

Low-Complexity Maximum-Likelihood Decoder for VBLAST-STBC Scheme Using Non-square OSTBC Code Rate 3/4

Van-Su Pham, Minh-Tuan Le, Linh Mai, and Giwan Yoon, *Member, KIMICS*

Abstract—This work presents a low complexity maximum-likelihood decoder for signal detection in VBLAST-STBC system, which employs non-square OSTBC code rate 3/4. Stacking received symbols from different symbol duration and applying QR decomposition result in the special format of upper triangular matrix R so that the proposed decoder is able to provide not only ML-like BER performance but also very low computational load. The low computational load and ML-like BER performance properties of the proposed decoder are verified by computer simulations.

Index Terms—Space-time coding, Multiple-Input Multiple-Output (MIMO) system, VBLAST, Wireless Communication System., maximum likelihood detection.

I. INTRODUCTION

The Vertical Bell Laboratories Layered Space Time (VBLAST) code [4] has been known as a high spectral efficiency space time code. However, similar to Orthogonal Space Time Block codes (STBCs), the optimal decoder for VBLAST as well as other high-rate Space Time Codes (STCs) is obviously Maximum-Likelihood (ML) decoder. Unfortunately, unlike the STBCs whose ML decoding scheme is very simple based on only linear processing of the received signal for separate single symbol, the complexity of ML decoder for VBLAST and other high-rate STCs grows exponentially with the increasing of number of transmit antennas, making it impractical when large number of transmit antennas and/or high-order modulation scheme are employed.

In literature, bunches of sub-optimal decoders have been

proposed for VBLAST system [1], [2], [6]. However, in VBLAST systems, using either Zero Forcing (ZF), Minimum Mean Square Error (MMSE) or QR decomposition (QRD) interference suppression, the diversity order of the first decoded layer, so-called the lowest layer, is $G = n_R - n_T + 1$. Thus, if VBLAST systems employ equal transmit and receive antennas, this diversity order is reduced to 1, leading to a very poor system performance. Although, MMSE-VBLAST and MMSE-SQRD [6] show remarkable system performance improvement in comparison with other sub-optimal decoders, the slope of the bit-error-rate (BER) curves indicate that they are able to improve diversity of the system only in low signal-to-noise power ratio (SNR) regime. To increase the diversity order of the first decoded symbol, and thus of the system, a combination of brute-force ML and QRD decoding algorithm was proposed [8]. Because of using brute-force ML, however, the system could hardly achieve high diversity order under the constraint of reasonable complexity. One alternative is the use of combination of VBLAST and STBC [3], that the diversity order is considerably increased compared to that of VBLAST systems, yet at the cost of spectral loss. In this work, by employing the operation principle of our previous work in [7], we proposed a new decoder for MIMO system employing VBLAST-STBC scheme. It is shown that the proposed decoder can significantly reduce the computational load while still obtain the ML-like BER performance.

The remaining parts of the paper are organized as follows. In section II, we describe the system model used in our consideration. The detail of the proposed decoder is presented in section III. The computer simulation results and discussion are given in section IV. Finally, the conclusion is given in section V.

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II. SYSTEM MODEL

Let us consider the VBLAST-STBC MIMO system as depicted in Fig. 1.

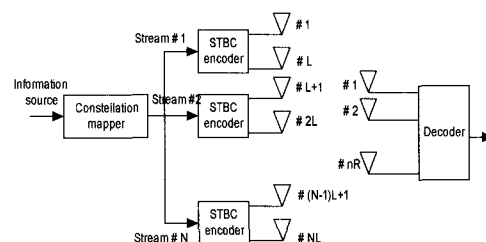


Fig. 1 VBLAST-STBC MIMO system model.

In our system, the number of receive antennas satisfies the condition $n_R \geq N$ and can be less than the number of transmit antennas. The transmitted signals is then written by stacking all STBC signal matrices resulted from STBC encoders of all layers as:

$$G(s) = \begin{pmatrix} G_1(s) \\ G_2(s) \\ \vdots \\ G_N(s) \end{pmatrix} \quad (1)$$

At the receiver, we have the received signal given as:

$$\mathbf{y} = \mathbf{H}G(s) + \mathbf{n} \quad (2)$$

where, \mathbf{n} is the $n_R \times T$ noise matrix of additive Gaussian noise with variance 0.5 per real dimension, T is the number of symbol duration to transmit one STBC. \mathbf{H} denotes the $n_R \times n_T$ complex channel matrix containing independent identical distribution complex fading gains $h_{i,j}$ from the j th transmit antenna to the i th receive antenna. We assume that the channel is Rayleigh flat fading, i.e. the magnitude of the elements of \mathbf{H} have a Rayleigh distribution and the channel is constant over one data frame and independently changed from one to another.

With the assumption that the channel matrix is perfectly known at the receiver, the transmitted signals can be ML decoded as follows:

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in S} \|\mathbf{y} - \mathbf{H}G(s)\|^2 \quad (3)$$

where S is set of signal constellation corresponding to the given modulation scheme.

