

Situated Theory and Two Kinds of Mathematics Instructional Beliefs of Teachers

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(Received July 4, 2005 and, in revised form, February 10, 2006)

The mathematics instructional beliefs of a teacher include the exterior mathematics instructional beliefs and the internal mathematics instructional beliefs. These two kinds of beliefs are formed in two kinds of different situations. The situated theory thinks that beliefs are related with the situations; so, the two kinds of beliefs are showed in the different situations. The internal instructional mathematics beliefs effect on the actual mathematics instruction, they ought to be noticeable.

Keywords: situated theory; mathematics instruction; belief; teachers

ZDM Classification: C60

MSC2000 Classification: 97C60

INTRODUCTION

The mathematics instructional beliefs of a teacher are teacher' subjective consciousness about the related phenomena of mathematics teaching and learning they can be fallen into the mathematics beliefs and the mathematics teaching and learning beliefs. Here, beliefs include views, conceptions, and attitudes. Many researches (Gonzales Thompson 1984, 1985, 1997, 2004; Hersh 1986; Cobb, Wood & Yackle 1991; etc.) related to mathematics instructional beliefs have been done, and you can see them in many papers and works, so I will not introduce them here. Different from those researches, in this paper, I will use the situated theory as the foundation to research the mathematics instructional beliefs of teacher.

THE SITUATED THEORY

The situated theory is a general learning theory, and its content is very rich. The following is part introduction of it.

Some ideas in the situated theory could be found in Dewey (1902), Piaget (1932), and Vygotsky, Cole, John-Steiner & Scribner (2006). As a definite and specific theory, the situated theory was put forward by Lave (1989) firstly. Afterward, because other scholars, working (Lave & Wenger 1991; Brown, Collins & Duguid 1989), the situated theory finally was formed.

In the situated theory, there are three important concepts that are *communities of practice*, *legitimate peripheral participation* and *knowledge's situation*. These concepts, in fact, embody the situated theory. Briefly, *communities of practices* thinks that people's actions (such as learning) occur in some community, and community's existent purpose is practice actions; *legitimate peripheral participation* expresses that people's progress is from newer to expert; *knowledge's situation* thinks that knowledge is related with special situation. These three concepts describe knowledge and people's learning.

In this paper, *knowledge's situation* is focused on. The situated theory thinks that learners must be put at a definite situation. In other words, knowledge and skills' study must be in a definite situation.

- To situate learning means to place thought and action in a specific place and time. To situate means to involve other learners, the *environment*, and the activities to create meaning. (Lave & Wenger 1991)
- The notion of learning knowledge and skills in contexts that reflect the way the knowledge will be useful in real life. (Collins 1989)

What is a situation? A situation is a certain context, an environment or a background. It reflects how knowledge is obtained and how knowledge is used. The situated theory thinks that any knowledge and skill is in a certain situation, knowledge and skills are related with situation. It is even said that situation is a part of knowledge, and neutral or no situated knowledge is nonexistent. Because knowledge's situated, it is easy to understand why learners must be put a certain situation, and it is easy to understand why the knowledge that is learned in a certain situation cannot be move to another situation easily.

People often pay attention to knowledge, skills and behaviors in the situated theory, but neglect ideas and beliefs' situated. But in this paper, the beliefs in the situated theory are focused. In fact, the researchers of the situated theory have mentioned it in their works. For example,

- To situate means to locate in a particular setting the *thinking* and doing processes used by experts to accomplish knowledge and skill tasks. (Lave & Wenger 1991)
- The theory of situated learning claims that every *idea* and human action is a generalization, adapted to the ongoing environment, because what people see and what they do arise together. (Clancey 1995)

Therefore, according to the situated theory, knowledge, skills and beliefs are related to situation. When we talk about the knowledge, skills, and beliefs, we should think about situations at a same time. We must realize definitely that knowledge, skills, and beliefs all have themselves situations. If the situation changes, the knowledge, the skills, and beliefs also change.

A people, as a middle school student, he/she has one view of mathematics and mathematics teaching and learning; when he/she becomes a normal school student, he/she has another view of mathematics and mathematics teaching and learning; when he/she is a mathematics teacher, he/she has some other view of mathematics and mathematics teaching and learning. When situation changes, the view of a mathematics' teacher's mathematics and mathematics teaching and learning change. It is what the situated theory tells us.

PUTTING FORWARD QUESTIONS

The author (*cf.* Zhang 2003, 2005) lectured to mathematics teachers in the summer and winter holidays for several years. The contents of lectures were the mathematics philosophy, the mathematics education philosophy, and mathematics education theories. All these courses include mathematics instructional beliefs. After lectures, I always want teachers tell me orally or in writing what their mathematics instructional beliefs are. Their answers accord with my lectures that is consistent with current mathematics education reform. So I satisfy with both my lectures and teachers' learning. But when I visited the classrooms of some teachers that attended my lectures, their behaviors in classrooms told me that the mathematics instructional beliefs they hold are different with their answers to me.

