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Implementation of Semi-infinite Boundary Condition for Dynamic Finite Element Analysis

동적 유한요소해석에서의 반무한 경계조건의 실행

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요 지

지반구조물의 동적해석은 모델의 영역이 커짐에 따라 에너지가 감소하는 현상을 표현할 수 있는 방법을 필요로 한다. 이러한 현상은 흔히 방사 감쇠(radiation damping) 또는 기하학적 감쇠(geometric attenuation)로 알려져 있으며, 탄성에너지가 점성 또는 이력현상에 의해 감소되는 재료 감쇠현상과는 구별된다. 따라서 수치해석으로 지반구조물의 동적거동을 해석할 경우 모델의 영역 구축은 특별한 고려를 필요로 한다. 인공적인 경계조건은 유한요소내의 지반상태를 무한상태로 변형시킬 수 있어야 하며, 경계에 도달하는 응력 파동을 모델내로 반사시키지 않고 흡수 할 수 있어야 한다. 본 논문에서는 간단한 점·탄성 반무한 불연속 요소를 이용하여 지반구조물의 동적해석을 수행할 경우 에너지를 투과하는 경계조건을 수립하는 방법을 보여준다. 반무한 요소의 실행은 OpenSees라는 유한요소 해석프로그램을 이용하여 수행되었으며, 예를 통하여 불연속 요소가 경계에 도달하는 응력 파동을 충분히 흡수하여 유한요소 모델을 반무한 상태로 전환 시킬 수 있다는 것을 보여준다. 본 논문에서 제시된 방법은 간단하게 실용적으로 사용할 수 있는 반무한 경계조건이지만, 입사각이 매우 예리할 경우는 에너지의 흡수정도가 충분치 않은 것으로 알려져 있다.

Abstract

Dynamic numerical analysis of geotechnical problems requires a way to simulate the decrease of energy as the domain of interest gets larger. This phenomenon is usually referred to as radiation damping or geometric attenuation and it is distinguished from material damping in which elastic energy is actually dissipated by viscous, hysteretic, or other mechanism. The fact that the domain of analysis in numerical modeling must be chosen, however, causes a need for special attention at the boundary. This observation leads directly to the idea of determining the dynamic response of the interior region from a finite model consisting of the interior region subjected to a boundary condition which ensures that all energy arriving at the boundary is absorbed. This paper presents a simple methodology to simulate transmitting boundaries condition using viscoelastic infinite elements within the recently developed "OpenSees" finite element code. The methodology used here provides that the level of absorption for traveling waves is efficient enough for practical purposes, but unsatisfactory for the case of sharp incident angles. The effectiveness of the infinite elements for the absorption of incident waves at boundaries is evaluated via example analysis.

Keywords : Dynamic finite element analysis, OpenSees, Semi-infinite element, Visco-elastic material

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1. Introduction

Many problemsde aling dealing with the dynamic response of geotechnical structure involve the wave propagation in semi-infinite domains. In that case, the numerical solution usually requires the introduction of an artificial boundary in order to render the domain finite and this artificial boundary must absorb the stress waves arriving at the boundary in order to simulate the physical fact that the energy decreases as the wave travels outer domain. This phenomenon is usually referred to as radiation condition or geometric attenuation and it is distinguished from material damping in which elastic energy is actually dissipated by viscous, hysteretic, or other mechanism (Kramer 1996). Thus, it is required that the incident waves do not reflect back into the numerical domain at the boundary and that the incident waves be transmitted freely through the boundary for the case of dynamic excitation. This observation leads directly to the idea of determining the dynamic response of the interior region from a finite model consisting of the interior region subjected to a boundary condition, which ensures that all energy arriving at the boundary is absorbed.

As one approach for seeking numerical solutions to this problem, the viscous boundary conditions have been formulated by Lysmer and Kuhlemeyer (1969), which damp out the spurious reflections. It has been shown that the level of absorption for traveling waves is in overall satisfactory for a wide range of incident angles and is efficient enough for practical purposes (Castellani 1974). In consequent research White et al. (1977) suggested a particular choice of parameters for the dashpot and showed that the amount of reflections can be reduced further.

