

예측필터를 이용한 확장칼만필터 고장검출 및 SDINS에의 적용

Fault Detection for Extended Kalman Filter Using a Predictor and Its Application to SDINS

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ABSTRACT

In this paper, a new fault detection method for the extended Kalman filter, which uses a N -step predictor, is proposed. The N -step predictor performs the only time propagations for N -step intervals without measurement updates and its output is used as a monitoring signal for the fault detection. A consistency between the extended Kalman filter and the N -step predictor is tested to detect a fault. A test statistic is defined by the difference between the extended Kalman filter and the N -step predictor. The proposed method is applied to strapdown inertial navigation system (SDINS). By computer simulation, it is shown that the proposed method detects a fault effectively.

주요기술용어(주제어) : Fault Detection, Extended Kalman Filter(EKF), N -Step Predictor, SDINS

1. INTRODUCTION

The statistical tests on filter innovations have been widely used for the detection of fault in stochastic dynamical systems^[1]. As a kind of statistical tests for real time fault detection in Kalman filter, the state chi-square test whose basic concepts were proposed by Kerr^[2,3] was used in the previous studies^[4,5]. In those methods, a state predictor was used to provide a monitoring

signal for fault detection, and the state chi-square test was used to check the consistency between the state estimate of the Kalman filter and that of the state predictor. But, if just one predictor is applied for generating the monitoring signal, the failure sensitivity may be gradually degraded as time runs because of the accuracy of the predictor. To improve the failure sensitivity of the state chi-square test, the use of a pair of state predictors that are alternatively reset with the Kalman filter was proposed by Ren Da^[6]. However, the use of a pair of state predictors causes the test statistic to change abruptly at the reset time of the predictors, which may

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increase the false alarm rate. Also the detection method is a little complicated because of using *two* predictors.

To overcome such problems, a new approach to the fault detection involving the extended Kalman filter is proposed in this paper. The proposed method involves the use of a N -step predictor, which is based on measurements made up to the $(k - N)$ th step, in order to produce a monitoring signal for the extended Kalman filter. The test statistics for fault detection are defined by the differences between the extended Kalman filter and the N -step predictor. The proposed fault detection method is applied to a strapdown inertial navigation system (SDINS) integrated with Global Positioning System (GPS), and its fault detection performance is examined and compared with the residual test by computer simulation.

2. State chi-square test for fault detection

The basic concept of the two-ellipsoid test for fault detection proposed by Kerr^[2] is to determine whether two estimates of the same parameter $x(k)$, agree within the expected uncertainties of the estimates. This fault detection method was modified into simpler closed form called for state chi-square test^[4]. In this section, the state chi-square test for fault detection in the extended Kalman filter is revisited.

Consider the following discrete-time linear system:

$$x(k+1) = \Phi(k+1, k)x(k) + G(k)w(k) \quad (1)$$

$$z(k) = H(k)x(k) + \nu(k) \quad (2)$$

where $x(k) \in R^n$ and $z(k) \in R^m$ represent the state and the measurement vector, respectively.

The process noise $w(k)$ and measurement noise $\nu(k)$ are assumed to be white Gaussian noise processes with zero mean which are mutually uncorrelated. The noise covariance kernels are $E\{w(i)w^T(j)\} = Q(i)\delta_{ij}$ and $E\{\nu(i)\nu^T(j)\} = R(i)\delta_{ij}$, respectively, where δ_{ij} is the Kronecker-delta function.

In the state chi-square test, the test statistics are defined by using two estimates: Kalman filter output $\hat{x}_f(k)$, and a monitoring signal $\hat{x}_m(k)$, which is the estimate from the N -step predictor discussed in next section. If the monitoring signal and its covariance matrix are given, then, the test statistics can be defined as follows.

