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THE ION ACOUSTIC SOLITARY WAVES AND DOUBLE LAYERS IN THE SOLAR WIND PLASMA

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ABSTRACT

Ion acoustic solitary wave in a plasma consisting of electrons and ions with an external magnetic field is reinvestigated using the Sagdeev's potential method. Although the Sagdeev potential has a singularity for $n < 1$, where n is the ion number density, we obtain new solitary wave solutions by expanding the Sagdeev potential up to δn^4 near $n = 1$. They are compressiv (rarefactive) waves and shock type solitary waves. These waves can exist all together as a superposed wave which may be used to explain what would be observed in the solar wind plasma. We compared our theoretical results with the data of the Freja satellite in the study of Wu et al. (1996). Also it is shown that these solitary waves propagate with a subsonic speed.

Keywords : space plasma, ion acoustic solitary wave, soliton

1. INTRODUCTION

Ion acoustic solitary waves in a collisionless plasma have long been a subject of extensive study (Yu et al. 1980, 2003, Rao et al. 1990, Nejoh & Sanuki 1995, Wu et al. 1995, 1996, Das et al. 1998, Shukla & Mamun 2002, 2003, Choi et al. 2004, 2005a,b). However most of these studies were about dust ion acoustic solitary waves (DIASWs) or dust acoustic solitary waves (DASWs) (Rao et al. 1990, Shukla & Mamun 2002, 2003). Nejoh & Sanuki (1995) studied the effect of the ion temperature in an unmagnetized electron beam plasma. Das et al. (1998) investigated the dynamics of solitary waves in an ion-beam plasma having multiple electron temperature. Choi et al. (2004) have studied the role of dust particles in a plasma composed of electrons and ions and also studied DIASWs in a magnetized dusty plasma. Among others, Wu et al. (1995, 1996) investigated solitary kinetic Alfvén waves (SKAWs) accompanied by a hump and dip density solitons, and applied it to the electromagnetic spikes observed by the Freja satellite. The soliton solution calculated by Wu et al. (1996) is the $sech^2$ type solution obtained by expanding the Sagdeev potential up to δn^3. However we obtained the higher order solutions than Wu et al. (1996) did. We will show that our results match well with the data of the Freja satellite. Therefore our results can be used to interpret ion acoustic solitary waves in the solar wind plasma.

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In the present work, we studied the solitary waves for ion density in a magnetized two-component plasma with hot electrons and cold ions but without dust particles. While Yu et al. (1980) obtained a solution of ion acoustic solitary wave in such a system by a small amplitude limit, here we re-examine the solitary wave in the same plasma, and find the existence of new types of solitary wave solutions that can be used to interpret similar waves in the solar wind plasma.

In Section 2 the basic equations and the calculation of the exact Sagdeev potential are presented. The calculated Sagdeev potential is analyzed and the existence of large amplitude localized solution is presented in Section 3. Using a small amplitude limit new solutions are obtained in Section 4. Finally, the conclusion is given in Section 5.

2. BASIC EQUATIONS AND CALCULATION OF THE SAGDEEV POTENTIAL

The basic equation describing the ion dynamics in a magnetized plasma composed of hot electrons and cold ions can be written as

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = 0, \tag{1}$$

and

$$\frac{\partial \mathbf{v}_i}{\partial t} + (\mathbf{v}_i \cdot \nabla)\mathbf{v}_i = -\frac{e\nabla\phi}{m_i} + \frac{eB_0}{m_i c}\mathbf{v}_i \times \hat{e}_z, \tag{2}$$

where subscript i and e stand for ions and electrons, n_i, v_i, m_i, and ϕ are the number density, velocity, mass, and the electrostatic potential, respectively. The background magnetic field $\mathbf{B} = B_0\hat{e}_z$ is a constant in z direction, where $\hat{e}_z$ is the unit vector in the z direction. We assume that the characteristic length is much larger than the Debye length λ_D. This assumption allows us to use the charge neutrality condition($n \approx n_i \approx n_e$) instead of the full Poisson equation, which greatly simplifies the algebra. The number density of the electrons is given by

$$n_e = n_{e0}\exp(e\phi/T_e). \tag{3}$$

The basic equation of the system can be rewritten in the normalized form as follows (Choi et al. 2004, 2005a)

$$\frac{\partial n}{\partial t} + \frac{\partial(nv_x)}{\partial x} + \frac{\partial(nv_z)}{\partial z} = 0, \tag{4}$$

$$\frac{\partial v_x}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_x = -\frac{\partial\Phi}{\partial x} + v_y, \tag{5}$$

$$\frac{\partial v_y}{\partial t} +] \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_y = -v_x, \tag{6}$$

and

$$\frac{\partial v_z}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_z = -\frac{\partial\Phi}{\partial z}, \tag{7}$$

which we normalized by ion gyro-frequency $\Omega = eB_0/m_i c$ and ion acoustic wave speed $C_s = (kT_e/m_i)^{1/2}$. Here we are considering ion acoustic wave, which is a longitudinal wave, in the presence of external magnetic field, and the two directions, perpendicular and parallel to the magnetic field, are taken as x and z coordinate, respectively. So all physical quantities are taken to be functions of x and z (Yu et al. 1980).