In this paper, the visco-elastic infinite element described above is reviewed and implemented in the recently developed OpenSees finite element code (OpenSees 2006) in which the Open System for Earthquake Engineering Simulation (OpenSees) is a finite element software framework for simulating the seismic response of structural and geotechnical systems developed by Pacific Earthquake Engineering Research (PEER). The effectiveness of the dashpot to mitigate reflections is addressed via comparison of example models created on unbounded and bounded domains. The procedure to create the model geometry and to generate the mesh for OpenSees is coded in Tcl/Tk script language and it is used to create the current models. Details on Tcl/Tk script is found in Flynt (2003). The work presented here may also be found in the website at http://www.ce.washington.edu/~geotech/opensees.

2. Viscous Boundary Condition

2.1 One-dimensional Equation of Motion

The one-dimensional wave equation is a partial differential equation of the form

$$\frac{\partial^2 \xi}{\partial t^2} = v^2 \frac{\partial^2 \xi}{\partial x^2} \tag{1}$$

where ξ and v represent the particle displacement function and the wave propagation velocity corresponding to the type of stress wave of interest. The solution of the above equation can be given by the form

$$\xi(x,t) = \xi_r(t - \frac{x}{v}) + \xi_i(t - \frac{x}{v}) \tag{2}$$

where ξ_r and ξ_i are any arbitrary functions of $(t-x/v)$ and $(t+x/v)$ that satisfy Eq. (1). Here the term $\xi_r(t-x/v)$ denotes the wave traveling at velocity v in positive x-direction while the term $\xi_i(t+x/v)$ does the wave traveling at the same speed in the negative x -direction. Therefore, ξ_i is the incident wave if it travels inwards into the computational domain and ξ_r is the reflected wave. Partial derivative of the Eq. (2) with respect to time produces

$$\frac{\partial \xi(x,t)}{\partial t} = \xi_r'(t - \frac{x}{v}) + \xi_i'(t + \frac{x}{v}) \tag{3}$$

The prime quantities represent the derivative of the associated function with respect to its arguments. Knowing that $\partial\xi = \epsilon_x \partial x$ (from the strain-displacement relationship), $\epsilon_x = \sigma_x / K$ where K is appropriate elastic moduli (from the elastic stress-strain relationship), and $\partial x = v \partial t$ (from the definition of wave propagation velocity), the linear elastic uniaxial stress-strain relationship can be shown to be

$$\sigma(x,t)=K\frac{\partial\xi(x,t)}{\partial x}=-\frac{K}{v}\xi_r'(t-\frac{x}{v})+\frac{K}{v}\xi_i'(t+\frac{x}{v}) \quad (4)$$

where $\sigma(x,t)$ is the elastic stress function. Arranging Eq. (3) for $\xi_r'(t-\frac{x}{v})$ and replacing it into Eq. (4), the elastic stress-strain relationship can be written in the form

$$\sigma(x,t)=-\frac{K}{v}\frac{\partial\xi(x,t)}{\partial t}+2\frac{K}{v}\xi_i'(t+\frac{x}{v}) \quad (5)$$

It is noted that $\partial\xi(x,t)/\partial x$ is the total particle velocity and $\xi'_i(x,t)/\partial x=\partial\xi_i(t+x/v)/\partial t$ is the velocity of incident wave. In other word, the first term on the right hand side of Eq. (5) is equivalent to the force per unit area to be employed at dashpot of coefficient K/v and the second term is equivalent to a force per unit area which is proportional to the incident wave velocity. Now, the reflected stresses at the boundary may be distinguished in terms of normal, $\sigma_n(x,t)$, and shear, $\tau(x,t)$, components to the plane. From Eq. (5) the dashpot stress in terms of normal and shear components are

$$\sigma_n(x,t)=-\frac{M}{v_p}\frac{\partial w(x,t)}{\partial t}=-\rho v_p\frac{\partial w(x,t)}{\partial t} \quad (6)$$

$$\tau(x,t)=-\rho\frac{G}{v_s}\frac{\partial u(x,t)}{\partial t}=-\rho v_s\frac{\partial u(x,t)}{\partial t} \quad (7)$$

where M and G represent the constrained and shear moduli, respectively; ρ is the mass density; ω and u are the normal and tangential particle displacement, respectively. The v_p and v_s are the volumetric P-wave and distortional S-wave velocities of soil, respectively, and defined by

$$v_s=\sqrt{\frac{G}{\rho}} \quad (8)$$

$$v_p=\sqrt{\frac{M}{\rho}}=\frac{1}{s}V_s \quad (9)$$

and s is a coefficient defined in terms of Poisson's ratio μ as

$$s^2=\frac{1-2\mu}{2(1-\mu)} \quad (10)$$

Lysmer & Kuhlemeyer (1969) proposed a simple absorbing boundary condition based on Eqs. (6) and (7) and usually referred to as the classical viscous boundary condition. Figure 1 shows this boundary element for the case of an angle of incidence $\theta=90°$. From Fig. 1 $\dot{\omega}$ and $\dot{u}$ are the normal and tangential velocities, respectively.

