TIME/FREQUENCY ANALYSIS
OF TERRESTRIAL IMPACT CRATER RECORDS

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ABSTRACT

The terrestrial impact cratering record recently has been examined in the time domain by Chang & Moon (2005). It was found that the $\sim$ 26 Myr periodicity in the impact cratering rate exists over the last $\sim$ 250 Myrs. Such a periodicity can be found regardless of the lower limit of the diameter up to $D \sim$ 35 km. It immediately called pros and cons. The aim of this paper is two-fold: (1) to test if reported periodicities can be obtained with an independent method, (2) to see, as attempted earlier, if the phase is modulated. To achieve these goals we employ the time/frequency analysis and for the first time apply this method to the terrestrial impact cratering records. We have confirmed that without exceptions noticeable peaks appear around $\sim$ 25 Myr, corresponding to a frequency of $\sim 0.04(\text{Myr})^{-1}$. We also find periodicities in the data base including small impact craters, which are longer. Though the time/frequency analysis allows us to observe directly phase variations, we cannot find any indications of such changes. Instead, modes display slow variations of power in time. The time/frequency analysis shows a nonstationary behavior of the modes. The power can grow from just above the noise level and then decrease back to its initial level in a time of order of 10 Myrs.

1. INTRODUCTION

Since Raup & Sepkoski (1984) have shown that a $\sim$ 26 Myr periodicity exists in the marine extinction record of the last $\sim$ 200 Myr, it becomes an interesting issue as to whether a similar periodicity can be found in the terrestrial impact crater record (Alvarez & Muller 1984, Davis, Hut, & Muller 1984, Rampino & Stothers 1984a,b, Whitmire & Jackson 1984, Rampino & Caldeira 1992, Clube & Napier 1996, Matsumoto & Kubotani 1996, Napier 2006, Stothers 1998, 2006). Their study on the temporal distribution of terrestrial impact craters has *impacts* on research of the ecological evolution in Earth history as well as the solar system formation and evolution (Bailey, Wilkinson & Wolfendale 1987, Clube & Napier 1984a,b, Davis, Hut, & Muller 1984, Grieve 1987, 1991, Rampino & Stothers 1984b, Stothers 1992, van den Bergh 1994, Nurmi, Valtonen, & Zheng 2001, Whitmire & Jackson 1984). The periodicity for impact craters, however, has remained controversial

for over decades. That is, the question of whether one can regard the period derived from records of major mass extinctions of species as being consistent with cratering records has remained unsettled. Several values for the period have been derived, depending on what data set one adopts for testing and how one analyzes the data set. Analyses so far carried out give a period ranging from ~ 26 to ~ 37 Myr (e.g., Rampino & Stothers 1984b, Stothers 1998, Yabushita 1991, 1992, 1996, 2002). Some authors claim that there is no periodicity at all (Jetsu 1997, Jetsu & Pelt 2000, Grieve & Shoemaker 1994, Tremaine 1986, Weissman 1985, 1990).

It is true that in order to account for claimed periodicity in biological mass extinction by Raup & Sepkoski (1984), several astronomical models were proposed which act as astronomical clocks. Although several astronomical models have been proposed which act as a clock, the only model which is based on astronomical observation and has theoretical plausibility is the one in which the suns' motion above and below the galactic mid-plane plays the required role. The half period of the suns' vertical motion is related to the galactic density. Recent investigations based upon Hipparcos satellite observations have yielded a value close to 0.1 in the solar neighborhood (see Holmberg & Flynn 2000). Compared with this, a period at ~ 35 Myr requires rho to be ~ 0.12 solar mass per cubic parsec along the solar orbit, which may just be compatible with the true galactic density. Therefore, longer periodicity seems with ease explained by an astronomical point of view.

Recently, the impact cratering record of the surface of the Earth has been re-examined in the time domain (Chang & Moon 2005). The authors have demonstrated there exists a ~ 26 Myr periodicity in the impact cratering rate over the last ~ 250 Myr. Such a periodicity can be found consistently in their subsamples regardless of the lower limit of the impact crater diameter up to $D \sim 35$ km. The technique accommodates an oscillating signal whose period is modulated slowly. Immediately following the paper, Napier (2006) and Stothers (2006) dispute that they found a ~ 24, ~ 35 or ~ 42 Myr periodicity and a ~ 30 or ~ 35 Myr periodicity, respectively, pro and con what was reported by the authors.