To obtain the linear dispersion relation for low frequency ($\omega \ll \Omega$) ion acoustic waves, we linearize the basic equations by assuming that all perturbed quantities vary as $\exp[i(k_x x + k_z z - \omega t)]$. Then, we find the dispersion relation to be written as

$$\omega = \frac{k_z}{\sqrt{1+k_x^2}}, \tag{8}$$

where k_x and k_z are the wave vectors in the x and z directions, respectively.

In order to derive the Sagdeev potential, we assume that all the dependent variables depend on a single independent variable $\xi = l_x x + l_z z - Mt$, where l_x and l_z are directional cosines satisfying $l_x^2 + l_z^2 = 1$, and M is the Mach number of localized wave. From Eqs. (4)-(7) and using the boundary condition($\xi \to \infty; n \to 1$ and $\phi = \mathbf{v} = \mathbf{0}$), we can obtain as

$$\frac{d^2}{d\xi^2}\left(\ln n + \frac{1}{2}\frac{M^2}{n^2}\right) = -\frac{l_z^2}{M^2}\left(n^2 - n\right) - (1-n). \tag{9}$$

Eq. (9) can be expressed as

$$\frac{1}{2}\left(\frac{dn}{d\xi}\right)^2 + V(n) = 0, \tag{10}$$

where the Sagdeev potential $V(n)$ is given by

$$V(n) = \frac{n^6}{2\left(n^2 - M^2\right)^2}\left\{\frac{l_z^2(n-1)^2}{M^2} + \frac{2}{n}\left((1-l_z^2)n\ln n - (n-l_z^2)(n-1)\right) + \frac{M^2}{n^2}(1-n)^2\right\}. \tag{11}$$

Equation (10) is in the form of an energy integral for a particle with a unit mass and a velocity $dn/d\xi$ at position n which oscillates in a potential $V(n)$ (Sagdeev 1966).

3. LARGE AMPLITUDE ION ACOUSTIC SOLITARY WAVES

We now examine the Sagdeev potential $V(n)$ to determine the conditions for the ion acoustic solitary wave to exist and the behavior of possible localized solutions. The conditions for the existence of localized solutions are given by

$$\begin{aligned} V(n)|_{n=1} &= \left.\frac{dV(n)}{dn}\right|_{n=1} = 0 \\ V(N) &= 0 \\ V(n) &< 0, \quad N < n < 1 \quad \text{or} \quad 1 < n < N, \end{aligned} \tag{12}$$

where N is the minimum or maximum ion density within the localized potential structure.

The condition for the large amplitude double layer further requires, in addition to the ones given above, that

$$\begin{aligned} V(n)|_{n=N_m} &= \left.\frac{dV(n)}{dn}\right|_{n=N_m} = 0, \\ V(n) &< 0, \quad \text{for} \quad N_m < n < 1 \quad \text{or} \quad 1 < n < N_m, \end{aligned} \tag{13}$$

where N_m is the height of a double layer.

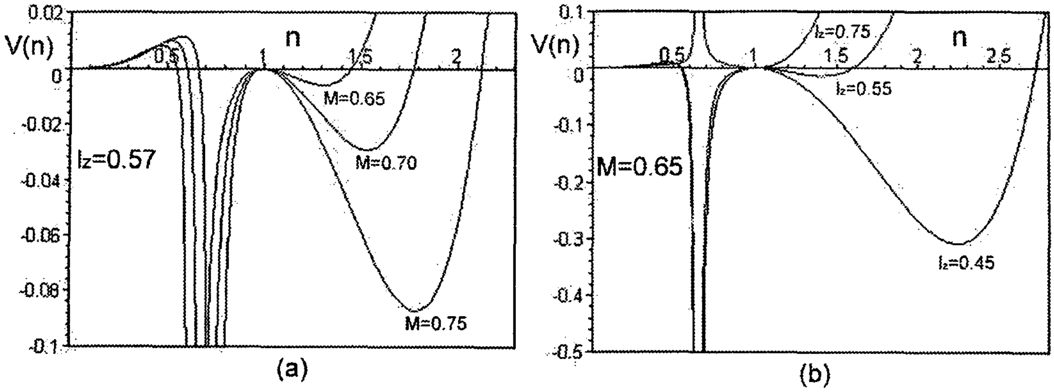

Figure 1. The Sagdeev potential: (a) for fixed $l_z = 0.57, M = 0.65, 0.70$, and 0.75, (b) for fixed $M = 0.65, l_z = 0.45, 0.55$, and 0.75.