Let us define the following variables to produce the test statistic for fault detection in Kalman filter:

$$e_f(k) = x(k) - \hat{x}_f(k) \quad (3)$$

$$e_m(k) = x(k) - \hat{x}_m(k) \quad (4)$$

$$\beta(k) = e_m(k) - e_f(k) = \hat{x}_f(k) - \hat{x}_m(k) \quad (5)$$

$$\begin{aligned} B(k) &= E\{\beta(k)\beta^T(k)\} \\ &= P_f(k) - P_{fm}(k) - P_{fm}^T(k) + P_m(k) \end{aligned} \quad (6)$$

where $x(k)$ is the true state vector, $e_f(k)$ is the estimation error from Kalman filter, $e_m(k)$ is the estimation error from the predictor, and $P_{fm}(k)$ is the cross covariance matrix between the N -step predictor and Kalman filter. Then, the test statistic is defined as

$$l(k) = \beta^T(k)B^{-1}(k)\beta(k) \quad (7)$$

The defined test statistic $l(k)$ is chi-square distributed with n degrees of freedom. The test

for fault detection is

$$\begin{cases} l(k) \geq \epsilon_\beta & \text{fault} \\ l(k) < \epsilon_\beta & \text{no fault} \end{cases} \quad (8)$$

where the threshold ϵ_β is determined from the chi-square distribution tables with the selected false alarm rate (FAR).

If the concerned system is linear and the initial conditions of the filter and the predictor are the same, then Eq. (6) can be simplified as follows^[4];

$$B(k) = P_m(k) - P_f(k) \quad (9)$$

However, if the system is nonlinear, such as in case of an SDINS, the state transition matrix of the predictor $\Phi_m(k)$ is not identical to that of the extended Kalman filter $\Phi_f(k)$ ^[7]. Therefore, the cross covariance matrix $P_{f_m}(k)$ should be reconsidered to calculate the test statistic $l(k)$.

The cross covariance matrix $P_{f_m}(k)$ is calculated by means of the following equations:

$$\begin{aligned} P_{f_m}(k+1|k) &= E\{e_f(k+1|k)e_m^T(k+1|k)\} \\ &= \Phi_f(k+1, k)P_{f_m}(k|k)\Phi_m^T(k+1, k) \\ &\quad + G_f(k)Q(k)G_m^T(k) \end{aligned} \quad (10)$$

$$\begin{aligned} P_{f_m}(k|k) &= E\{e_f(k|k)e_m^T(k|k)\} \\ &= E\{[(I - K_f(k)H_f(k))e_f(k|k-1) \\ &\quad - K_f(k)\nu(k)]e_m^T(k|k-1)\} \\ &= [I - K_f(k)H_f(k)]P_{f_m}(k|k-1) \end{aligned} \quad (11)$$

where $K_f(k)$ is Kalman gain, and $H_f(k)$ is measurement matrix.

Now, if a well-defined monitoring signal, $\hat{x}_m(k)$, its covariance, $P_m(k)$, and the cross covariance, $P_{f_m}(k)$ are given, the test statistic can be

calculated to detect a fault in the extended Kalman filter. In next section, the monitoring signal for fault detection in the extended Kalman filter is discussed.

3. Monitoring signal by N -step predictor

If a state predictor is used to get a monitoring signal for consistency test, the sensitivity of a fault is gradually degraded because the covariance of the signal increases in magnitude as time runs. To avoid such a problem, the application of *two* predictors, which are alternatively reset by Kalman filter output, was proposed by Ren Da^[6]. In that method, however, the test statistics may change abruptly when the predictors are reset. In this section, N -step predictor is proposed for generating a monitoring signal for fault detection in the extended Kalman filter, in order to overcome the problem.

Let us consider the following nonlinear state prediction equation:

$$\dot{\hat{x}}(t) = f(\hat{x}(t)) \quad (12)$$

where the predicted time interval is $t_{k-1} \leq t < t_k$ with initial condition $\hat{x}(t_{k-1}) = \hat{x}(t_{k-1}|t_{k-1})$. We will use the time index k instead of t_k for convenience, from now on.