The aim of this paper is two-fold. Firstly, with an independent method we attempt to test periodicities found by Chang & Moon (2005). We employ the time/frequency analysis. This analyzing technique is now used in many fields of physics and engineering, such as, acoustics, geophysics, helioseismology, image processing, etc. It is a very powerful tool to obtain information on the temporal behavior of an oscillatory signal. We have applied this method, for the first time, to terrestrial impact cratering records. Secondly, we attempt to follow any period variations over the time. The possible origin of such an apparent periodic behavior of impact cratering was ascribed to a periodic comet influx caused by Oort cloud disturbance by oscillations of the Solar system in the Galactic plane, which may also be subject to a periodic modulation. Thus, the period of the terrestrial impact rate may be modulated with time. Under that circumstance, measuring the period by traditional power spectrum methods is likely to be restricted by phase wandering. Hence, one cannot resort for the data set to a traditional Fourier analysis, such as, the Lomb-Scargle analysis. This situation can be improved with the time/frequency analysis. The time/frequency analysis, as for instance wavelet transform, is able to give variations of power or frequency of a mode with time. This information is obtained in a direct way, allowing an easier interpretation.

This paper begins with descriptions of the analysis method in section 2. We present periods of the impact cratering rate obtained with the time/frequency analysis, and discuss results on the impact rate in section 3. Finally, we summarize and conclude in section 4.

2. WAVELET TRANSFORM

The best way to understand the method is comparing with the Fourier transform FT, which is

defined by

$$FT(\nu) = \int S(t) \exp(-i2\pi\nu t)dt, \tag{1}$$

where $S(t)$ is the signal as a function of time t and ν is the cyclic frequency. Compared to the Fourier transform, the Gabor transform is effectively an harmonic analysis with a temporal window having a Gaussian shape, centered at a running time t_0. The Gabor transform GT is thus defined as

$$GT(\nu, t_0) = \int S(t)G(t - t_0) \exp[-i2\pi\nu(t - t_0)]dt, \tag{2}$$

where G is the window function. Actually, many forms of the wave train can be chosen for the wavelet. In the above form the wavelet transform can be considered as a transform of a signal derived from an original wavelet by dilation in time. That is, it is defined as

$$WT(d, t_0) = \int S(t)W\left(\frac{t - t_0}{d}\right) dt, \tag{3}$$

where d is a dilation coefficient.

The wavelets are built from an original wavelet: they are dilated in time by a coefficient d, and then cover different frequency ranges with varying frequencies (and thus temporal resolution). Naively speaking, the wavelet transform is the correlation of the signal $S(t)$ and a running "wavelet" $W(t)$. The temporal information is obtained by computing the correlation, which is a function of finite duration in time. Several wavelet-forms can be selected, but in practice the most popular choice is the Morlet wavelet (Grossmann & Morlet 1984). It is a sinusoidal wave with a Gaussian window W:

$$W\left(\frac{t}{d}\right) = \exp\left[-\pi\left(\frac{t}{d}\right)^2\right] \exp\left(-2\pi i \frac{t}{d}\right) \tag{4}$$

Then, the wavelet transform is an harmonic analysis with a running temporal window varying with frequency. A wavelet coefficient WT depends on two parameters: the time t_0 and the dilation coefficient d. Thus it will provide information around time t_0 and around frequency $\nu = 1/d$. Its squared modulus is proportional to the energy (or power) of the signal. This information is displayed in two-dimensional plots, showing variations of the energy with time and frequency.

One of the issues of this method to take care of is to optimize the resolution, both in time and in frequency domains. The resolution length has to be precisely defined in order to make quantitative comparisons. Time and frequency resolutions (ΔT and $\Delta\nu$) are the half-width at half maximum of the window in respective domains. The parameter of the wavelet allows the resolution to be set to required values. There is a well-known trade-off between frequency resolution and time resolution, mathematically known as the Heisenberg relation, which can be written:

$$\Delta T \Delta\nu \geq C, \tag{5}$$

where the constant C depends on the definition of width (Bracewell 1965). The more widely a function spreads in time, the less widely its Fourier transform spreads in the frequency domain, and vice-versa.