For a negative finite value of the Sagdeev potential, it has to satisfy the condition $d^2V(n)/dn^2|_{n=1} < 0$. Then there can exist the ion acoustic solitary wave. In this case, the Mach number of the solitary wave lies in the range of

$$l_z^2 < M^2 < 1 \tag{14}$$

implying the solitary wave propagates with a subsonic speed. We then find the critical Mach number $M_c = l_z$. In the limit of $M \to M_c$, the amplitude of the compressive ion acoustic solitary wave becomes zero.

However, we find that, in addition to this conventional compressive wave, there exists other types of solitary wave solution in the region $n < 1$. In order to demonstrate this, we plotted the Sagdeev potential for $M = 0.65$, 0.70, and 0.75 with l_z being fixed to 0.57 in Fig. 1a. It is seen that the amplitude of compressive ion acoustic solitary wave increases in the region $1 < n < N$ as M increases, but also discontinuity in ion density appears as a shock in the region $N < n < 1$. In Fig. 1b, the amplitude of compressive ion acoustic solitary wave decreases as l_z increases for a fixed $M = 0.65$. Namely, the ion acoustic solitary waves become difficult to propagate along to the external field.

4. SMALL AMPLITUDE ION ACOUSTIC SOLITARY WAVES

Now we will examine the structure of the ion density solitary wave solutions by a small amplitude limit. If we expand the Sagdeev potential $V(n)$ up to δn^3 as,

$$V(n) \approx A\delta n^2 + B\delta n^3, \tag{15}$$

where $\delta n = n - 1$ and the coefficients are defined by

$$A = \frac{-M^2 + {l_z}^2}{2M^2(1 - M^2)}, \tag{16}$$

$$B = \frac{1}{3}\frac{-6M^4 - M^2(-7l_z^2 - 2) - 3l_z^2}{-M^2(1 - M^2)^2}. \tag{17}$$

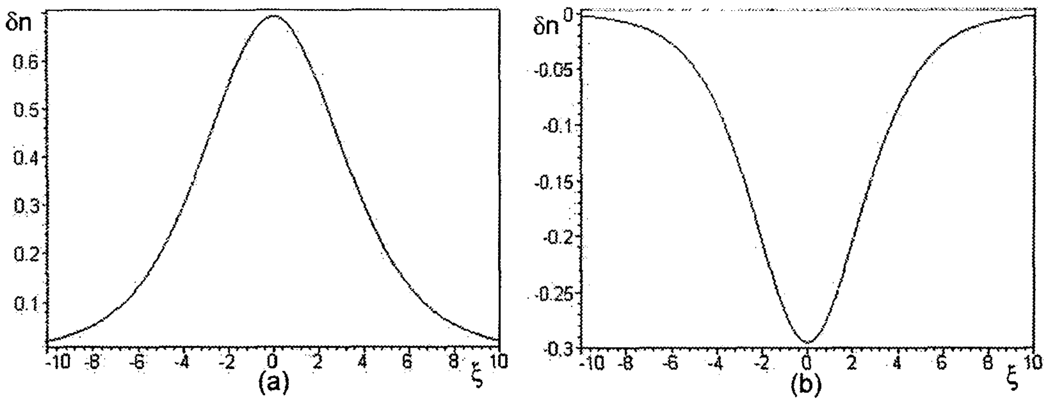

Figure 2. The $sech^2$ type ion acoustic solitary wave solutions by expanding the Sagdeev potential up to δn^3; (a) the hump type for $l_z = 0.847$ and $M = 0.776$, (b) the dip type for $l_z = 0.61$ and $M = 0.75$.

By substituting Eq. (15) into Eq. (10), we obtain

$$\delta n = \delta N sech^2(\xi/\lambda), \tag{18}$$

where $\delta N = A/B$ and $\lambda^2 = 4/A$. We plotted the $sech^2$ type (Yu et al. 1980, Wu et al. 1996) solution of the hump type IASW for $l_z = 0.847$ and $M = 0.776$ in Fig. 2a and the dip type IASW for $l_z = 0.61$ and $M = 0.75$ in Fig. 2b.

