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Creative Potential of Olympiad Problems[1]

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The present article is dedicated to discussing the methods of teaching schoolchildren to solve creative problems. A few types of Olympiad problems were chosen as a didactic content of the article. The relevance of the choice receives proper grounding. Explanations are followed by examples from pedagogic practice.
The article was written for the use of school teachers and educationalists that are researching the problem of improving creative thinking with schoolchildren.

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INTRODUCTION

It is a well-known fact that the system of classes and lessons, due to its conventionality forces teacher to stick generally to teaching ready-made schemes and solving typical exercises. Consequently, because of it a pupil is very rarely granted an opportunity of comprehending mathematics in its diversified manifestations as the integral and unified science. And the intricate and breath-taking process of mathematical discovery is at all discarded during mental education and development. As a result, many school graduates

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recollect mathematics as a *science of obsolete rules and remembering-resisting instructions*. Such school "intellectual trauma" can avert an individual from studying mathematics for a very long time. Consequently, doubtless becomes the very possibility of successful school instruction.

An apparent problem.

Its essence stems from considerable difficulties associated with conveying researcher experience from a researcher to a teacher, from a teacher to a student. Sometimes, it is extremely hard to overcome these hardships. And the reason is as follows.

In real life, we often encounter the situations when something, even seemingly simple, cannot be made instead of some other person. For instance, you cannot sleep, eat, and recover from illness instead of somebody. In all these cases, the result of action cannot be separated from its doer. It belongs to him and him only. Student's learning and research activity is also in the list.

In other words, success of teaching is in the least measure dependable on teacher's professional skill to present the material. And, in the highest measure – on student's ability of self-training, and is ability of becoming immersed in the intellectual self-excitation state.

It is good to remind that the Olympiad tasks are traditionally called such tasks, which at different times have been offered to schoolchildren at various competitions, contests, mathematical combats, etc. These tasks differ in quality from the standard textbook ones, primarily, by the presence of vivid mathematical idea, peculiar folklore grace, hard-to-achieve but surprisingly brilliant, sometime insight solution.

Polya (1965) noted that in the complex of conventional problems a teacher is imperative to include several of them, which, though slightly more challenging and time-consuming, nevertheless, possess genuine mathematical grace and profound contents.

The Olympiad problems nearly always meet these requirements. Moreover, many mathematicians and specialists think that the meditation over the solution of an Olympiad problem is nearly always a piece of "genuine" mathematics which provides sufficiently vivid, though a little limited, image of the science idea, method, and esthetics.

In this connection, Boris Delonney, a bright representative of the Moscow mathematical school of thought, said once: "Great research discovery differs from a difficult Olympiad problem only in that five hours are sometimes sufficient to solve the problem, whereas it takes you the whole life to obtain a notable research result."

On the other hand, we should not, however, think that the integrity of the Olympiad tasks can serve as an adequate model of "great" mathematics. More probably, we deal here with the "research natural reserve." In reality, a mathematician generally speculates over the never-before-solved problem, which means that he is even unaware whether the problem has any solution at all. Add here the fact that he would probably be forced to devise his own techniques and theoretical skills to find the solution. Still, on the other hand, the best tasks of the mathematical Olympiads are genuine pieces of art. During meditations and search for their solutions a student passes principally the same route and enjoys the same feeling so familiar to any professional mathematician.

It is evident that if a young researcher can experience the entire spectrum of feelings peculiar to any trailblazer, and can enjoy the pleasure of powerful intellectual self-excitement, he can be anticipated to actively continue his research activities. Nevertheless, numerous facts from the biographies of many outstanding mathematicians convince us that rigid and in most cases, unnatural, conditions of the mathematical contest are not necessary for the acquaintance with certain interesting problems and achievement of high scientific results.

Their top achievements in the science stem from prolonged and profound inner contemplation rather than from fast operating ingenuity (Agakhanov, Kuptsov & Nesterenko 1997).

The practice shows that modern curricula pay insufficient attention to the development of this quality in schoolchildren.

OUR DIDACTIC PRINCIPLES

The major didactic principles in our work are:

The variativity principle in teaching mathematics.

The principle essence is described by Polya (1965) who states that it is better (from the viewpoint of development of a personality thinking) to solve one problem by a number of various techniques than to solve several similar or even dissimilar problems.

In this process, we have a rare opportunity of comparing different solutions from various viewpoints:

- Conventionality or ingenuity;
- Is the amount of calculations large (much to count and write) or not?
- Is the solution available to all and everybody, or only to the population familiar with a certain theory?
- Is the chosen allegory of the problem restated conditions clear to all students, or only to the author? And so on...