I have another example. The TIMSS' project of the videotape classroom study (Stigler Gonzales, Kawanaka, Knoll & Serrano 1999) was to explore the eighth-grade mathematics instruction in German, Japan, and the United States. In the research of the United States' mathematics instruction, the researches firstly asked the teachers their mathematics instructional beliefs; the major of them (96%) said their beliefs were consistent with the demand of National Council of Teachers of Mathematics (NCTM) standards (either the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) or the *Professional Standards for Teaching Mathematics* (NCTM 1991)); But after analyzing the videotapes of the mathematics classrooms, the researches found that the behaviors of the teachers in classrooms reflected their mathematics instructional beliefs differ completely with the demands of the standards.

These two examples tell us it is seem that there are two different mathematics

instructional beliefs. Then, how do these two different beliefs form? Where do teachers put up the two different beliefs? Why? And how these two different beliefs work on teacher's teaching practice in the classrooms?

TWO KINDS OF MATHEMATICS INSTRUCTIONAL BELIEFS AND THEIR FORMING

I believe that there are two kinds of mathematics instructional beliefs. One is conscious and the other is unconscious. In the examples above, what NCTM's standards and I told teachers became their conscious mathematics instructional beliefs, and teachers have themselves unconscious mathematics instructional beliefs. In fact, some researchers had taken note of this phenomenon.

- Beliefs – to be interpreted as an individual's understandings and **feelings** that shape the ways that the individual conceptualizes and engages in mathematical behavior... (Schoenfeld 1992, p. 358)
- A teacher's conceptions of the nature of mathematics may be viewed as that teacher's conscious or **subconscious** beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline of mathematics (Thompson 1992, p. 132)

For the sake of the study's convenience, we introduce two conceptions: the exterior mathematics instructional beliefs and the internal mathematics instructional beliefs. Of course, the exterior mathematics instructional beliefs include the exterior mathematics beliefs and the exterior mathematics teaching and learning beliefs, and the internal mathematics instructional beliefs include the internal mathematics beliefs and the internal mathematics teaching and learning beliefs.

1. Exterior and internal mathematics beliefs

We notice a fact, which is that a teacher holds some mathematics beliefs when he/she is being interviewed or talking to class about mathematics, such mathematics beliefs maybe see mathematics as dynamic and fallible. But he/she holds another mathematics belief when he/she in actual mathematics instructional activities, such mathematics beliefs may see mathematics as absolute truth. Therefore, we bring forward two kinds of mathematics beliefs: the internal mathematics beliefs and the exterior mathematics beliefs.

The exterior mathematics beliefs are a person (for example, a mathematics teacher) tell other people orally or in paper his/her mathematics beliefs.

In public, especially in formal occasion, he/she tells others that how he/she see

mathematics. The exterior mathematics beliefs of a teacher often are consistent with the mainstream viewpoints of current mathematics philosophy. For example, the modern mathematics philosophy sees mathematics as non-absolute truth, dynamic and fallible, etc. If you now interview a mathematics teacher, what he/she tells you is his/her exterior mathematics beliefs.

The internal mathematics beliefs are basic viewpoints about mathematics that are held in his/her heart.

Usually, the exterior mathematics beliefs of a person can be expressed clearly, but the internal mathematics beliefs even cannot be expressed clearly. That is to say, when you ask someone what his/her real mathematics beliefs are he/her often cannot answer your questions. To know someone's internal mathematics beliefs, you must observe and analyze his/her daily (not once or twice) behaviors in mathematics classroom. Obviously, the internal mathematics beliefs are more important than the exterior mathematics beliefs, because the internal mathematics beliefs decide the behaviors of a mathematics teacher, and the exterior mathematics beliefs affect hardly on the behaviors of the teacher.

The exterior mathematics beliefs of a teacher are from two sources, one is from the colleagues and teachers, including universities' mathematics teachers, and in-service education teachers, the other is from some papers and works. In fact, teacher gets the exterior mathematics beliefs easily, because he/her can get it in some lesson or a paper. The exterior mathematics beliefs change easily, it can be changed in short time. For example, after a teacher reads a paper about mathematics nature, he/she thinks the ideas of writer of this paper are more reasonable than that he/she now holds, and then he/she may change his old exterior mathematics beliefs to new ones.