From the equation (2), let us defined the transmitted signal vector as:

$$\mathbf{s} = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_i \\ \vdots \\ s_{NL} \end{pmatrix} \quad (4)$$

With the definition of \mathbf{s} , the receive signal can be rewritten as:

$$\tilde{\mathbf{z}} = \tilde{\mathbf{H}}\mathbf{s} + \tilde{\mathbf{n}} \quad (5)$$

where $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{n}}$ are the equivalent channel matrix, noise vector respectively. With the equivalent channel matrix, we can directly apply the QRD-based detection [5]-[6] or ZF-VBLAST decoder [4]. However, it is worth emphasizing that those approaches are sub-optimal ones, that obtain low computational load at the expense of

remarkable degradation in bit-error-rate (BER) performance compared to the optimal ML decoder.

We start with the QR decomposition of the equivalent channel matrix $\tilde{\mathbf{H}}$ as $\tilde{\mathbf{H}} = \mathbf{Q}\mathbf{R}$, where the $n_R T \times n_T$ matrix \mathbf{Q} has orthogonal columns with unit norm and the $n_T \times n_T$ matrix \mathbf{R} is an upper triangular matrix.

By pre-multiplying the equation (5) with \mathbf{Q}^H , where $(\cdot)^H$ denotes the Hermitian transform, we obtain the following equation:

$$\tilde{\mathbf{z}} = \mathbf{Q}^H \mathbf{z} = \mathbf{R}\mathbf{s} + \boldsymbol{\eta} \quad (6)$$

In (6), since \mathbf{Q} is a unitary matrix, the noise term $\boldsymbol{\eta} = \mathbf{Q}^H \tilde{\mathbf{n}}$ has the same statistical properties as \mathbf{n} .

Consequently, the ML solution of vector \mathbf{s} in (4), $\hat{\mathbf{s}}$, can be obtained by utilizing either the decision rule in (3) or the following rule:

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in S} \|\tilde{\mathbf{z}} - \mathbf{R}\mathbf{s}\|^2 \quad (7)$$

Due to the upper triangular structure of matrix \mathbf{R} , we can express the k^{th} element of $\tilde{\mathbf{z}}$ as follows:

$$\tilde{z}_k = r_{k,k}s_k + \sum_{i=k+1}^{n_T} r_{k,i}s_i + \eta_k \quad (8)$$

where $r_{i,j}$ is the element in the i^{th} row and j^{th} column of matrix \mathbf{R} .

III. PROPOSED DECODER

For brevity, we exemplify the working of the proposed decoder with the (6,2) system using STBC code rate 3/4 for three transmit antennas given in equation (3.49) of [2]. Thus the equation (6) can be rewritten as follows:

$$\tilde{\mathbf{z}} = \mathbf{R}\mathbf{s} + \boldsymbol{\eta} \quad (9)$$

In (9), the upper triangular matrix \mathbf{R} has the following format:

$$\mathbf{R} = \begin{pmatrix} r_{1,1} & 0 & 0 & r_{1,4} & r_{1,5} & r_{1,6} \\ 0 & r_{2,2} & 0 & r_{2,4} & r_{2,5} & r_{2,6} \\ 0 & 0 & r_{3,3} & r_{3,4} & r_{3,5} & r_{3,6} \\ 0 & 0 & 0 & r_{4,4} & r_{4,5} & r_{4,6} \\ 0 & 0 & 0 & 0 & r_{5,5} & r_{5,6} \\ 0 & 0 & 0 & 0 & 0 & r_{6,6} \end{pmatrix} \quad (10)$$

Let us define the Euclidean distance of the k th element as:

$$d_k = \left| \tilde{z}_k - \sum_{i=k}^{n_r} r_{k,i} s_i \right|^2 \quad (11)$$

From (7) and (11) we have:

$$\hat{s} = \arg \min_{s \in S} \sum_{k=1}^{n_r} d_k \quad (12)$$

From (12), the proposed decoder can be stated as follows. In this statement, \mathbf{s} is the vector containing all the signal points of the transmission constellation, whose size is S . DECSORT() is a function that uses \mathbf{s} , the equation (11) to compute d_k and to sort the values of d_k as well as the corresponding signal points in \mathbf{s} in an ascending order of d_k . The output of DECSORT() are vectors d_k and x_k , which contain the sorted values of d_k and \mathbf{s} respectively.

1. (Initialization) Set $k := n_r$, $D_{\min} := 0$, $D_t := 0$, $D_k := 0$, $D_{n_r+1} := 0$.