Didactical content of individual and group work is selected by the teacher, basing on concrete training objectives and abilities of the group members.

The creativity principle.

If a teacher wants to follow this principle in his work, he should not only train his student in understanding of the problem-underlying idea but to make an attempt of exercising with the idea, of coining a new phrase, a new plot for it. In other words, the teacher has to try and formulate new problems on the idea's basis. It might happen that a student will not be able to solve the problem you have made, without assistance. This fact is not a rarity in the history of mathematics. Up to now, there are the problems formulated over a hundred years ago but still unsolved, though many great brains attempted to do it.

Besides, the problem offered to a student can be only a «top of an iceberg» for the more general issue. And having solved the problem, it is worth contemplating whether the result can be generalized?

The sequence of operations schematically looks like:

A new problem
$\Rightarrow$ analysis of the problem conditions
$\Rightarrow$ solution of the problem
$\Rightarrow$ an attempt to generalize the result
$\Rightarrow$ coining of the idea underlying the problem conditions or its solution
$\Rightarrow$ search for new possibilities of the idea application
$\Rightarrow$ statement of a new problem.

The above described creativity concept can be called "the aerobatics flying" of student activity in his self-education progress. It is hard to be implemented, particularly in its advent. However, the expediency of its realization is beyond all doubts.

We can demonstrate on below given examples our experience in this direction.

The group of schoolchildren included mathematically inclined children.

EXAMPLES

Example 1 (8th grade. Olympiads Zota 2000, Rehovot, Israel).

It is known that the sum n of various positive integers $S = a_1 + a_2 + \cdots + a_{n-1} + a_n$ equals the sum of squares of the same integers.

Which of the two following sums is bigger:

$$S^{(4)} = a_1^{\ 4} + a_2^{\ 4} + \cdots + a_{n-1}^{\ 4} + a_n^{\ 4} \qquad (1)$$

the sum of the 4th powers of given n integers, or

$$S^{(3)} = a_1^{\ 3} + a_2^{\ 3} + \cdots + a_{n-1}^{\ 3} + a_n^{\ 3} \tag{2}$$

the sum of the 3rd powers of given n integers?

Teacher's strategy

We have observed, and not once, the phenomenon of psychological alienation experienced by a student when he first sees the problem conditions. Both letters and words seem familiar to him. But he does not see a single conventional catch, any logical support. We cannot say how many potential mathematicians fail to overcome this obstacle in school years, but we can exactly appreciate the role of teacher's psychological support to the student.

"If one does not have a high self-concept with respect to one's ability in mathematics then hard problems are not even attempted, and this "defeatist attitude" seems to go way beyond school mathematics. With respect to girls, this is often called the phenomenon of learned helplessness. Their perception of their own ability is an extremely important factor in the mathematics they do. A low self-opinion of one's ability in school mathematics can be devastating." (Eisenberg 1991)

The strategy of teacher's work anticipates rendering assistance to a student by subdividing the problem into a chain of supposedly easy for him lemmas, along with the possibility of assessment and analysis of the solution intermediate results both by the student and the teacher.

Teacher's tactics

There is the necessity of being able to control the situation when a student has already made several unsuccessful attempts and is contemplating the idea: should he give in or go on struggling? In this case, the most expedient teacher's step is to start, together with the student, scheduling the problem study. Here is the example of the initial plan of study.

1) Accumulation of statistical experience.

- State the problem for the case $n = 1$, $n = 2$.
- Try to find the answer to the question of the problem of two positive integers satisfying the condition:

$$a + b = a^2 + b^2.$$

Try to choose one negative integer for this condition.

- To advance a hypothesis for given n and to check it directly on the chose $n = 4,\ \ n = 6$ integers.
- To try to continue the induction rise. To check whether the problem solution can be obtained for $n = 3$, similarly to the solution for $n = 1,\ \ n = 2$? If a positive reply is obtained, the generalization techniques should be sought for. If not, then...

2) Repeated return to the examination of the problem conditions.

- To advance a hypothesis for given n and to check it directly on the chose $n = 4,\ \ n = 6$ integers.
- To find an answer to the problem question for all equal numbers;
- To study the base when all $a_i = 1,\ \ i = \overline{1, n}$ numbers exceed a unity;
- Can all a_i numbers be less than a unity?
- To try to find the answer to the problem question for three positive integers satisfying the condition.

$$a + b + c = a^2 + b^2 + c^2.$$

To try to choose one or two negative integers such the given condition.

