

# Design of MTLMS Based Decision Feedback Equalizer

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**Abstract**—A key issue toward mobile multimedia communications is to create technologies for broadband signal transmission that can support high quality services. Such a broadband mobile communications system should be able to overcome severe distortion caused by time-varying multi-path fading channel, while providing high spectral efficiency and low power consumption. For these reasons, an adaptive suboptimum decision feedback equalizer (DFE) for the single-carrier short-burst transmissions system is considered as one of the feasible solutions. For the performance improvement of the system with the short-burst format including the short training sequence, in this paper, the multiple-training least mean square (MTLMS) based DFE scheme with soft decision feedback is proposed and its performance is investigated in mobile wireless channels throughout computer simulation.

**Index Terms**—Decision Feedback Equalizer, multiple-training, DFE

## I. INTRODUCTION

The recursive least squares (RLS)-type algorithms have been used commonly because these algorithms provide a fast converging property. But, these algorithms require high computational complexity and also provide a numerical instability when the eigenvalue spread of the input correlation matrix is large [1]. As a consequence, the RLS based equalizer consumes a large amount of the computational power at the receiver. By contrast, the least mean square (LMS) algorithm has low computational complexity but the convergence is very slow when the eigenvalue spread of the input correlation matrix is large. A multiple-training LMS (MTLMS) algorithm has been known as an effective adaptive algorithm [1] that can provide the desired converging performance with a competitive computational complexity in such short-burst transmissions with a short training sequence. This algorithm has mitigated the problem of the slow convergence by using the multiple-training method, i.e., the reuse of the received training

symbols and of the numerical instability by regularizing the solution of the adaptive coefficient vector such that the sensitivity to small eigenvalues is minimal while this capability is absent from the conventional LMS algorithm. Recently this MTLMS algorithm has been applied to the mobile wireless communications system, especially IS-136 receiver [2].

In this paper, to mitigate the effect of error propagation and provide robustness at low SNRs, we propose MTLMS based DFE with a simple soft decision feedback device and investigate the performance of the equalizer according to the iterations parameter, the length of the training sequence and the Doppler frequency in mobile wireless channels throughout the computer simulations.

## II. MULTIPLE-TRAINING LMS BASED DFE

Let the burst format be composed of the training sequence and the message sequence. If the DFE using the MTLMS algorithm is in the training mode, the received training sequence is repeatedly trained up to a pre-assigned iteration number,  $K$ . Then the tracking mode is operated to acquire the equalized message sequence. In the training mode, the DFE tap coefficients are acquired from the last iteration. In the tracking mode, the message symbols are equalized with these converged DFE tap coefficients as its initials. In [2], the MTLMS algorithm was also used extensively in the tracking mode for exploring fully the decision information. However, the performance improvement is very slight but the complexity is increased. So, this operation is not considered.

In the MTLMS based DFE, the DFE output  $\hat{a}^q(n)$  at the  $q$  th iteration,  $1 \leq q \leq K$  for training and  $q=1$  for tracking, is given by

$$\hat{a}^q(n) = \sum_{i=0}^{N_f-1} g_f^q(n; i) r(n-i) + \sum_{j=1}^{N_b} g_b^q(n; j) d^q(n-j) \quad (1)$$

where  $g_f^q(n; i)$  and  $g_b^q(n; j)$  represent the feed forward filter (FFF) and feedback filter (FBF) tap coefficients at  $q$  th iteration, respectively.  $N_f$  and  $N_b$  are the length of FFF and FBF, respectively.  $d^q(n-j)$  represents the feedback symbol which is the known symbol  $a(n-j)$  for training mode and the previously detected hard- or soft-decision symbol  $\tilde{a}^q(n-j)$  for tracking mode. Note that  $r(n-i)$  is the received signal which has the same value for