The shape of the window is also an important parameter. It will allow optimization in time and in frequency of the information extracted. The Gaussian window is no longer the best tradeoff in some cases. An exponential window and its Fourier transform, so called the Lorentzian, provide better resolutions.

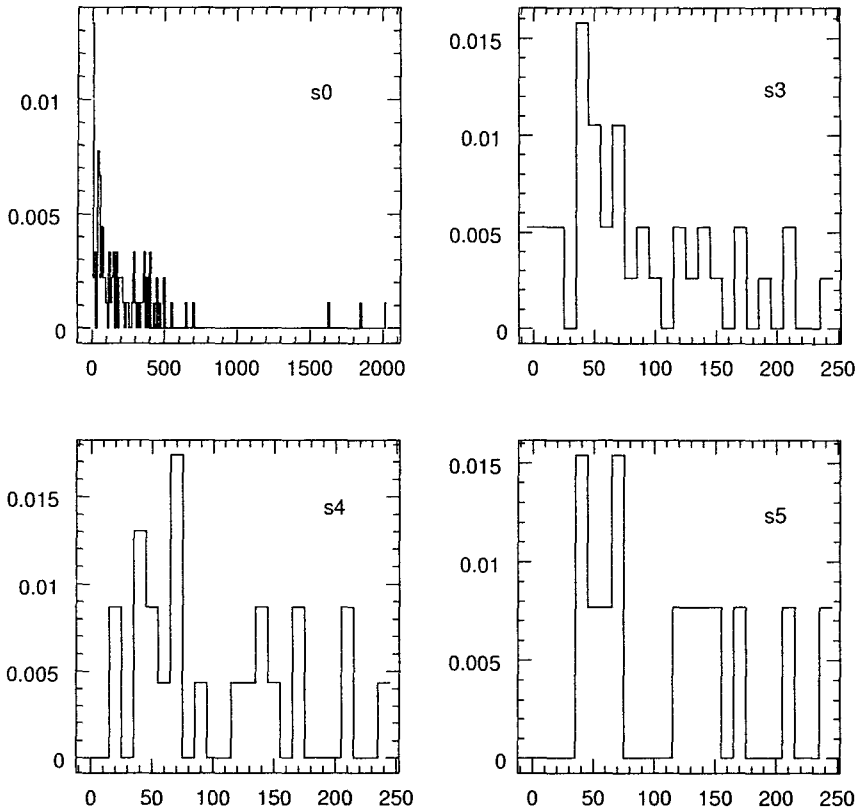$$W\left(\frac{t}{d}\right) = \exp\left(-a\frac{|t|}{d}\right) \exp\left(2\pi i \frac{t}{d}\right). \tag{6}$$

Figure 1. Impact cratering rate per interval of 10 Myrs. The horizontal axis represents time in Myr, and the vertical axis the impact cratering rate. In each panel, data sets used in the analysis are indicated at the upper right corners.

In this case, the same frequency resolution will yield a temporal resolution twice as good in the case of the exponential window. In this paper, we adopt the Lorentzian wavelet transform for the analysis.