From Eq. (12), the predicted state vector at k -th step can be approximated as

$$\begin{aligned} \hat{x}(k|k-1) &= f(\hat{x}(k-1|k-1))\Delta T \\ &\quad + \hat{x}(k-1|k-1) \end{aligned} \quad (13)$$

where ΔT is the measurement update interval and $\hat{x}(k|k-1)$ is the state vector based on the measurements $Z_{k-1} = \{z_1, z_2, \dots, z_{k-1}\}$.

Then, the state vector $\hat{x}(k)$ based on the measurement set $Z_{k-2} = \{z_1, z_2, \dots, z_{k-2}\}$ can be approximately written as

$$\begin{aligned} \hat{x}(k) &= f(\hat{x}(k-1|k-2))\Delta T + \hat{x}(k-1|k-2) \\ &= f(\hat{x}(k-1|k-1))\Delta T + \hat{x}(k-1|k-1) \\ &\quad - [I + F_{k-1}\Delta T]\delta\hat{x}_{k-1} \\ &= \hat{x}(k|k-1) - \Phi(k, k-1)\delta\hat{x}_{k-1} \end{aligned} \quad (14)$$

where $\delta\hat{x}_{k-1}$ is the estimated error of $x(k-1)$ using z_{k-1} , that is, $K_{k-1}[z_{k-1} - h(\hat{x}(k-1|k-2))]$, and $F_{k-1} = \left. \frac{\partial f(x)}{\partial x} \right|_{x=\hat{x}(k-1|k-1)}$,

$$\Phi(k, k-1) = [I + F_{k-1}\Delta T].$$

From Eq. (14), the state vector $\hat{x}(k-N+1)$ at the $(k-N+1)$ th step without using the measurements $\{z_{k-N}, z_{k-N+1}\}$ can be approximated as

$$\begin{aligned} \hat{x}(k-N+1) &= \hat{x}(k-N+1|k-N+1) - \delta\hat{x}_{k-N+1} \\ &\quad - \Phi(k-N+1, k-N)\delta\hat{x}_{k-N} \end{aligned} \quad (15)$$

where $\Phi(k-N+1, k-N)$ can be similarly defined as Eq. (14). The state $\hat{x}(k-N+2)$ without using the measurements $\{z_{k-N}, z_{k-N+1}\}$ can be approximated as

$$\begin{aligned} \hat{x}(k-N+2) &= f(\hat{x}(k-N+1))\Delta T + \hat{x}(k-N+1) \\ &\quad - \Phi(k-N+2, k-N+1)[\delta\hat{x}_{k-N+1} \\ &\quad - \Phi(k-N+1, k-N)\delta\hat{x}_{k-N}] \\ &= \hat{x}(k-N+2|k-N+2) - \delta\hat{x}_{k-N+2} \\ &\quad - \Phi(k-N+2, k-N+1)\delta\hat{x}_{k-N+1} \\ &\quad - \Phi(k-N+2, k-N)\delta\hat{x}_{k-N} \end{aligned} \quad (16)$$

If the procedure mentioned above is repeated, then the predicted state $\hat{x}(k-1)$ at the $(k-1)$ th step without using the measurements $\{z_{k-N}, z_{k-N+1}, \dots, z_{k-2}\}$ can be written as

$$\begin{aligned} \hat{x}(k-1) &= \hat{x}(k-1|k-1) - \delta\hat{x}_{k-1} \\ &\quad - \sum_{j=k-N}^{k-2} \Phi(k-1, j)\delta\hat{x}_j. \end{aligned} \quad (17)$$

Therefore, the predicted state $\hat{x}(k)$ at the k -th step without using the measurements $\{z_{k-N}, z_{k-N+1}, \dots, z_{k-1}\}$ in a nonlinear system can be approximated as

$$\hat{x}(k) = \hat{x}(k|k-1) - \sum_{j=k-N}^{k-1} \Phi(k, j)\delta\hat{x}_j \quad (18)$$

where $\hat{x}(k|k-1)$ is the estimated state obtained from the extended Kalman filter. The predicted state $\hat{x}(k)$ given by Eq. (18) can be used as the monitoring signal $\hat{x}_m(k)$ defined in previous section. Therefore, we will use the state vector $\hat{x}_m(k)$ instead of $\hat{x}(k)$ for convenience.