In Lysmer's formulation, the absorption of incidence wave is accomplished by entering normal and tangential stresses of the form

$$\sigma_n=a\rho v_p\dot{w} \quad (11)$$

$$\tau=b\rho v_s\dot{u} \quad (12)$$

where σ_n and τ are the normal and shear stresses in the boundary, respectively, and a and b are dimensionless parameters. The parameters a and b are chosen to minimize the reflected energy corresponding to an incident plane wave reaching the boundary at a given angle of incidence. The parameter selection is considered separately for an incident longitudinal wave, an incident transverse wave, and a surface wave. Lysmer & Kuhlemeyer (1969) have shown that the viscous boundary defined with $a=b=1$ is very effective if the angle of incidence is greater than 30 degree with absorbing 98% of P-waves and 95% of S-waves. As shown in Fig. 1, if the angle of incidence angle is perpendicular to the plane, the stress that can be absorbed by the dashpot is equivalent to the stress produced by the incident wave both in the parallel and normal directions.

White et al. (1977) proposed the following a and b values

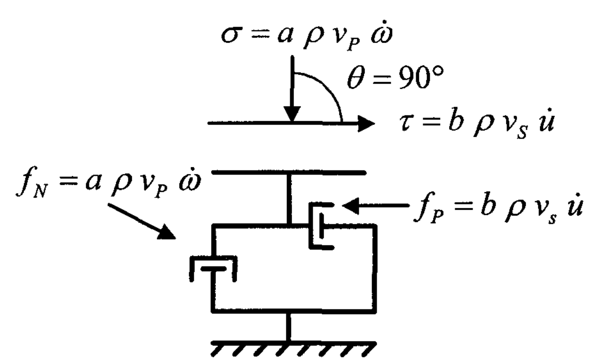

Fig. 1. Simple infinite visco-elastic dashpot proposed by Lysmer & Kuhlemeyer (1969)

to maximize the absorption efficiency at the boundary:

$$a = \frac{8}{15\pi}(5+2s-2s^2) \tag{13}$$

$$b = \frac{8}{15\pi}(3+2s) \tag{14}$$

where s is given in Eq. (10). Thus, the viscous boundary parameters depend only on poisson's ratio. Above expressions were obtained from the solution of dynamic stress-strain relations in terms of the normal and shear velocities for isotropic materials. It was shown that using these a and b parameters the absorption efficiency slightly increases for both P-and S-waves (White, et al. 1977).

Even though the simple viscous boundary condition presented here yields spurious reflections in certain conditions as in the case of a sharp incident angle, it has been proven to be satisfactory and useful for most practical problems.

3. Numerical Scheme in OpenSees

The advantage of using viscous boundaries to represent the absorbing condition is that it can be simply coded in any numerical tool. In this particular work, the existing OpenSees Zero-Length element was used to include the artificial boundary condition (Mazzoni, et al. 2005). The Zero-Length element is represented by two nodes defined with the same geometric location (line 5 and 6 in Table 1). The nodes are connected through multiple Uniaxial-Material (Simo & Hughes 1998) objects that represent the elemental force-deformation relationship. Table 1 shows a simple example where a Zero-length element is used. The viscous damping coefficients correspond to the parallel and normal direction to boundary in which

$$C_P = a\rho v_s \tag{15}$$

$$C_N = b\rho v_p \tag{16}$$

are defined by using material properties, ρ, v_s, and v_p, and dimensionless parameters a and b. These coefficients are used to create the material object in lines 3 and 4 of Table 1. Then, the artificial element is defined using the material object and the orientation.

This element absorbs the incident elastic energy through the force-displacement relationship shown in Fig. 1. The degree of absorption is governed by C_P and C_N which are defined in terms of the material properties given to the interior region.