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all iterations and so the superscript  $q$  can be dropped. The DFE tap update equation using MTLMS algorithm at the  $q$  th iteration can be represented as

$$g_f^q(n+1; i) = g_f^q(n; i) + \mu_f e^q(n) x^*(n-i) \quad \text{for } i = 0, 1, \dots, N_f - 1 \quad (2)$$

$$g_b^q(n+1; j) = g_b^q(n; j) + \mu_b e^q(n) d^{q*}(n-j) \quad \text{for } j = 0, 1, \dots, N_b \quad (3)$$

where the superscript  $*$  denotes the complex conjugation and  $\mu_f$  and  $\mu_b$  represent the FFF step size and the FBF step size, respectively.  $x(n-i)$  is the  $j$  th power normalized output element of the received sequence and given by

$$x(n-i) = r(n-i) / \sqrt{\varepsilon + P(n; i)} \quad (4)$$

where  $P(n; i)$  is the instantaneous power estimate of  $r(n-i)$  and  $\varepsilon$  is a small constant that eliminate overflow when the value of  $P(n; i)$  are very small. For computing the values of  $P(n; i)$ , the exponential weighted method was used as follows

$$P(n; i) = \beta P(n-1; i) + (1-\beta) |r(n-i)|^2 \quad (5)$$

where  $\beta$  is the forgetting factor between 0 and 1. The error signal is computed by

$$e(n) = \hat{a}^q(n) - d^q(n) \quad (6)$$

In the MTLMS algorithm, note that the initial weight vector at  $q$  th iteration is the same as the last updated weight vector at  $(q-1)$  th iteration. In addition, the term “normalized” was dropped for convenience. The performance of the MTLMS based DFE becomes better with the increase of the iterations number  $K$ . However, the computational complexity also increases linearly with  $K$ . The MTLMS algorithm has several merits over other algorithms such as LMS and RLS [1]. The MTLMS algorithm can permit the faster tracking performance than the LMS algorithm in the time-varying channel. In addition, because the MTLMS algorithm performs regularization in solving for the adaptive coefficients, it is more robust to noise for spectrally nulled data than LMS algorithm. The RLS algorithm was shown to have instability and noise amplification properties that are traceable to the large eigen value spread of the data correlation matrix. But, the MTLMS algorithm does not suffer from these problems.

The problem of a DFE approach is the error propagation. In this section, the simple soft decision feedback device is described. While the optimum soft feedback is estimated using maximum a posteriori probability (MAP) algorithms, the simple soft decision method is acquired by approximating the optimum approach and requires only the simple operation of

passing the DFE output through a (soft) nonlinear function.

Note that although the a posteriori probability of  $a(n)$  is, in general, a function of all available observations, it is the current observation  $\hat{a}(n)$  that contributes the most to the value of this probability (since  $\hat{a}(n)$  is the equalized output corresponding to  $a(n)$ ). Thus, it is assumed that the soft feedback  $\tilde{a}(n)$  is a function only of the current observation  $a(n)$ . Accordingly,

$$\tilde{a}(n) = E[a(n) | \hat{a}(n)] = \sum_{a(n)} a(n) P(a(n) | \hat{a}(n)) \quad (7)$$

and

$$P(a(n) | \hat{a}(n)) = \frac{P(a(n), \hat{a}(n))}{P(\hat{a}(n))} = \frac{P(\hat{a}(n) | a(n)) P(a(n))}{\sum_{a(n)} P(\hat{a}(n) | a(n)) P(a(n))} \quad (8)$$

where the a priori probability  $P(\hat{a}(n) | a(n))$  can be given as

$$P(\hat{a}(n) | a(n)) = \frac{1}{\sqrt{\pi/\gamma}} \text{Exp}(-\gamma |\hat{a}(n) - a(n)|^2) \quad (9)$$

where  $\gamma$  is the signal to ISI-plus-noise ratio. Using Eq. (7), (8) and (9), and assuming that  $a(n)$  is QPSK, the following soft decision function is obtained:

$$\tilde{a}(n) = f(\hat{a}(n)) = \frac{1}{\sqrt{2}} \left( \tanh(\sqrt{2}\gamma \text{Re}(\hat{a}(n))) + j \tanh(\sqrt{2}\gamma \text{Im}(\hat{a}(n))) \right) \quad (10)$$