To derive the covariance of the state vector $\hat{x}_m(k)$, let us define the prediction error $e_m(k)$ as

$$\begin{aligned} e_m(k) &= x(k) - \hat{x}_m(k) \\ &= e(k|k-1) + \sum_{j=k-N}^{k-1} \Phi(k, j)\delta\hat{x}_j \end{aligned} \quad (19)$$

Then, the covariance matrix $P_m(k)$, which is the covariance of the predicted state vector $\hat{x}_m(k)$ given by Eq. (18), can be written as

$$\begin{aligned}
 P_m(k) &= E\{e_m(k)e_m^T(k)\} \\
 &= E\left\{e(k|k-1) + \sum_{j=k-N}^{k-1} \Phi(k,j)\delta\hat{x}_j\right\} \\
 &\quad \left\{e(k|k-1) + \sum_{j=k-N}^{k-1} \Phi(k,j)\delta\hat{x}_j\right\}^T \quad (20)
 \end{aligned}$$

In Eq. (20), it can be demonstrated that $E\{\delta\hat{x}_{k-j}e^T(k|k-1) \mid j=k-N, \dots, k-1\}$ is equal to zero by the orthogonal projection lemma^[8]. Therefore, the covariance equation (20) can be written as

$$\begin{aligned}
 P_m(k) &= P(k|k-1) \\
 &\quad + \sum_{j=k-N}^{k-1} \Phi(k,j)E\{\delta\hat{x}_j\delta\hat{x}_j^T\}\Phi^T(k,j) \\
 &= P(k|k-1) + \\
 &\quad \sum_{j=k-N}^{k-1} \Phi(k,j)\{P(j|j-1) - P(j|j)\}\Phi^T(k,j) \quad (21)
 \end{aligned}$$

where $P(j|j-1)$ and $P(j|j)$ are the covariance matrix of the state vector $\hat{x}(j|j-1)$ and $\hat{x}(j|j)$, respectively.

The cross covariance matrix $P_{fm}(k)$ between the N -step predictor and the extended Kalman filter must be considered to define the test statistics. From the definition, $P_{fm}(k)$ can be written as

$$\begin{aligned}
 P_{fm}(k) &= E\{e(k|k)e_m^T(k)\} \\
 &= E\{e(k|k)e^T(k|k-1)\} \\
 &\quad + E\left\{e(k|k)\left[\sum_{j=k-N}^{k-1} \Phi(k,j)\delta\hat{x}_j\right]^T\right\} \quad (22)
 \end{aligned}$$

In Eq. (22), the second term on the right hand side is equal to zero according to the orthogonal projection lemma^[8]. Therefore, the cross covariance matrix $P_{fm}(k)$ can be written as

$$\begin{aligned}
 P_{fm}(k) &= E\{e(k|k)e^T(k|k-1)\} \\
 &= [I - K_f(k)H_f(k)]P_f(k|k-1) \\
 &= P_f(k|k) \quad (23)
 \end{aligned}$$

where $P_f(k|k)$ is the covariance matrix of the state vector $\hat{x}(k|k)$, and $K_f(k)$ is the Kalman gain. From Eq. (23), it is found that the cross covariance matrix $P_{fm}(k)$ is equal to the covariance matrix of Kalman filter output $P_f(k|k)$.

Consequently, the covariance matrix represented by Eq. (6) can be simplified as Eq. (9), though the output of the N -step predictor is used as a monitoring signal in the extended Kalman filter. Now, the test statistic for the fault detection of the extended Kalman filter can be computed from Eqs. (18), (21), (23) as follows.

$$\begin{aligned}
 l_N(k) &= [\hat{x}_f(k|k) - \hat{x}_m(k)]^T [P_m(k) - P_f(k|k)]^{-1} \\
 &\quad \cdot [\hat{x}_f(k|k) - \hat{x}_m(k)] \quad (24)
 \end{aligned}$$

where $\hat{x}_f(k|k)$ and $P_f(k|k)$ represent the state vector of the extended Kalman filter and its covariance matrix, respectively.