Although the Sagdeev potential $V(n)$ includes a singularity, the double-well shape of the potential as shown in Fig. 1 strongly suggests that it needs to be expanded at least up to δn^4 near $n = 1$. Thus we approximate $V(n)$ as (Choi et al. 2005a)

$$V(n) \approx A\delta n^2 + B\delta n^3 + C\delta n^4, \tag{19}$$

where the coefficient C is

$$C = \frac{1}{12}\frac{-36M^6 + M^4(51l_z^2 - 3) + M^2(-17l_z^2 - 1) + 6l_z^2}{M^2(1 - M^2)^3}. \tag{20}$$

In order to find the solutions for ion acoustic solitary waves, we substitute Eq. (19) into Eq. (10). This leads to

$$\frac{1}{2}\left(\frac{d\delta n}{d\xi}\right)^2 + A\delta n^2 + B\delta n^3 + C\delta n^4 = 0. \tag{21}$$

From the requirement that $d\delta n/d\xi > 0$ and by defining $\delta n = 1/y$, we obtain $B^2 - 4AC > 0$, which gives rise to a condition $A < 0$ and $C > 0$. As a result, the solution for the solitary wave emerges

$$\delta n = \frac{1}{-B/2A \pm |a|\cosh(\zeta)}, \tag{22}$$

where $a^2 = (B^2 - 4AC)/4A^2$ and $\zeta = \sqrt{|-2A|}\,\xi$. In the case of the δn^3 expansion, $C = 0$, and thus only $sech^2$ type solitary wave solution can be found (Yu et al. 1980, Wu et al. 1996).

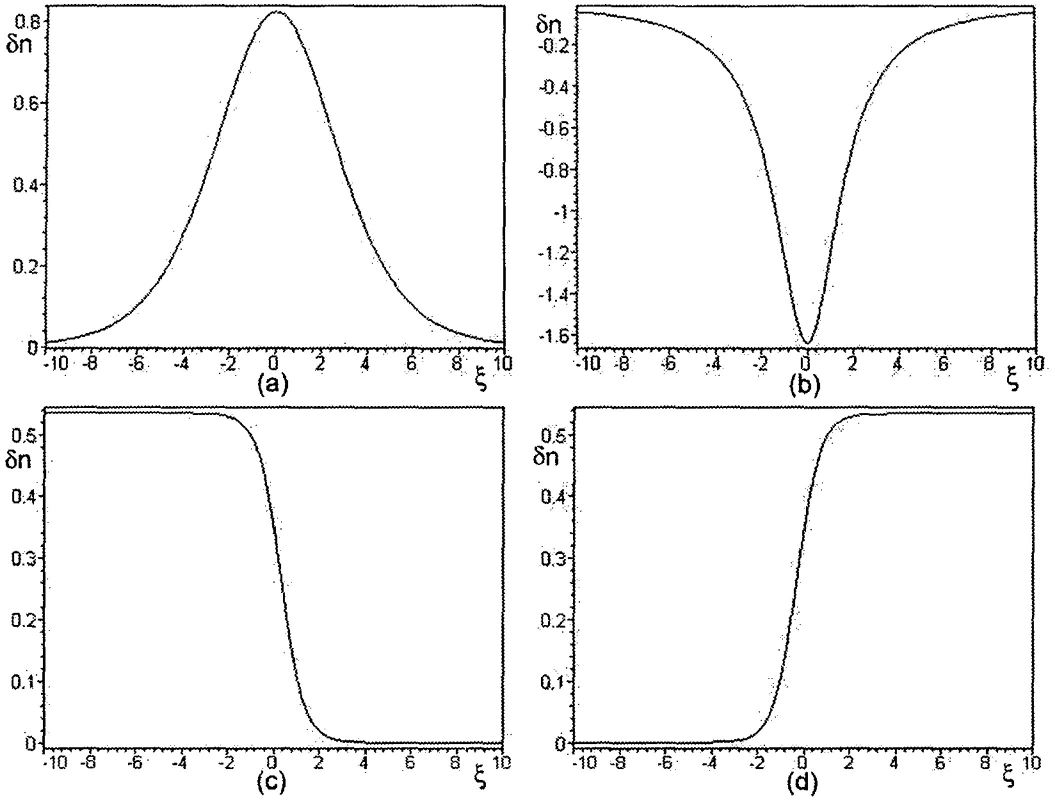

Figure 3. Small amplitude ion acoustic solitary wave solutions; (a) the compressive type ion acoustic solitary wave for $l_z = 0.414$ and $M = 0.483$, (b) the rarefactive ion acoustic solitary wave for $l_z = 0.451$ and $M = 0.451$, and (c)-(d) the shock type ion acoustic solitary waves for $l_z = 0.553$ and $M = 0.603$.