The internal mathematics beliefs are produced by socialization, the process is slow and long. It is mainly formed through the practice activities of mathematics learning of a teacher; the practice exerts a subtle influence on him/her in a long period. The forming process is stage by stage and the teacher forms it unconsciously. It is can be said that the internal mathematics beliefs of a teacher are formed in the process of mathematics learning from kindergarten to university. For example, if all mathematics problems that a teacher solved were routine or procedural, and then he/she may have such a belief that is all mathematics problems can be solved by using routine methods.

If his/her mathematics learning was always under infusion teaching methods, then he/she may have such a belief, which is that mathematics is innate, and so on. Because of long forming process (some scholars use "thousands of hours' apprentice observation" (Lortie 1975) to describe such process), the internal mathematics beliefs of teachers are hard to be changed.

The internal mathematics beliefs of a teacher hide in his/her heart. Maybe he/she

cannot express it clearly, but it is strong and powerful. We can even use “bigotry” to describe it. It can manifest itself by a teacher’s behaviors in mathematics classroom, and the teacher is unconscious to one self’s internal mathematics beliefs. The internal mathematics beliefs of a teacher maybe are consistent with his/her exterior mathematics beliefs but both maybe consist partly or differ completely.

2. Exterior and internal mathematics teaching and learning beliefs

The mathematics teaching and learning beliefs are a person’s basic perspectives about mathematics teaching and learning and related phenomena. Like mathematics beliefs, in order to understand wholly a teacher’s mathematics teaching and learning beliefs, we must bring forward the exterior mathematics teaching and learning beliefs and the internal mathematics teaching and learning beliefs.

The mathematics teaching and learning beliefs a teacher claims orally or in writing are his/her exterior mathematics teaching and learning, and the ones the teacher holds in the heart and are manifested through his/her behaviors in classroom are his/her internal mathematics teaching and learning beliefs.

Like the exterior mathematics beliefs, the exterior mathematics teaching and learning beliefs forms easily, it may be learned in teacher education, or be told by his/her colleagues, even saw in a paper. Because formed easily, they can also be changed easily. Teachers’ exterior mathematics teaching and learning beliefs hardly affect on the mathematics classroom, so we will not discuss them. In the following, we just discuss the internal mathematics teaching and learning beliefs. We think that the internal mathematics teaching and learning beliefs of a teacher are produced by his/her socialization, especially the social practice of the mathematics learning that he/she participated in. For example, he/she liked some mathematics teacher’s teaching very much in some time, then he/she maybe thought that one is a good mathematics teacher, good mathematics teacher must like that teacher, good mathematics teaching must like that teacher’s teaching, good mathematics learning must like what the teacher told.

The internal mathematics teaching and learning beliefs of a teacher form slowly, and after formed, it is difficult to change them. In K–12, a student spends much time (a thousand of hours) learning mathematics. In such a long time, mathematics becomes the important part his/her life. We think that before entering university, a person’s internal mathematics teaching and learning beliefs has been doubtless. In a future teacher’s heart, he/she has been possessed the beliefs of mathematics teaching and learning, such as how to teach and learn mathematics, what is good mathematics teaching and learning, how to learn mathematics effectively, etc. Some researches hold out above perspectives.

Wright & Tuska (1968) did a research; its aim was to find whether the important

people of a student (for example, parents and teachers) affect his/her decision-making on becoming a teacher and whether these people affect on a teacher's teaching practices. The research objects are 508 female teachers. Using teaching attitude questionnaires, the researches investigated them in the beginning and the end of the practices and after a year of the formal teaching.

The investigated teachers were requested to fill in their self-cognitions and the teacher's roles in the questionnaires, then the results were compared. The investigation indicates that early experiences effect on their teaching and teacher understanding in their hearts, and these early imaginations to teaching and teachers will go down to their formal teacher education. This investigation also indicates that these future teachers will use the similar methods their teachers used to teach in classroom unless the formal teacher education can change their early imaginations to teaching and teachers.

In other researches, the researchers (Crawford & Deer 1993; Lerman 1997; Klein 1997) found that despite many students in teachers colleges can write good papers when studying some education curricula, their teaching methods are not different with the ones who do not study teacher curricula when they begin actual teaching. When a person enters the mathematics department of a teachers college, he/she will study some curricula, including mathematics, general education theories and mathematics education theories.

These curricula tell the future teacher how to be a teacher, especially, the mathematics education theories tell the future mathematics teacher how to teach, how to guide students' mathematics leaning, what methods are good, how to use strategies, what are sound mathematics teaching and learning beliefs, etc. But the mathematics teacher education has a underlying hypotheses, that is that the mathematics teaching and learning beliefs of a student are a piece of white paper.