4. Comparison with Residual Test

In this section, the performance of fault detection of the proposed N -step predictor method is analytically compared to the conventional method using residual. The miss detection rate (MDR) is chosen as the performance index.

If there is a fault in the system, the previous defined test statistics have non-central chi-square distributions with unknown non-central parameter whose expected value is zero under no fault conditions. For the purpose of reducing MDR at

given threshold ϵ_β , it is reasonable that non-central parameter has large value in fault conditions.

In the residual test, the test statistic at k -th step is generally defined as following:

$$l_r(k) = \sum_{j=k-N+1}^k r^T(j) V^{-1}(j) r(j) \quad (25)$$

where N is the size of moving windows, $r(j)$ and $V(j)$ represent the j -th residual and its covariance, respectively. If there is an additive fault in measurement sensors, then, the expected value of the non-central parameter $l_r(k)$ given by Eq. (25) at k -th step can be approximated as

$$E\{l_r(k)\} = E\left\{ \sum_{j=k-N+1}^k f^T(j) V^{-1}(j) f(j) \right\} \quad (26)$$

where $f(j)$ is a fault vector at j -th step in sensors.

On the other hand, the expected value of the test statistics in N -step predictor method can be calculated as follows. If there is a fault in the system, the expected value of the difference between the state vector of N -step predictor and that of the extended Kalman filter can be approximately written as

$$\begin{aligned} E\{\widehat{x}_f(k|k) - \widehat{x}_m(k)\} &= E\left\{ \sum_{j=k-N}^k \Phi(k, j) \delta \widehat{x}_j \right\} \\ &\approx E\left\{ \sum_{j=k-N}^k \Phi(k, j) K_f(j) f(j) \right\} \end{aligned} \quad (27)$$

where $\delta \widehat{x}_j$ and $K_f(j)$ are the estimated error vector and the gain matrix of the extended Kalman filter at j -th step, respectively. Therefore, if the expected value of the test

statistics given in Eq. (24) can be written as

$$\begin{aligned} E\{l_N(k)\} &= E\left\{ \left[\sum_{j=k-N}^k \Phi(k, j) K_f(j) f(j) \right]^T B^{-1}(k) \right. \\ &\quad \left. \cdot \left[\sum_{j=k-N}^k \Phi(k, j) K_f(j) f(j) \right] \right\} \end{aligned} \quad (28)$$

where $B(k) = P_m(k) - P_f(k|k)$.

In Eqs. (26) and (28), if the extended Kalman filter and N -step predictor are assumed in steady state, the covariance matrices V , B , and the gain matrix K_f can be considered as constant. In Eq. (26), the fault vector $f(j)$ ($j = k - N, \dots, k$) directly affects the test statistic $l_r(k)$ in residual test method. However, in Eq. (28), the vector $\Phi(k, j) K_f(j) f(j)$ affects the test statistics $l_N(k)$ in the N -step predictor method. The following equation, which is held in the extended Kalman filter, is considered for the comparison of sensitivity of the fault:

$$\lambda_{\max} \{I - K_f H_f\} < 1 \quad (29)$$

where λ_{\max} represents maximum eigenvalue. If we assume that the measurement matrix H_f is identity matrix, some characteristics can be told from the above equations. In the early stage after a fault, the residual test method may be more sensitive to fault than the proposed method because of Eq. (29). Therefore, if the size of the fault is sufficiently large, for example, hard fault, the residual test is more advantageous than the proposed method from a detection time point of view. However, the proposed method is more desirable than the residual test when the size of the fault is not so large as to detect the fault in a few seconds, because the influence of the fault in the N -step predictor method is propagated with an accumulated form owing to the state

transition matrix $\Phi(k, j)$ on the other hand, the influence of the fault in residual test is uniform within a interested interval. In the next section, these characteristics are confirmed by computer simulation.

