac{1}{v_{SH}}=\sqrt{\frac{\rho}{c_{44}\sin^2\theta+c_{66}\cos^2\theta}}$$
$$=\sqrt{\frac{\rho}{7.1\sin^2\theta+3.1\cos^2\theta}}*10^{-3}mm$$

(2) quasilongitudinal velocity and quasitransverse velocity

$$R_{QL}(\theta)=\frac{1}{v_{QL}}=\sqrt{\frac{2\rho}{M+\sqrt{M^2-4N}}}$$

$$R_{QT}(\theta)=\frac{1}{v_{QT}}=\sqrt{\frac{2\rho}{M-\sqrt{M^2-4N}}}$$

where,

$$M = c_{11}\cos^2\theta + c_{33}\sin^2\theta + C_{44}$$

$$N = (c_{11}\cos^2\theta + c_{44}\sin^2\theta)(c_{11}\cos^2\theta + c_{44}\sin^2\theta) - (c_{13} + c_{44})^2\sin^2\theta\cos^2\theta$$

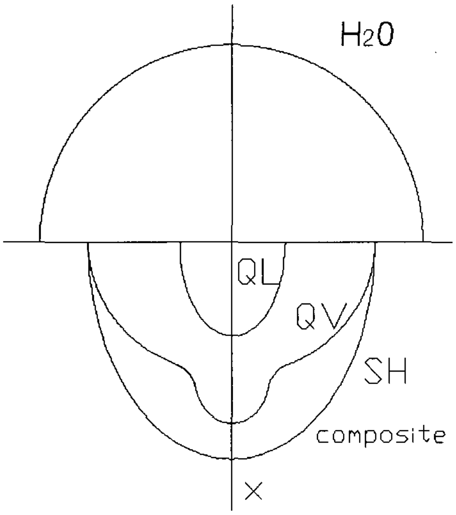

Fig. 4 The slowness surface (parallel to fiber)

To apply the above result, polymethyl methacrylate (PMMA) was used. It was assumed that the water was not dispersive ($\rho = 0.997g/cm^2, c = 1480m/s$). The density of PMMA was 1.191g/cm^3. Sample size was 2x2 in. with the thickness ranging from 0.1 to 1.0 in. Date acquisition was achieved using a Biomation 8100 transient digitizer to convert the analog transducer output into digital form and a minicomputer to store the data. The transient digitizer is capable of sample rates as high as 100 MHz with a horizontal resolution of 2048 points and a vertical resolution of 256 points. Using longitudinal wave slowness radius, plot of phase velocity versus frequency was presented in Fig. 5.

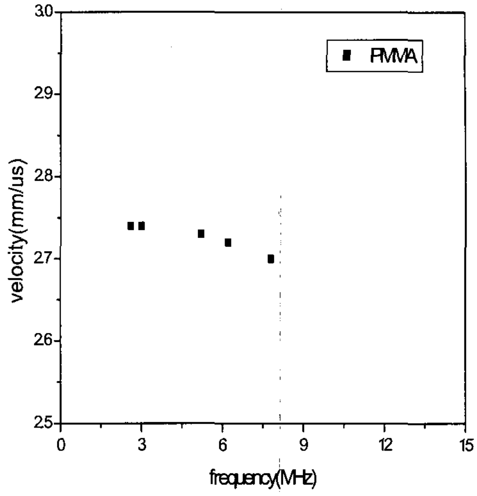

Fig. 5 Velocity versus frequency(PMMA)

7. Conclusions

Wave propagation features in the transversely isotropic materials were evaluated for a microcomputer based technique. Using oblique angles of incidence, important informations (i. g. the velocities, the particle directions and the amplitudes of reflected and transmitted waves) about laminate properties could be obtained. This approach may be used in a scanning mode to detect local flaws of a big composite motor case if the shell effect does not play an important role.

References

1. Kline, R. A. "The Origin of Localized Weak in SMC Composites," Materials Evaluation, Vol. 40, 1982, pp.847-879
2. Kriz, R. D., and W. W. Stinchcomb, "Elastic Moduli of Transversely Isotropic Graphite Fibers and Their Composites,"

Experimental Mechanics, Vol. 19, 1979, pp. 41-49

3. Hyungwon Kim, and Youngkyu Choi, "The Study on Axisymmetric Deformation of Thin Orthotropic Composite Pressure Vessel," J. of KSPE, Vol. 7, 2003, pp.36-43
4. Zimmer, J. E., and J. R. Cost., "Determination of the Elastic Constants of Unidirectional Fiber Composites Using Ultrasonic Velocity Measurements," J. Acoustic. Soc. Am, Vol. 41, 1970, pp.795-799
5. Dean, G. D., and P. Turner, "The Elastic Properties of Carbon Fibers and Their Composites," Composites, Vol.4, 1973, pp.174-180