Soft feedback is obtained simply by passing the real part and the imaginary part of the DFE output through a hyperbolic tangent function. This method requires the knowledge of the signal to ISI-plus-noise ratio,  $\gamma$ . However, an appropriate fixed value of  $\gamma$  can be chosen without greatly affecting the achievable performance of the soft-feedback DFE. Fig. 1 shows the soft decision feedback architecture.

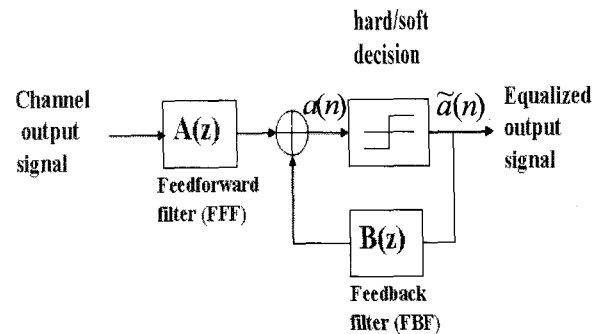


Fig. 1 Soft decision feedback

### III. CHANNEL MODEL FOR SIMULATION

Mobile radio channels can be modeled as multipath Rayleigh fading channels having an impulse response.

$$h(t; \tau) = \sum_{l=1}^{L-1} \alpha_l(t) \delta(\tau - \tau_l) \quad (11)$$

where the coefficient  $\alpha_l(t)$  is the  $l$ th multipath gain which is modeled as complex Gaussian random processes with zero mean. Coefficients of power delay profiles are COST-207 channel coefficients used in [3] (some modification is done for urban model).

A QPSK signal is transmitted. Each transmitted burst contains the training sequence of variable length and the message sequence of the length of 144 (only for the purpose of the simulation). The length of the training sequence are 12, 16, 20, 24, 28, 32 and 64 which correspond to the overhead of about 7.7%, 10%, 12.2%, 14.3%, 16.3%, 18% and 31%, respectively. Note that the small overhead represents high spectral efficiency.

The carrier frequency is 5GHz and the channel bandwidth is 1MHz. The symbol interval is  $1\mu s$ . For channel model the FFF length was set to be 11 and the FBF length was set to be 9. The FFF step size was 0.05 and the FBF step size was 0.005 for both channels. For the soft decision feedback DFE,  $\gamma=5dB$  is used [4].

#### IV. SIMULATION AND RESULTS

In Table 1, the computational complexities of the MTLMS, RLS, fast RLS (FRLS), LMS, and power-normalized LMS (NLMS) algorithms are compared in terms of the number of operations per input sample for training mode.  $N$  denotes the number of equalizer tap coefficients. The extensive computer simulations have been carried out to compare and evaluate the performance of the MTLMS based DFE with hard or soft decision feedback.

Table 1. Complexity comparisons of various algorithms

Algorithm	Complex multiplications	Complex division
MTLMS	$K(2N+1)$	$N$
RLS	$2.5N^2 + 4.5N$	2
FRLS	$20N+5$	3
LMS	$2N+1$	0
NLMS	$2N+1$	$N$

Since the MTLMS algorithm is the block-iterative algorithm, the performance of a MTLMS based DFE depends on the iterations parameter ( $K$ ) and the length of the training sequence ( $A$ ). Therefore, the effects of the iterations parameter ( $K$ ) and the length of training sequence ( $A$ ) on the BER performance are investigated. The slow fading channel with a normalized Doppler frequency of 0.00012 ( $f_d=120$ ) was assumed. This implies that the minimum time between the two fading nulls is 4.2ms ( $1/2 f_d$ ), which is much longer than the considered maximum burst length of 208 $\mu s$  (this is in the case of  $A=64$ ). The SNR ( $E_b/N_0$ ) was 18dB.