5. APPLICATION FOR SDINS

In order to verify the performance of the proposed fault detection method, it is applied to an SDINS with aided sensor, global position system (GPS). The performance of the method is examined by computer simulation, and compared with the residual test which is conventionally used for detecting a fault in Kalman filter.

The error models for the SDINS are referred to [9], and the inertial sensor errors are simply modeled as the sum of the random constant bias and white noise. Also, the GPS errors are simply modeled as additive white Gaussian noise. The filter model used in the simulation has sixteen state variables: three position errors, three velocity errors, four quaternion errors^[10], and six gyro and accelerometer biases. The measurement models includee the velocity and position variables obtained from the GPS with a frequency of 1Hz. In Table 1, the standard deviations of the sensor errors for the simulation are given.

The reference trajectory for the simulation is such that the speed of the vehicle is constant at 250m/s with the exception of the initial stage, and the vehicle changes its attitude with a turning rate of 30deg/s in yaw at 100sec.

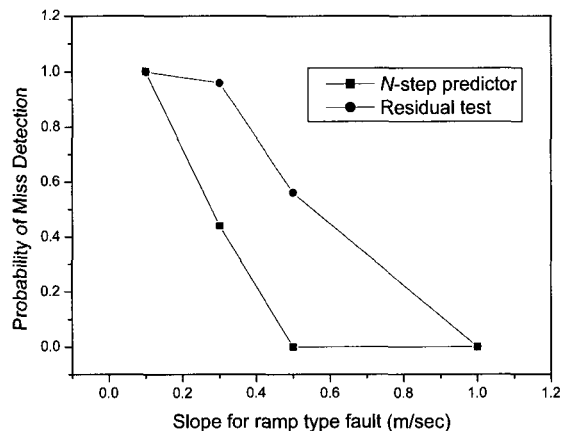
The position and velocity states are considered for the test statistics; hence the test statistic has a chi-square distribution with 6degrees of freedom in the N -step predictor method, 120degrees of freedom in residual test where the size of moving window, N , equals 20, respectively. The chosen

[Table 1] Characteristics of Sensors noise

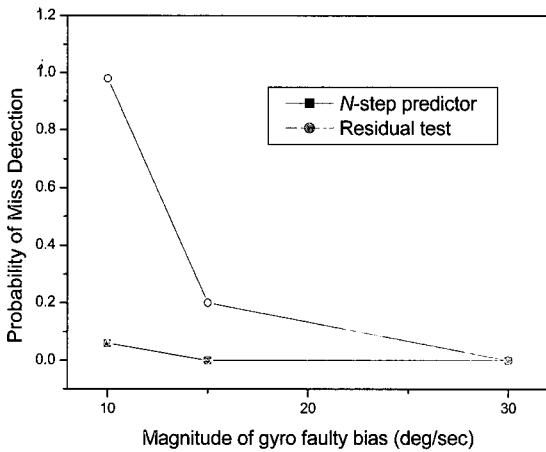
	Characteristic	Magnitude (1σ)
Gyro	Bias	3deg/h
	White noise	0.35deg/h
Accelerometer	Bias	1mg
	White noise	50 μ g
GPS (Velocity) (Position)	White noise	0.2m/s
	White noise	10m

false alarm (FAR) rate in simulation is 10^{-5} . Two kinds of faults are assumed to occur, that is, an additive ramp type fault in latitude data for GPS and an additive bias in gyro sensor fault at 130sec. The performance of the fault detection is evaluated by averaging over 50 Monte Carlo runs for every case.

Fig. 1 shows MDR obtained from the simulation results for the ramp type fault in GPS. Several slopes are used for the ramp type fault. From Fig. 1, it is shown that MDR is all zero in two detection methods when the slope of ramp type fault is 1m/sec. But, when the slope is between more than 0.1m/s and less than 1m/s, MDR of the proposed method (marked by squares) is smaller than that of the residual test (marked by



[Fig. 1] MDR for GPS fault



[Fig. 2] MDR for gyro fault

circles).