We also note that, for $A < 0$ and $B^2 - 4AC = 0$, there can exist another type of solitary wave, i.e., shock type solitary wave (or kink type). From (21), the shock type solitary wave can be obtained as

$$\delta n = \frac{1}{-B/2A + \exp(\pm\zeta/\sqrt{-2A})}. \tag{23}$$

The height and width of this type of wave are given by $|-2A/B|$ and $\sqrt{-2A}$, respectively, as $\zeta \to \infty$. The two types of solitary wave solutions are shown in Figs. 3: (a) the compressive solitary wave for $l_z = 0.414$ and $M = 0.483$, (b) the rarefactive one for $l_z = 0.451$ and $M = 0.451$, (c)-(d) the shock type solitary waves for $l_z = 0.553$ and $M = 0.603$.

It should be noted that the solitary wave may appear as a superposition of $sech^2$ type and the two other types of solutions. Figure 4a and 4b show observations from Freja satellite on March 3, 1993 (Wu et al. 1996) where two density fluctuations dn/n were interpreted as hump and dip solitons (See Holbak et al. 1994, for details about the Freja measurements). The amplitudes of the density hump and dip are $\sim$ 20% and $\sim$-20%, respectively, the characteristic width of the solitons is about $\sim$300 m being estimated based on the velocity of the Freja satellite 6.7 km/s, and the dip soliton seems

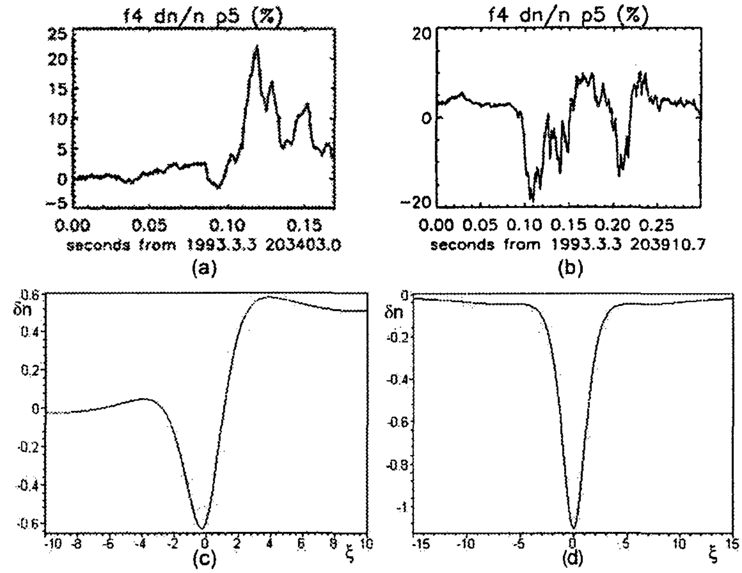

Figure 4. (a) and (b) The Freja satellite measurements of SKAWs on March 3, 1993 (taken from Wu et al. (1996)), (c) and (d) the superposed solutions from our theoretical calculations. The solution in (c) was obtained from solutions in Figure 2a and Figure 3b and 3c, and the solution in (d) from Figure 2b and Figure 3a and 3b, respectively.

to be slightly wider than the hump (Wu et al. 1996). We suggest that these waves can be better interpreted in terms of the superposed waves of the solutions that we have found above. Specifically, the wave forms in Figures 4a and 4b can be reproduced by superpositions of solutions shown in Figures 2a,b,c and Figures 2b, 3a,b, respectively. Figures 4c and 4d show the results. We believe that our interpretation based on the superposed solutions is an improvement from that suggested by Wu et al. (1996) who used hump and dip solutions only.

5. CONCLUSION

In the present paper, we have studied nonlinear ion acoustic solitary waves in a magnetized plasma consisting of electron and ion, obliquely propagating to the external magnetic field. Although the Sagdeev potential always has a singularity in the region $n < 1$, it gives rise to a double-well shaped potential. Thus this necessitates the small amplitude expansion up to δn^4 near $n = 1$. It is found that there can coexist the shock (or kink) type ion acoustic solitary wave and the compressive (or rarefactive) ion acoustic solitary wave. Then it is possible to have a linear combination of the $sech^2$ type, shock (or kink) type, and compressive (or rarefactive) solitary wave. Therefore we suggest that the superposed waves may be used to explain what would be observed in the solar wind plasmas.

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