2. Set $l_k := 1$, $m := k+1$, compute $\xi_k := \tilde{z}_k - \sum_{j=m}^{n_r} r_{k,j} x_j(l_j)$.

3. Find $[d_k, x_k] := \text{DECSORT}(\xi_k, \mathbf{s})$.

4. If $k = n_r$ then $D_k := d_k(l_k)$ else $D_k := d_k(l_k) + D_m$.

5. Set $x_k(l_k)$ as a solution for s_k , if $k > 4$ then $k := k-1$ and goto Step 2; else compute

$$\xi_k := \tilde{z}_k - \sum_{j=4}^{n_r} r_{k,j} x_j(l_j) \quad \text{and find } [d_k, x_k] := \text{DECSORT}(\xi_k, \mathbf{s}) \quad (k=1,2,3) \quad \text{and}$$

$$D_t := d_1(1) + d_2(1) + d_3(1) + D_4(l_4).$$

6. (Searching) Set $D_{\min} := D_t$, $k := 4$, $l_k := l_k + 1$ and goto Step 8.

7. If $k > 3$ find $[d_k, x_k] := \text{DECSORT}(\xi_k, \mathbf{s})$, $l_k := 1$; else find $[d_k, x_k] := \text{DECSORT}(\xi_k, \mathbf{s})$, $l_k := 1$ for $k = 3, 2, 1$.

8. If $k > 3$ then $D_k := d_k(l_k) + D_{k+1}$ else $D_t := d_3(1) + d_2(1) + d_1(1) + D_4$

9. If $D_t \geq D_{\min}$ or $l_k > S$ then: if $k = n_r$ then terminate and report the result; else set $k := k+1$, $l_k := l_k + 1$ and goto Step 8.

10. If $k > 3$ then let $m := k$, $k := k-1$, compute

$$\xi_k := \tilde{z}_k - \sum_{j=m}^{n_r} r_{k,j} x_j(l_j) \quad \text{and goto Step 7; else set}$$

$$D_{\min} := D_t, \quad \text{update } x_j(l_j) \quad (j=1, \dots, n_r), \quad \text{let } k := k+1, \quad l_k := l_k + 1 \quad \text{and goto Step 8.}$$

IV. COMPUTER SIMULATION RESULTS

To verify performance and complexity of the proposed decoder, we apply it to a VBLAST-STBC scheme in a (6,2) system. In addition, the modulation scheme is QPSK. The frame length is set to equal to 100 STBC symbol periods, i.e. 400 symbol periods.

Figure 2 shows the BER performances versus average bit-energy-to-noise ratio (EbN0) per receive antenna of the proposed decoder, the ZF-QRD decoder and the optimal brute-force ML decoder. It can be seen from the Figure 2 that the proposed decoding scheme obtains almost the same BER performance as the optimal ML decoder does, whereas it significantly outperforms the ZF-QRD decoder in term of BER performance.

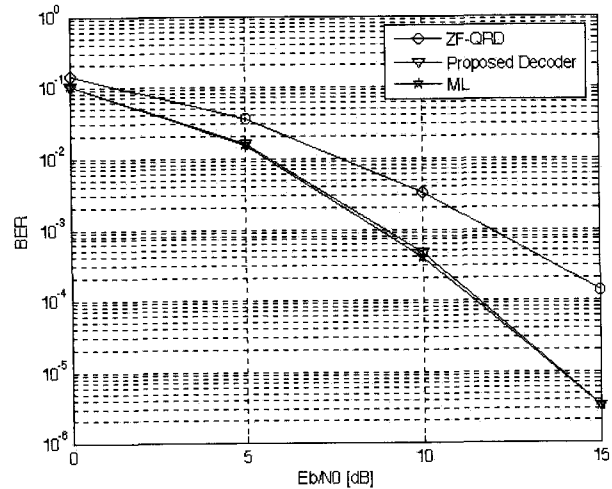


Fig. 2 BER performances of proposed decoder, ML decoder and ZF-QRD decoder for (6,2) VBLAST-STBC MIMO system using non-square OSTBC code rate 3/4; QPSK modulation.

The average computational loads per symbol period of our proposed decoder, ZF-QRD decoder and optimal brute-force ML decoder in term of complexity addition and multiplication operations are given in the Table I. In this comparison, we only concern the complexity in searching stage. The parameters for the simulation are the same as those used for Figure 2. In addition, the EbN0 is kept constant at 10dB, the number of iterations is set to 5×10^3 .

Table 1. Comparison of average complexity per symbol period of the proposed decoder and its counterparts

Decoder	Number of additions	Number of multiplications
ZF-QRD	20	26
Proposed Decoder	59	51
ML	3520	3584

It can be seen from the Table I that the complexity of the proposed decoder is comparable to that of ZF-QRD decoder while it is remarkably smaller than that of the optimal ML decoder.

V. CONCLUSIONS

In this work, the new low-complexity ML-like performance decoder for VBLAST-STBC MIMO system employing non-square OSTBC code rate 3/4 has been proposed. The proposed decoder is shown to be capable of not only providing the systems with ML-like performance but also obtaining extremely low complexity. Although it suffers from spectral efficiencies loss by nature of VBLAST-STBC scheme, the significant low complexity and the possibility of usage in case number of receive antennas less than number of transmit antennas make it to be a very promising decoder for practical applications.

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