3. DATA AND RESULTS

We have applied the method described in the previous section to impact crater data, as collected in Chang & Moon (2005). They used the ages and diameters of impact craters from the updated Earth impact database (Earth Impact Database 2004)[1], in which recently discovered impact craters are constantly included and some recalculated age estimates are updated. They chose the impact craters with error estimates in age, and discarded ones with only upper or lower limits for their age
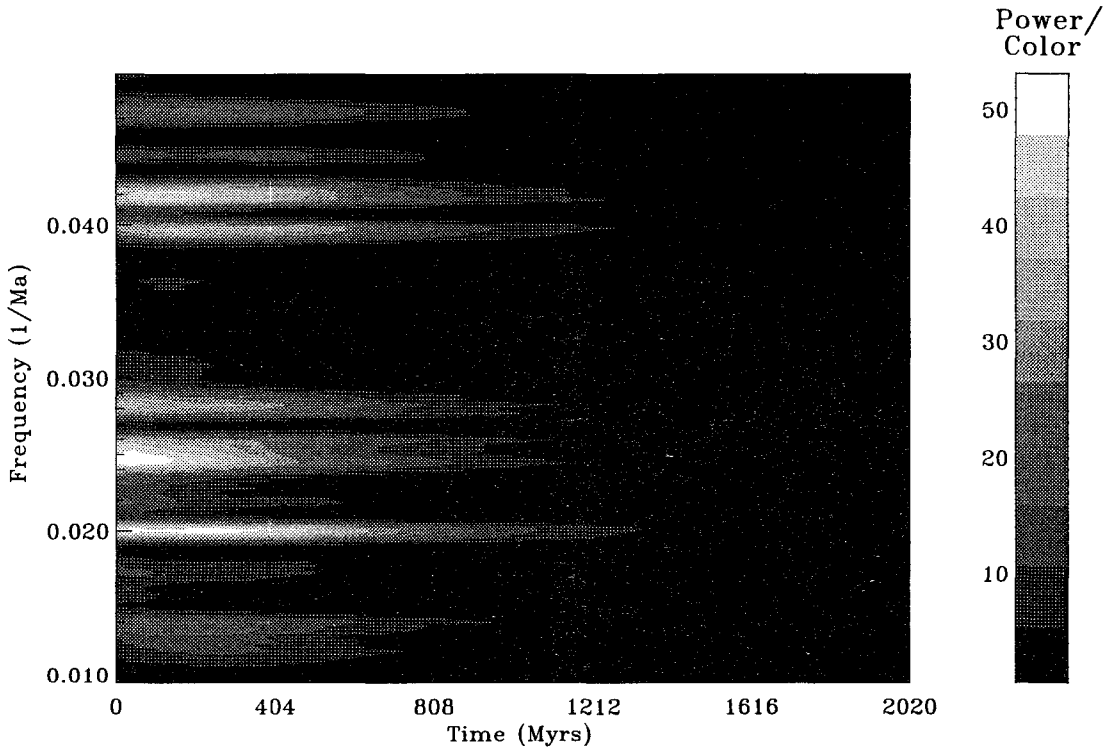
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[1] http://www.unb.ca/passc/ImpactDatabase/index.html

Figure 2. Time/frequency analysis of the data set s0. Power is normalised by the mean value of the noise power.

(see also Moon et al. 2001). By applying these criteria, they ended up with 90 craters among 173 impact craters in the original database, with an age of $t_i \pm \sigma_{t_i}$ [Myr] and a diameter of D_i [km]. They called this data set s0, which has no constraints on size yet. One may find the craters selected for the data set s0 in their Table 1. Other subsets were selected from data set s0 according to the age and diameter, and named as follows:

s1 $t_i \leq 150, \sigma_{t_i} \leq 10, D_i \geq 5, n = 32,$

s2 $t_i \leq 150, \sigma_{t_i} \leq 10, D_i \geq 20, n = 18,$

s3 $t_i \leq 250, \sigma_{t_i} \leq 10, D_i \geq 5, n = 38,$

s4 $t_i \leq 250, \sigma_{t_i} \leq 10, D_i \geq 20, n = 23,$

s5 $t_i \leq 250, \sigma_{t_i} \leq 10, D_i \geq 35, n = 13,$

where n indicates the number of input craters belonging to each data set. In this paper, we analyze data sets s0, s3, s4, s5. Basically, data sets s1, s2 are subsets of s3, s4, respectively. In Figure 1, we show impact cratering rate histograms per interval of 10 Myr for our data sets that we analyze. It was normalized such that the total area under the histogram would equal to unity. The horizontal axis represents time in Myr, and the vertical axis the impact cratering rate. In each panel, data sets are indicated at the upper right corners. Data sets s3, s4 and s5 respectively represent separate sets
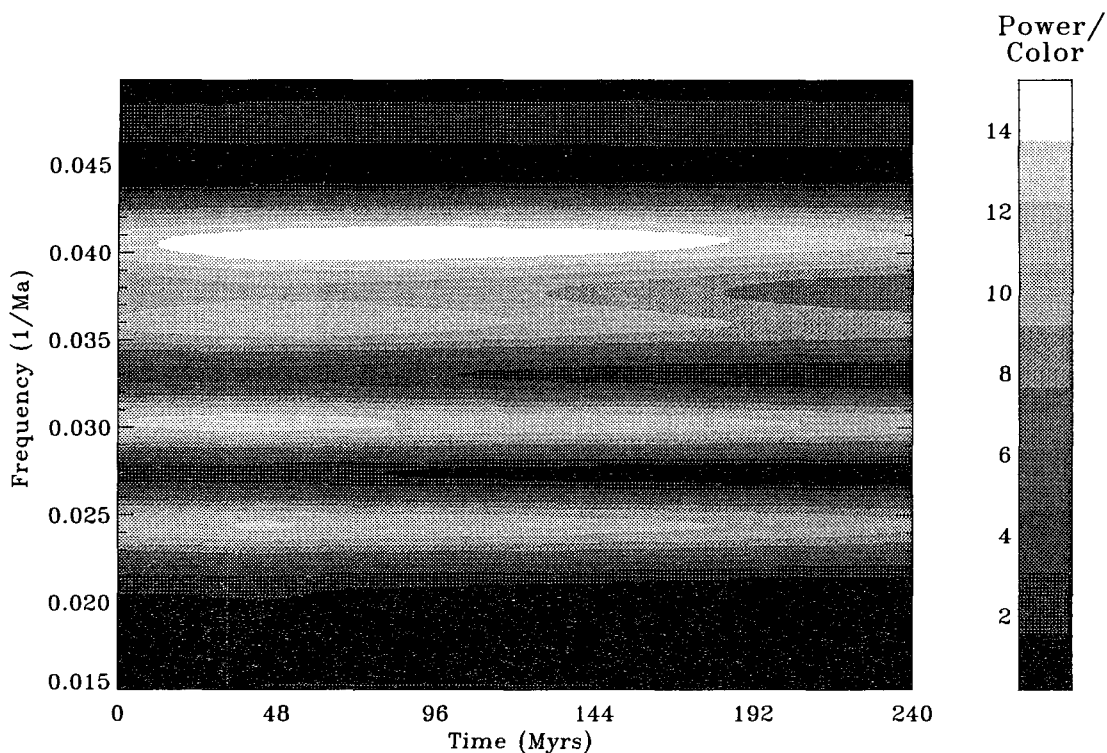
Figure 3. Time/frequency analysis of the data set s3. Power is normalised by the mean value of the noise power.

of impact craters with a different lower limit of diameters that a different average size of impactors would make. Impact craters with $D \sim 5$ km are due to an impact with an asteroid of size $d \sim 250$ m, while impact craters with $D > 20$ km are due to an impact with $d > 1$ km (Jansa, Aubry, & Gradstein 1990).