Fig. 2 shows MDR for the N -step predictor method and the residual test, when the abrupt change of gyro bias is used as the fault model. From the results, it is also shown that MDR of the proposed method is smaller than that of the residual test, which is similar to the assumed GPS fault case.

As another performance index for fault detection, the detection delay is considered for the assumed two fault cases. Table 2 shows the averaged detection delay when MDR is zero, that is, the slope of ramp type in GPS fault is 1m/s and the magnitude of bias in gyro fault is 30deg/s, respectively. From Table 2, it is known that the N -step predictor method is more advantageous from a point of detection delay than the residual test.

From the simulation results, it is confirmed that the proposed method detect a fault of the extended Kalman filter, effectively. Also, it is shown that the N -step predictor method is more appropriate for fault detection than the residual test from the points of MDR and detection delay, which is expected from the comparison in the previous section.

[Table 2] Comparison of fault detection delay

	N -step predictor	Residual test
GPS fault	25.1sec	33.2sec
gyro fault	33.1sec	40.9sec

6. CONCLUSION

In this paper, we proposed the new approach to fault detection method with the N -step predictor which is used for obtaining a monitoring signal of the extended Kalman filter. A consistency between the extended Kalman filter and the N -step predictor is tested to detect a fault, which is similar to the previous studies. But, the proposed method overcomes the problems such as gradually degraded fault sensitivity and abruptly change of test statistic. The fault detection method with the N -step predictor is compared to the residual test, and some characteristics of the method are discussed.

The proposed fault detection method is applied to an SDINS with GPS, and its performance is evaluated by Monte Carlo simulation. From the simulation results, it is shown that the proposed method exhibits good performance in terms of its fault detection capacity. Also, the N -step predictor method is compared to the residual test by computer simulation, and the characteristics of the proposed method are verified.

REFERENCES

- [1] A. S. Willsky, "A Survey of Design Methods for Failure Detection in Dynamic Systems", *Automatica*, Vol. 12, No. 6, pp.601~611, 1976.
- [2] T. H. Kerr, "Real-Time Failure Detection : A

- Nonlinear Optimization Problem That Yields a Two-Ellipsoid Overlap Test”, *Journal of Optimization Theory and Applications*, Vol. 22, No. 4, pp.509~536, 1977.
- [3] T. H. Kerr, “Statistical Analysis of a Two-Ellipsoid Overlap Test for Real-Time Failure Detection”, *IEEE Transactions on Automatic Control*, Vol. 25, No. 4, pp.762~773, 1980.
- [4] B. D. Brumback and M. D. Srinath, “A Chi-Square Test for Fault-Detection in Kalman Filters”, *IEEE Transactions on Automatic Control*, Vol. 32, No. 6, pp.552~554, 1987.
- [5] Ali Zolghadri, “An Algorithm for Real-Time Failure Detection in Kalman Filters”, *IEEE Transactions on Automatic Control*, Vol. 41, No. 10, pp.1537~1539, 1996.
- [6] Ren D Da, “Failure Detection of Dynamical Systems with the State Chi-Square Test”, *Journal of Guidance, Control and Dynamics*, Vol. 17, No. 2, pp.271~277, 1994.
- [7] P. S. Maybeck, *Stochastic Models, Estimation, and Control*, Vol. 1, Academic Press, 1982.
- [8] A. H. Jazwinski, *Stochastic processes and filtering theory*, Academic Press, 1970.
- [9] D. H. Titterton and J. L. Weston, *Strapdown inertial navigation technology*, Peter Peregrinus Ltd., 1997.
- [10] M. J. Yu, J. G. Lee and H. W. Park, “Comparison of SDINS In-Flight Alignment Using Equivalent Error Models”, *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 35, No. 3, pp.1046~1054, 1999.