It is a little complicated to formulate this boundary element within a soil domain in OpenSees since the "so called preprocessing" procedure to generate the mesh for a plane problem is not defined in OpenSees. Thus, a Tcl routine, which creates the input file for OpenSees, was developed.

Table 1. Example of Zero-Length element construction in OpenSees

```
1. set DampP 93
2. set DampN 173
3. uniaxialMaterial Elastic  1  14000  $DampP
4. uniaxialMaterial Elastic  2  14000  $DampN
5. node 1 16.0 0.0
6. node 2 16.0 0.0
7. element zeroLength 1  1  2  -mat 8  10 -dir 1 2
```

4. Implementation Viscous Boundary Infinite Model

To examine the effectiveness of the viscous boundary and to demonstrate that it can be easily included in OpenSees, two models were used to analyze a semi-infinite domain. The response of each model using fixed and viscous boundaries, respectively, is compared. The model properties are given in Table 2.

As a first verification case a rectangular domain was selected as shown in Fig. 2. By forcing the nodes on the left side boundary to behave in the same way as the right

Table 2. Material properties of example models

ρ $tonf/m^3$	v_s m/sec	v_p m/sec	E kN/m^2	μ	C_P $tonf/m^2sec$	C_N $tonf/m^2sec$
1.6	58	108	14000	0.3	93	173