Two alternative solutions of the problem are given here for a teacher. Having them in mind, the teacher can easily supplement the study schedule, if necessary.

Technique 1

1. From the data of the problem conditions we have:

$$a_1 + a_2 + \cdots + a_{n-1} + a_n = a_1^2 + a_2^2 + \cdots + a_{n-1}^2 + a_n^2$$

$$a_1 \cdot (a_1 - 1) + a_2 \cdot (a_2 - 1) + \cdots + a_n \cdot (a_n - 1) = 0 \qquad (3)$$

2. Since given positive integers are all different, we can assume, without any loss of generality, that $\left(a + b + c = a^2 + b^2 + c^2\right)$ $\ \ 0 < a_1 < a_2 < \ldots < a_n$
3. We shall prove that

$$S^{(4)} - S^{(3)} > 0$$

$$\Leftrightarrow a_1^4 + a_2^4 + \cdots + a_{n-1}^4 + a_n^4 > a_1^3 + a_2^3 + \cdots + a_{n-1}^3 + a_n^3$$

$$\Leftrightarrow a_1^3 \cdot (a_1 - 1) + a_2^3 \cdot (a_2 - 1) + \cdots + a_n^3 \cdot (a_n - 1) > 0$$

$$\Leftrightarrow a_1^2 \cdot a_1 \cdot (a_1 - 1) + a_2^2 \cdot a_2 \cdot (a_2 - 1) + \cdots + a_n^2 \cdot a_n \cdot (a_n - 1) > 0 \qquad (4)$$

4. Compare (3) and (4)

5. It is evident that all given numbers cannot exceed a unity. Similarly, all given numbers cannot be less than a unity.

It contradicts (3).
Suppose that

$$0 < a_1 < a_2 < \cdots < a_k < 1 \le a_{k+1} < \cdots < a_n \tag{5}$$

Then, we have from (4).

$$\frac{a_1^2}{a_{k+1}^2} \cdot a_1 \cdot (a_1 - 1) + \frac{a_2^2}{a_{k+1}^2} \cdot a_2 (a_2 - 1) + \cdots + a_{k+1} \cdot (a_{k+1} - 1) + \frac{a_n^2}{a_{k+1}^2} \cdot a_n \cdot (a_n - 1) > 0 \tag{6}$$

Denote

$$\frac{a_1^2}{a_{k+1}^2} = \beta_1 < 1, \quad \frac{a_2^2}{a_{k+1}^2} = \beta_2 < 1, \ldots, \quad \frac{a_k^2}{a_{k+1}^2} = \beta_k < 1,$$

$$\frac{a_{k+2}^2}{a_{k+1}^2} = \lambda_1 > 1, \quad \frac{a_{k+3}^2}{a_{k+1}^2} = \lambda_2 > 1, \ldots, \quad \frac{a_n^2}{a_{k+1}^2} = \lambda_{n-k-1} > 1.$$

Then, we have from (4) – (6).

$$S^{(4)} - S^{(3)} > 0$$

$$\Leftrightarrow a_1^2 \cdot a_1 \cdot (a_1 - 1) + a_2^2 \cdot a_2 \cdot (a_2 - 1) + \cdots + a_n^2 \cdot a_n \cdot (a_n - 1) > 0$$

$$\Leftrightarrow \frac{a_1^2}{a_{k+1}^2} \cdot a_1 \cdot (a_1 - 1) + \frac{a_2^2}{a_{k+1}^2} \cdot a_2 (a_2 - 1) + \cdots + a_{k+1} \cdot (a_{k+1} - 1) + \frac{a_n^2}{a_{k+1}^2} \cdot a_n \cdot (a_n - 1) > 0$$

$$\Leftrightarrow \beta_1 \cdot a_1 \cdot (a_1 - 1) + \beta_2 \cdot a_2 \cdot (a_2 - 1) + \cdots + \beta_k \cdot a_k \cdot (a_k - 1) + a_{k+1} \cdot (a_{k+1} - 1) +$$
$$\lambda_1 \cdot (a_{k+2} - 1) + \lambda_2 \cdot (a_{k+3} - 1) + \cdots + \lambda_{n-k-1} \cdot a_n \cdot (a_n - 1)$$
$$> a_1 \cdot (a_1 - 1) + a_2 \cdot (a_2 - 1) + \cdots + a_n \cdot (a_n - 1) = 0 \tag{7}$$