The hypotheses think that the teachers (or curricular) tell students what the mathematics teaching and learning are, and then students possess corresponding (internal) mathematics teaching and learning beliefs. The hypotheses also think that even though student has certain (internal) mathematics teaching and learning beliefs, they will be change into what the teachers (or curricular) tell him/her. But the effects of a few curricula's sermon are not as good as the ones of more than ten years' subtle influence from social experiences. So we can infer easily that the effects current teacher educations are not good. Some researches' works sustain this viewpoint. For example, Lerman (1997) pointed that the teachers colleges students have possessed the consciousness of their future imaginations.

The ideas the teachers colleges provide and the papers the students write hardly impact on the students' original ideas, in despite of teachers estimating the students' papers highly. Meanwhile, good teaching methods and the information a future teacher must know that teacher school's teachers try to inform are also suppressed, these methods

and information are thought very valuable by teachers. In a sense, Lerman denies the function of teacher colleges and their teachers.

How the internal and exterior instructional beliefs work on teacher's teaching in the classrooms? The internal instructional beliefs of a teacher will affect directly the teacher's teaching practice. The teacher's behaviors will accord with his/her internal instructional beliefs, and the teacher's internal instructional beliefs embody by his/her teaching behaviors. About this, many researches have been done. So, the researching a teacher's instructional beliefs must through his/her day-to-day teaching' behaviors.

The exterior instructional beliefs of a teacher have hardly any effect to his/her actual teaching practice, but sometimes, when he/she tells students what are mathematics and mathematics teaching and learning, he/she can use his/her exterior instructional beliefs. This is meaningless if a teacher's internal and exterior instructional beliefs are different completely.

USING THE SITUATED THEORY TO EXPLAIN TWO KINDS OF BELIEFS AND SOME ILLUMINATIONS

Now, we firstly see in what situations two kinds of beliefs are shaped. The exterior mathematics instructional beliefs are shaped in the teacher education schools, in-service education schools, and some lectures. So, according to the situated theory, the exterior mathematics instructional beliefs are related with these situations. The internal mathematics instructional beliefs are shape in the mathematics classrooms of elementary, middle, and high schools. Then, the internal mathematics instructional beliefs are accompanying with the mathematics classroom situations.

Because beliefs are related with the corresponding situation, the different beliefs will appear the different situations. Beliefs can just transfer between the same or similar situations. When the situations' difference is very obvious, then, the transference is difficult.

Then, we can explain the two kinds of beliefs. When a teacher studies in the teacher education school, in-service education school, and some lectures, he/she forms a kind of mathematics instructional beliefs, which we call it the exterior mathematics instructional beliefs. Once the teacher appears in the similar situations, he/she will manifest these beliefs. For example, when the teacher makes a lecture, he/she will tell the audiences the exterior mathematics instructional beliefs. Because the lecture's situation is similar with the situation in which he/she formed these beliefs.

But if the teacher teaches in the mathematics classroom, he/she will show another mathematics instructional belief that were formed in the mathematics classroom as a

pupil. These beliefs are called the internal mathematics instructional beliefs. Because two situations are similar, the teachers will have similar behaviors.

The exterior mathematics instructional beliefs are formed by learning; the forming process is short, easy, and self-conscious. By contrary, the forming process of the internal ones is slow and unconscious. So, the former is easy to change and the latter is difficult to change. To mathematics teaching and learning, only internal mathematics instructional beliefs exert effects on them. So we can neglect completely the exterior ones.

From the analysis of the mathematics instructional beliefs, we can get some illuminations.

Firstly, in our researches, we must distinguish between the internal mathematics instructional beliefs and the exterior ones; this division is very important to our researches and can make us avoid unwanted mistakes.

Secondly, it is very important that teachers' internal mathematics instructional beliefs are difficult to change because the internal mathematics instructional beliefs often are dated and are not coincident with current mathematics education and mathematics instructional reforms.

We have known that the internal mathematics instructional beliefs of a teacher can affect on his/her behaviors in classroom directly, so it is very important to change a teacher's dated internal mathematics instructional beliefs. Because it is stubborn, the changing is very difficult.

The chief tasks of mathematics education reform (for example, currently mathematics curricula reform in china) are teachers training, and the training ought to put changing teachers' internal mathematics instructional beliefs in first place. If the dated internal mathematics instructional beliefs of teachers cannot be changed, and if new ones cannot be formed, we believe that the mathematics education reform will fail. There are two points noticeable in the training.

The first point is that the training time must be enough. Because the internal mathematics instructional beliefs of a teacher are formed for a long time, it is impossible to change the dated beliefs and form new ones in a short time, for example, by a few classes or reports.

The second point is that the trainer must use some appropriate strategies in the training the stubborn mathematics instructional beliefs cannot be changed by simple sermon, and simple sermon just change the exterior ones.

Now because the researchers work, some effective strategies have been found, for example, reflection is one of the effective strategies.

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