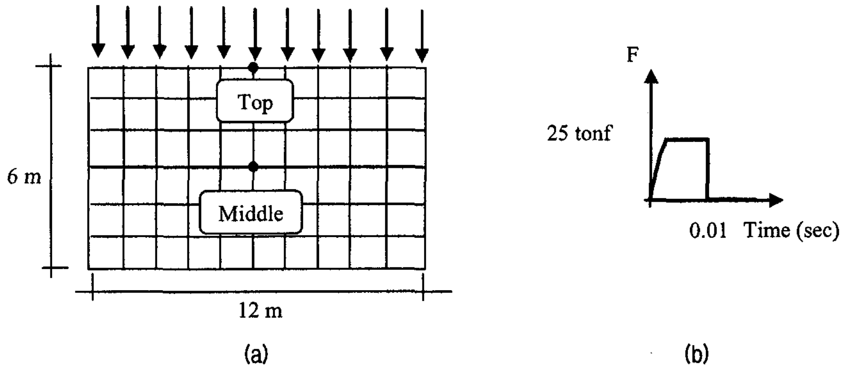

Fig. 2. (a) One-dimensional model and (b) loading condition

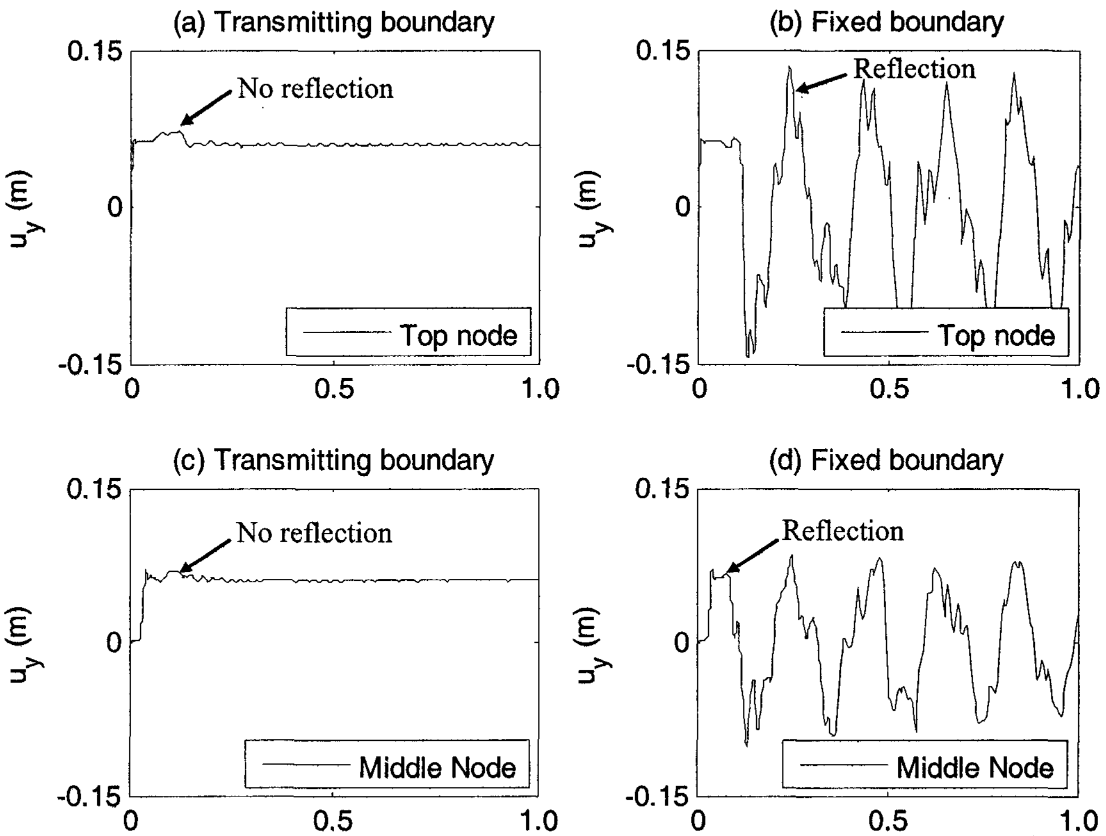

Fig. 3. Vertical displacement at the top and middle nodes for one-dimensional model: (a) top node of transmitting boundary, (b) top node of fixed boundary, (c) middle node of transmitting boundary, and (d) middle node of fixed boundary

nodes, the one-dimensional wave propagating condition was achieved by use of the OpenSees command "equalDOF". The fixed and transmitting boundaries were applied along the base and sides of the model. A pulse load defined using sine and rectangular loading patterns allows to examine how waves reflect at the boundary. Figure 3 compares the vertical displacement at the top and middle nodes for the fixed and transmitting boundary conditions. The reflection occurs in the fixed case, but it disappears when the transmitting boundary is used.

As a second verification example, a more realistic two-dimensional model was simulated. Figure 4 shows the geometry and loading condition. As in the previous example, the fixed and transmitting boundaries were used along the base and sides of the model. The response shown in Fig. 5 indicates that the transmitting boundary absorbs the incident waves satisfactorily while the fixed boundary generates spurious reflections.

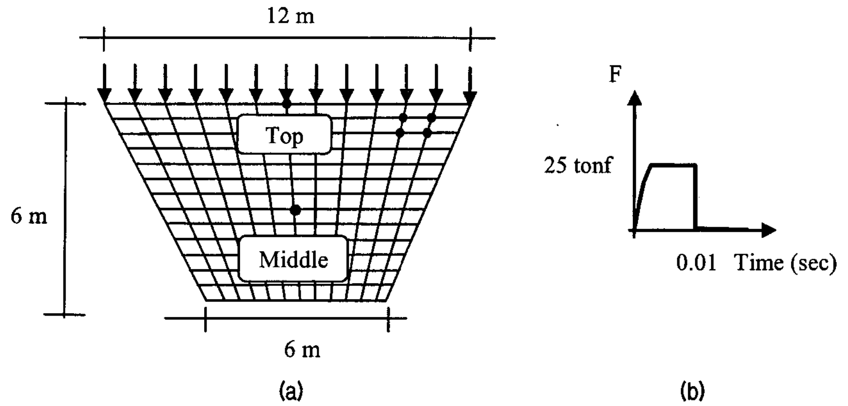

Fig. 4. (a) Two-dimensional model and (b) loading condition.