Actually, (7) is right, since, for $0 < i < k+1$,

$$\begin{cases} 0 > \beta_i \cdot a_i \cdot (a_i - 1) > a_i \cdot (a_i - 1) \\ 0 < i < k+1 \\ 0 < a_i < 1 \end{cases}$$

$$\Leftrightarrow \quad 0 < \beta_i < 1$$

Also, *for* $k+1 < j < n$, *and* $a_{k+1+j} > 1$,

$$\begin{cases} \lambda_j \cdot a_{k+1+j} \cdot (a_{k+1+j} - 1) > a_{k+1+j} \cdot (a_{k+1+j} - 1), \ \lambda_j > 1 \\ \lambda_j > 1 \\ k+1 < j < n \\ a_{k+1+j} > 1 \end{cases}$$

$$\Leftrightarrow \quad \lambda_j > 1$$

Technique 2

1. From the data of the problem conditions we have:

$$a_1 + a_2 + \cdots + a_{n-1} + a_n = a_1^2 + a_2^2 + \cdots + a_{n-1}^2 + a_n^2$$

$$a_1 \cdot (a_1 - 1) + a_2 \cdot (a_2 - 1) + \cdots + a_n \cdot (a_n - 1) = 0 \qquad (3)$$

2. Let us prove that

$$S^{(4)} - S^{(3)} > 0$$

$$\Leftrightarrow \left(a_1^4 + a_2^4 + \ldots + a_{n-1}^4 + a_n^4\right) > \left(a_1^3 + a_2^3 + \ldots + a_{n-1}^3 + a_n^3\right)$$

$$\Leftrightarrow \left(a_1^3 \cdot (a_1 - 1) + a_2^3 \cdot (a_2 - 1) + \cdots + a_n^3 \cdot (a_n - 1) > 0\right)$$

$$\Leftrightarrow a_1^2 \cdot a_1 \cdot (a_1 - 1) + a_2^2 \cdot a_2 \cdot (a_2 - 1) + \cdots + a_n^2 \cdot a_n \cdot (a_n - 1) > 0 \qquad (4)$$

3. Let us compare (3) and (4).
   Subtract equality (3) from both parts of inequality (4).
   We have inequality of the same sign.

$$a_1 \cdot (a_1 - 1) \cdot (a_1^2 - 1) + a_2 \cdot (a_2 - 1) \cdot (a_2^2 - 1) + \cdots + a_n \cdot (a_n - 1) \cdot (a_n^2 - 1) > 0$$

$$\Leftrightarrow a_1 \cdot (a_1 + 1) \cdot (a_1 - 1)^2 + a_2 \cdot (a_2 + 1) \cdot (a_2 - 1)^2 +$$

$$\cdots + a_n \cdot (a_n + 1) \cdot (a_n - 1)^2 > 0$$

Since not all given positive integers are similar, the last inequality is evident. What else is worth speculating about?

It is expedient to recommend the students to continue searching the problem.

- Does the solution method permit to answer the question: For an integer z,

$$S^{(2\cdot z)} - S^{(2\cdot z-1)} > 0\,?,\quad S^{(2\cdot z)} = a_1^{\,2\cdot z} + a_2^{\,2\cdot z} + \cdots + a_{n-1}^{\,2\cdot z} + a_n^{\,2\cdot z}$$

- Is it correct that for non-negative (a,b) the following inequality holds?

$$(a+b)\cdot\left(a^4+b^4\right) \ge \left(a^2+b^2\right)\cdot\left(a^3+b^3\right)$$

- The same question we can ask for positive (a,b).
- Can the answer to the preceding question be obtained, using the basic problem conclusions? (Hint: $\dfrac{a^4+b^4}{a^3+b^3} \ge \dfrac{a^2+b^2}{a+b} = 1$)

An attempt to solve the initial problem for the cases:

a) $a_1^{\,2} + a_2^{\,2} + \cdots + a_{n-1}^{\,2} + a_n^{\,2} > a_1 + a_2 + \cdots + a_{n-1} + a_n$

b) $a_1^{\,2} + a_2^{\,2} + \cdots + a_{n-1}^{\,2} + a_n^{\,2} < a_1 + a_2 + \cdots + a_{n-1} + a_n$

Attracting students' attention to creative activity.

Stimulation of student's creative mathematical activity is rather natural at this stage. We present an example of such activity with our pupils.