In Figure 2, we show the time/frequency analysis result of the data set s0. The horizontal axis represents time in Myr, and the vertical axis the cyclic frequency in 1/Myr. The power is shown in gray scale. The power is scaled by the mean value of the noise. The frequency resolution is 2×10^{-4} in that unit, which is much better than required to see any slight frequency variations. Several high power peaks can be seen, whose periods correspond to $\sim 24, \sim 25, \sim 33, \sim 40, \sim 50$ Myrs. It should be noted that peaks are only visible during the first ~ 800 Myrs. It is due to the fact that its *signal* actually exists over the same period as shown in Figure 1. In some sense, the Fourier power spectrum is an average of the time/frequency power spectrum over the time axis. In other words, the Fourier power spectrum is a 2 dimensional one rather than a 3 dimensional time/frequency power spectrum. Hence it is unable to show in which part of the data in time are dominated by a particular *mode* of signals. That is, unlike the Fourier analysis, the time/frequency analysis does show where the *modes* of the signal exists. This is a strong point of the time/frequency analysis over the traditional Fourier-based analysis. By following the high power peaks one may say over which duration the signal show what periodicity. For instance, the peak at ~ 40 Myrs (corresponding to a frequency of $\sim 0.025 \mathrm{Myr}^{-1}$) appears somewhat impulsive. That is, such a *mode* decays so fast
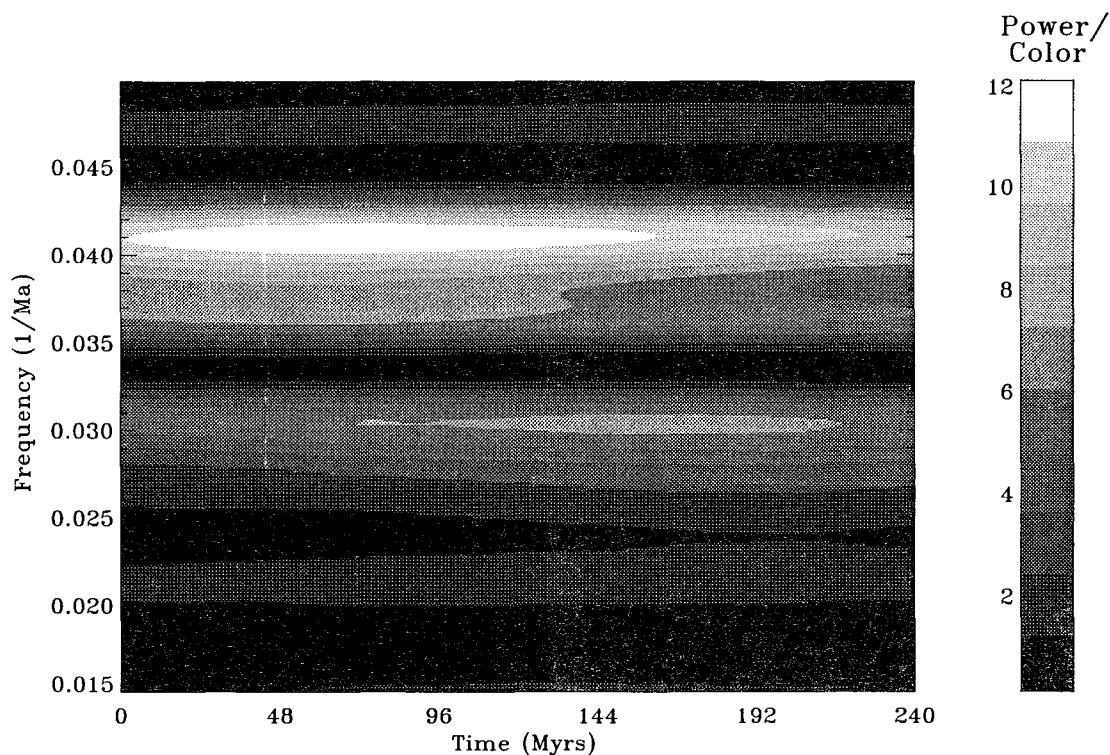
Figure 4. Time/frequency analysis of the data set s4. Power is normalised by the mean value of the noise power.

after $\sim$ 120 Myrs and remains weak over the total duration of $\sim$ 2000 Myrs. On the other hand, the peak at $\sim$ 50 Myrs (corresponding to a frequency of $\sim$ 0.02Myr^{-1}) appears up to $\sim$ 400 Myrs. According to Figure 2, the impact craters covering all sizes show a periodicity of $\sim$ 40 and $\sim$ 50 Myrs, as well as $\sim$ 24 Myrs. They do show the periodicity.