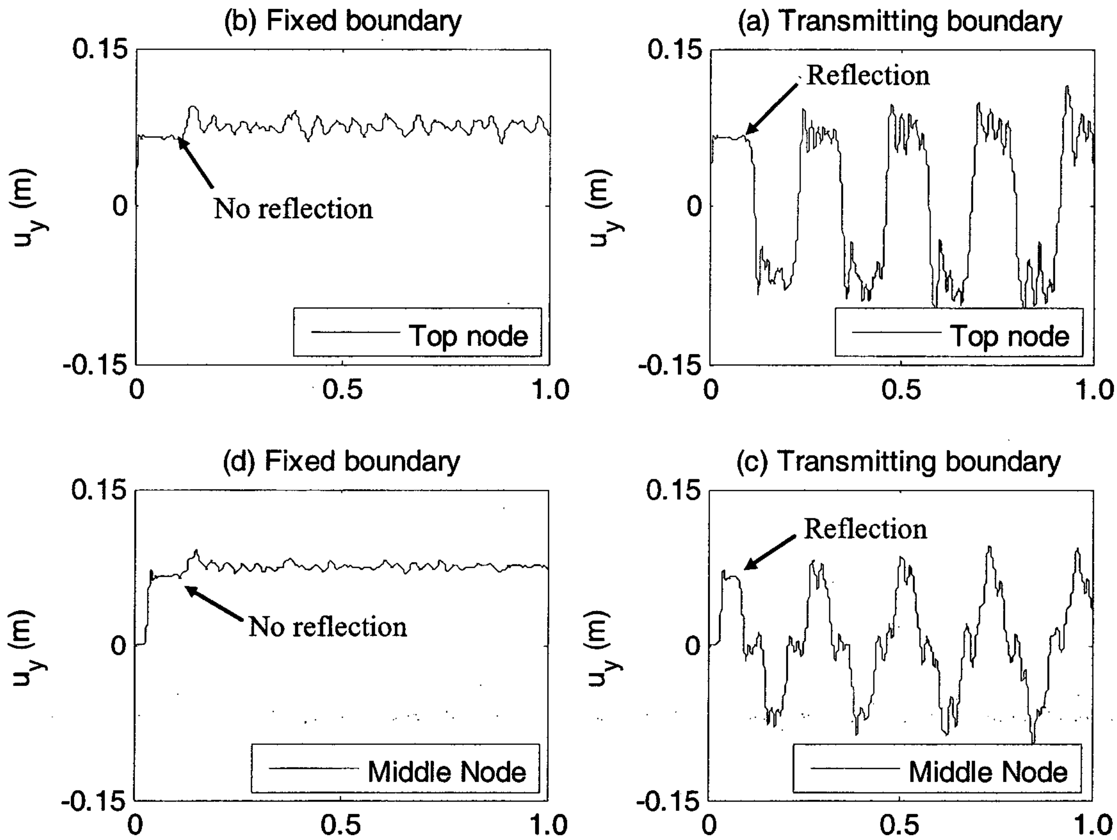

Fig. 5. Vertical displacement at the top and midddle nodes for two-dimensional model : (a) top node of transmitting boundary, (b) top node of fixed boundary, (c) middle node of transmitting boundary, and (d) middle node of fixed boundary.

5. Conclusions

The primary objective of this study was to demonstrate the capabilities of OpenSees to simulate a semi-infinite domain for geotechnical purposes. A simple viscous dashpot was implemented to consider geometric attenuation. One-and two-dimensional models were simulated using OpenSees. The comparison made using fixed and transmitting boundaries shows that visco-elastic transmitting element provides a method to simulate the semi-infinite domain within OpenSees program. The advantages of the viscous boundary type are that (1) it can be easily included into finite element code, OpenSees (2) it absorbs the incident wave satisfactory for most practical purposes.

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References

1. Castellani, A. (1974), Boundary conditions to simulate an infinite space: *Meccanica, Journal of the Italian Association of Theoretical and Applied Mechanics, ASCE*, Vol.9, No.3. 199-205.
2. Flynt, C. (2003), *Tcl/Tk, Second Edition : A Developer's Guide*, Elsvier science.
3. Kramer, S. L. (1996), *Geotechnical Earthquake Engineering*, Prentice Hall, New Jersey, 179 pp.
4. Lysmer, J. M. and Kuhlemeyer, R. L. (1969), Finite dynamic model for infinite media, *Journal of the Engineering Mechanics Division, ASCE*, Vol.98, No.EM4. 859-877.
5. Mazzoni, S, McKenna, F, Fenves, G.L. (2005), *OpenSees command language manual*, http://opensees.berkeley.edu.
6. OpenSees (2006), Pacific Earthquake Engineering Research Center, http://peer.berkeley.edu.
7. Simo, J. C. and Hughes, T. J. R. (1998), *Computational Inelasticity*, Springer.
8. White, W., Valliappan, S. and Lee, I. K., (1977), Unified boundary for finite dynamic models, *Journal of the Engineering Mechanics Division, ASCE*, Vol.103, No.EM5. 949-964.

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