New problem 1*. (Here and further on we give the examples of problems invented by our students.) It is known that sum n of numbers which are not equal to one another,

$$S = a_1 + a_2 + \cdots + a_{n-1} + a_n,$$

is not less than the sum of squares of the same numbers.

It is known that for any integer i, $1 \le i \le n$:

$$a_i \in (-\infty,-1)\cup(0,+\infty)$$

is held. Compare fraction with a unity: For an integer z,

$$\frac{S^{(2\cdot z)}}{S^{(2\cdot z-1)}} = ?\ \text{ and }\ S^{(2\cdot z)} = a_1^{\,2\cdot z} + a_2^{\,2\cdot z} + \cdots + a_{n-1}^{\,2\cdot z} + a_n^{\,2\cdot z}.$$

Example 2. (8th grade. Olympiads Zota 2000, Rehovot, Israel) Let us consider a number $N = 1 + 2^n + 3^n + 4^n$.

What is the potential amount of zeros for the termination of number N decimal representation?

Solution. Only the methodological analysis of the problem is given here.

- Scientific experiment (Throwing a "touchstone")

n	2^n	3^n	4^n	$N = 1+2^n+3^n+4^n$
1	2	3	4	10
2	4	9	16	30
3	8	27	64	100
4	16	81	256	354

Undoubtedly, the examples given are insufficient. However, we can immediately define from the table that the desired quantity of zeroes can be equal to zero, to a unity and to two. Let us try to continue our experiment.

n	2^n	3^n	4^n	$N=1+2^n+3^n+4^n$
5	32	243	1024	1300
6	64	729	4096	4890
7	128	2187	16384	18700

No changes for the answer.

- Plausible hypothesis:
  The desired maximal potential quantity of zeros is exactly two.
- The hypothesis validation.
  a) With $n = 2 \cdot k + 1,\ k \in \mathrm{N}$, we have:

$$N = 1 + 2^n + 3^n + 4^n = 1 + 2 \cdot 4^k + 3 \cdot 9^k + 4 \cdot 16^k \equiv 1 + 0 + 3 \cdot 1 = 4 \pmod 8 \qquad (8)$$

  b) With $n = 2 \cdot k,\ \ k \geq 2$, we have:

$$N = 1 + 2^n + 3^n + 4^n = 1 + 4^k + 9^k + 16^k \equiv 1 + 0 + 1 = 2 \pmod 8 \qquad (9)$$

It means that number $N = 1 + 2^n + 3^n + 4^n$ is not divisible by 8. Therefore, this number N can under no n values be divisible by 1000, *i.e.*, to have three zeros at the end of decimal representation. The hypothesis is substantiated by the proof.

Hence, and supported by the table data we arrive at the conclusion: **Two zeroes.**

For advanced students

Let us try to determine the amount of numbers of the desired nature.

Prove that there is an infinite amount of natural n values for each of which $N = 1 + 2^n + 3^n + 4^n$ number will be divisible by 100.

Proof.

$N = 1 + 2^n + 3^n + 4^n = 1 + 2 \cdot 4^k + 3 \cdot 9^k + 4 \cdot 16^k \equiv 1 + 2 \cdot 0 + 3 \cdot 1 + 4 \cdot 0 \equiv 0 \pmod 4$ (10)

Let us analyze the possibilities of division of

$$N = 1 + 2^n + 3^n + 4^n = \left(1 + 4^{2\cdot k+1}\right) + \left(2^{2\cdot k+1} + 3^{2\cdot k+1}\right) \text{ by } 25.$$

We will use the formula:

$$a^{2\cdot k+1} + b^{2\cdot k+1} = (a+b)\cdot\left(a^{2\cdot k} - a^{2\cdot k-1}\cdot b + a^{2\cdot k-2}\cdot b^2 - \cdots - a\cdot b^{2\cdot k-1} + b^{2\cdot k}\right),$$

Since $1 \equiv 1 \pmod 5$, $4 \equiv -1 \pmod 5$
We have:

$$1^{2\cdot k+1} + 4^{2\cdot k+1} = (1+4)\cdot\left(1^{2\cdot k} - 1^{2\cdot k-1}\cdot 4 + 1^{2\cdot k-2}\cdot 4^2 - \cdots - 1\cdot 4^{2\cdot k-1} + 4^{2\cdot k}\right)$$
$$\equiv 5\cdot(1\cdot(2\cdot k+1)) \pmod 5$$