In Figure 3, we show a similar plot as Figure 2, but resulting from the data set s3. Several high power peaks can be seen, whose periods correspond to $\sim$ 24, $\sim$ 28, $\sim$ 32, $\sim$ 40 Myrs. However, three peaks with longer periods are relatively weak. Moreover, the duration over which peaks reach at the highest power is rather shorter. The only peak of a period $\sim$ 24 Myrs is dominant all over the duration of 250 Myrs. Figure 4 shows results from the data set s4 where the limit size of the impact crater gets larger than in s3. Compared with the result from s3 peaks around the periodicity of $\sim$ 40 and $\sim$ 50 Myrs dramatically become insignificant. On the other hand the strong power remains around the period of $\sim$ 24 Myrs. One more interesting point we would remark is that the epoch showing its highest power of peaks of $\sim$ 24 and $\sim$ 40 Myrs appears different in time. It looks as if, in terms of the mode analysis, the power of the shorter period mode are transferred to the longer period one. In Figure 5, the power spectrum of the time/frequency transform of the largest impact craters, that is, the data set s5. It shows high power peaks of the periodicity $\sim$ 27 and $\sim$ 33 Myrs, of which the first one is yet stronger. As noted earlier, however, the significance is low due to the lack of the data. There are only 13 impact craters bigger in diameter than 35 km.
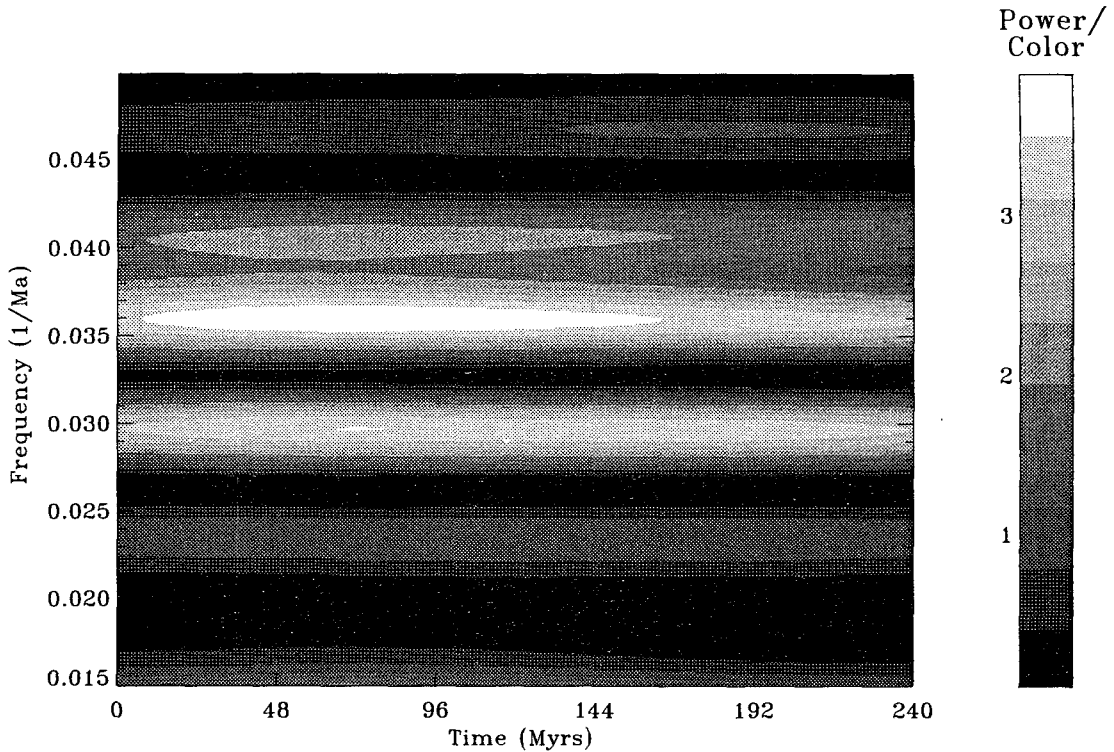
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Figure 5. *Time/frequency analysis of the data set s5. Power is normalised by the mean value of the noise power.*

4. SUMMARY AND CONCLUSION

It is a quite interesting issue whether a periodicity does exist in the terrestrial impact crater records and, if true, what the period would be. The wavelet analysis is applied to the terrestrial impact cratering rate. In this paper we present results of the time/frequency analysis. It is for the first time to apply this technique to the cratering records of the surface of the Earth.

To sum up: (1) Having applied the technique to 4 data samples, selected from the original database according to the upper limit of the age and the lower limit of the diameter, we have found the presence of a ~ 25 Myr periodicity in the terrestrial record over the last ~ 250 Myr. Contrary to what one might expect, our conclusion is apparently insensitive to the cut-off of the diameter up to $D \sim 35$ km. In addition, these periods are of course identical to the frequencies we obtained with the classical Fourier transform. (2) As for the period modulation, we cannot find any indications of serious changes in frequency, as a confirmation of what Chang & Moon found (2005). Hence, the impact cratering rate is found to be as unimodal. (3) The results display the variation of the spectral power as a function of time. The power of modes is shown to vary slowly over a timescale of ~ 200 Myrs on average. Nevertheless, the power can grow from just above the noise level and then decrease back to its initial level in a time of order of 10 Myrs.

In conclusion, we would like to point out the following. Stothers (2006) claims that only the long periodicity ~ 35 Myr survives for the larger craters ($D > 35$ km). The longer period solutions seem

preferred since they are consistent with Galactic density estimates and with the current passage of the Solar system through the plane of the Galaxy. However, it should be stressed that other episodes with shorter periodicity may also be a result of sporadic encounters with spiral arms, or stars.

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