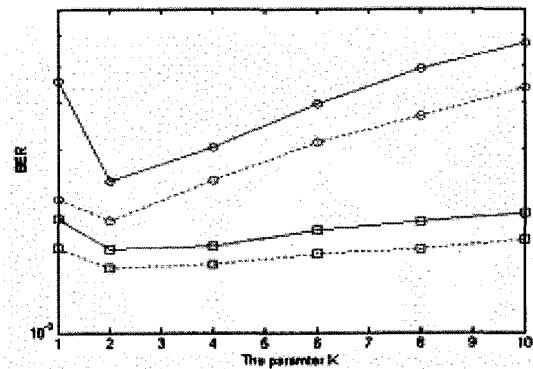


Fig. 2 BER performance of a MTLMS based DFE as a function of the iterations parameter ( $K$ ). SNR=18dB,  $f_d T_s=0.00012$ , Solid line: hard decision; Dotted line: soft decision, Circle: 16 training; Square: 32 training

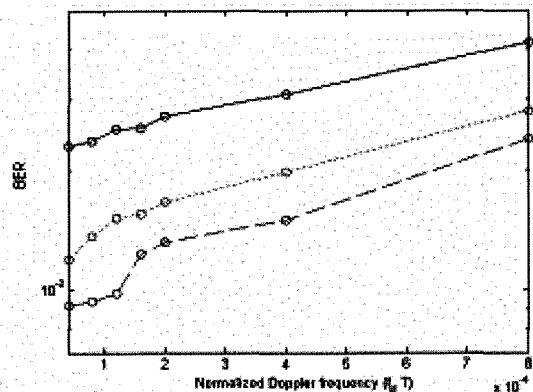


Fig. 3 BER performance of a MTLMS based DFE as a function of the normalized Doppler frequency.  $K=4$ , SNR=18dB, Soft decision feedback, Solid line: 16 training; Dotted line: 32 training; Dashed line: 64 training.

Fig. 2 shows the BER performance of a MTLMS based DFE as a function of the iterations parameter ( $K$ ). It is shown that the performance is improved as the value of  $K$  increases while the computational complexity is proportional to  $K$ . Since the FFF step size of 0.05 is not small enough to achieve the stable MSE performance the BER performance is also not stable. To achieve the stable performance, a smaller FFF step size is needed. However, since the large FFF step size can give the faster converging performance and the smaller complex multiplications, the FFF step size of 0.05 is hold. A MTLMS based DFE with the soft decision feedback shows the better performance than a MTLMS based DFE with the hard decision feedback at SNR of 18dB.

Fig. 3 shows the BER performance of a MTLMS based DFE as a function of a slot-normalized Doppler frequency. As a normalized Doppler frequency increases, the BER performance becomes worse. Note that the performance of a DFE with the largest  $K$  goes through the faster degradation. The reason is that the TDMA slot with the larger  $K$  has a more chance of a deep fade during a given transmit time and the TDMA slot of a deep fade cannot be equalized reliably.

## V. CONCLUSIONS

The MTLMS algorithm has mitigated the problem of the slow convergence by using the multiple-training method with a competitive computational complexity in such short-burst transmissions by using a short training sequence. Soft decision feedback device can mitigate the effect of error propagation and provide robustness at low SNR. With these attractive features, in this paper, a MTLMS based DFE method with soft decision feedback was proposed and its performance was investigated in mobile wireless channels throughout the computer simulations. Simulation results show that the better performance can be achieved as the length of the training sequence increases, but the spectral efficiency is lowered and the system becomes weaker to time-varying fading. The more training sequences are required in the higher normalized Doppler frequency, and MTLMS with soft decision feedback shows better BER performance than the case of hard decision.

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