Similarly, as $3 \equiv -2 \pmod 5$, we have:

$$2^{2\cdot k+1} + 3^{2\cdot k+1} = (2+3)\cdot\left(2^{2\cdot k} - 2^{2\cdot k-1}\cdot 3 + 2^{2\cdot k-2}\cdot 3^2 - \cdots - 2\cdot 3^{2\cdot k-1} + 3^{2\cdot k}\right)$$
$$\equiv 5\cdot\left(2^{2\cdot k}\cdot(2\cdot k+1)\right) \pmod 5$$

It remains to prove that integer P,
$P = 2\cdot k + 1 + (2\cdot k+1)\cdot 2^{2\cdot k} = (2\cdot k+1)\cdot\left(1+4^k\right)$, is not always divisible by 5. For instance, when $k = 4$.
However, with odd k or with $(2\cdot k+1)\vdots 5$, we have:

$$N = 1 + 2^n + 3^n + 4^n = \left(1 + 4^{2\cdot k+1}\right) + \left(2^{2\cdot k+1} + 3^{2\cdot k+1}\right) \vdots 25 \qquad (11)$$

It follows from (10) – (11) that $N = 1 + 2^n + 3^n + 4^n$ can terminate with two zeroes.
It takes place every time when $k = \frac{5\cdot z - 1}{2}$, $z \in \mathbb{N}$ or when k is odd.

New problem 2*. Solve the equation in integers: $1 + 2^n + 3^n + 4^n = 10^z \cdot t$

New problem 2.** Find at least three solutions to the integer equation:

$$1 + 2^n + 3^n + 4^n + \cdots + p^n = \frac{1+p}{2}\cdot p\cdot x, \text{ where } n, x, p \text{ are integers.}$$

Hint: Prove that the left-hand side of the equation is divided by $1 + 2 + 3 + \cdots + p$ with odd n.

DEVELOPMENT OF INTELLECTUAL ENDURANCE IN AN INDIVIDUAL

Sharygin (1991, p. 3) has substantiated the essential necessity of using super-complicated problems and exercises in training schoolchildren. "We must constantly work at limiting and even over-limiting heights. Long and intensive work over a sufficiently hard problem and subsequent (both in case of success and in failure) investigation of the solution are more beneficial than tens of one-step and stereotype examples."

Our practical work, remarks of our colleagues, and corresponding observations conform that even purely technical training skills are more efficient when they are taught through the attractive material.

We can note that when the problem complexity level exceeds conventional average school level then another problem is added to purely methodological ones, *i.e.*, the pedagogical problem. Its essence – how can student's attention be retained on the problem under study for a sufficiently long period?

It is a well-known fact that many pupils have absolutely insufficient intellectual endurance for mathematical studies. Such students cannot cope with the proposed problem, firstly, not because of the lack of requisite mathematical faculties but because the corresponding personal qualities have not been developed to the proper extent. In such a student, the most valuable and essential feature is lost during the problem solution – the stage of search for a given problem solution.

We have questioned 75 pupils – graduates of the school highest level of education (5 points). Each of them was offered to recollect whether there was an event in their lives when:

(a) They had to speculate over the problem offered by the teacher for above three hours?
(b) They had to revert to the search of a definite problem solution for several days?

Only 8% of those questioned said that fact (a) did occur in their learning experience. Four percent of students replied positively to question (b). In doing this, two of the students could actually recollect and state the problem essence.

CONCLUSION

As we have seen in the above examples, employing Olympiad problems in teaching conduces to the development of creative thinking.

Many authors have come to the conclusion that "the developments of intellect and outlook on life in the younger age are closely related to the development of creative abilities based upon intellectual initiative and creative activity in making something new, rather than simple taking in of information."

The main difference in the intellectual development of a high-school student and a teen-ager seems to lie in the gap between their scientific views. However, the formation a scientific views in youth is closely related to the development of creative abilities based upon intellectual initiative and creative activity in making something new, rather than simple taking in of information. Though at the same time it is apparent, that any act of creation or a product whereof are results of development-determined features of a specific individual, related to gaining and differentiating useful experience for a certain field of activity over a long period of time, rather than of an intuitive insight or natural genius.

The immense potential for scientific research, that Olympiad problems hold, enables the students to percept the mathematics as a whole in all the variety of its form. The sophisticated and absorbing process of making a mathematical discovery may influence the choice of the future occupation.

By the virtue of our teaching experience we believe that:

- Employing the potential of Olympiad problems for scientific research should serve as a means of activating and stimulating the free creative intellectual activity of students.
- A young learner wielding the methods of the demonstrative and probable speculation is capable of displaying maturity in creative constructive activity.

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