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A New Ship Scheduling Set Packing Model Considering Limited Risk

Si-Hwa Kim · Hee-Su Hwang***

**Division of Maritime Transportation Science, Pusan 606-791, Korea*
***Department of Industrial & Manufacturing Systems Engineering, University of Texas at Arlington, USA*

Abstract : *In this paper, we propose a new ship scheduling set packing model considering limited risk or variance. The set packing model is used in many applications, such as vehicle routing, crew scheduling, ship scheduling, cutting stock and so on. As long as the ship scheduling is concerned, there exits many unknown external factors such as machine breakdown, climate change and transportation cost fluctuation. However, existing ship scheduling models have not considered those factors apparently. We use a quadratic set packing model to limit the variance of expected cost of ship scheduling problems under stochastic spot rates. Set problems are NP-complete, and additional quadratic constraint makes the problems much harder. We implement Kelley's cutting plane method to replace the hard quadratic constraint by many linear constrains and use branch-and-bound algorithm to get the optimal integral solution. Some meaningful computational results and comments are provided.*

Key words : *0-1 integer set packing model, Quadratic programming, Branch-and-cut, Kelley's cut, Limited risk*

1. Introduction

The set packing or partitioning problem has wide applications in the area of vehicle routing, crew scheduling, airline and ship scheduling, cutting stock and so on. Most vehicle routing and airline crew scheduling, in particular, have utilized so far the deterministic set packing or partitioning model to minimize the expected cost of routes and crew trips. However, such unpredictable external factors as terrorism, war, and some global sources of insecurity make the decisions for existing logistics models so vulnerable that decision makers need to consider risk or variance somehow.

This paper propose a new ship scheduling set packing model for real-world industrial and tramp shipping practices considering limited risk or variance. The main idea of new model is to capture the variability by restricting variance. However, incorporating limited variance into the existing set packing or partitioning model usually requires quadratic optimization model, which can be found in portfolio selection (Winston, 1994). Set packing problem is NP-complete, and using quadratic terms to restrict variability makes the problem much harder. This nature prohibits set packing models from incorporating quadratic constraints, and rather leads them to scenario based two-stage stochastic models (Mulvey, 1995).

Ship scheduling problems model the transportation of commodities from points of supply to points of demand, so they are vital to world trade and military logistics. A ship requires a multi-million dollar capital investment, and the daily operating costs of a ship can be tens of thousands of dollars. Consequently, improved fleet utilization can yield significant financial results. Ship scheduling problems have been studied extensively in the literature, and Christiansen et al. (2004) provide an excellent survey. Industrial operators deliver their own cargo on their own ships at minimal cost, while tramp shippers transport cargo for other companies on demand. Tramp operators have some cargo under contract that they must ship. Both industrial and tramp operators often transport additional cargo from the spot market when capacity is available. When an entire ship is available, they will place it on the spot charter market, so other shipping companies can charter it. If an industrial or tramp operator has cargo that they cannot transport, then they ship it on the spot market. This can be modeled as set packing problem (Christiansen et al., 2004).

Each year the International Tanker Nominal Freight Scale Association Ltd. (ITNFSAL) announces the cost of shipping between any combination of ports using standard vessels, called Worldscale (WS) 100. Current market value of shipping freight (spot rate) is represented by a direct percentage of the WS 100. Past experiences show that spot rate is highly variable and can fluctuate between WS 15 and WS 500 within one week. Consequently, the cost of shipping on the spot market is extremely volatile. Fig. 1 shows the fluctuation of WS, or spot tanker freight rates,

* Corresponding Author : Si-Hwa Kim, Shalom@mail.hhu.ac.kr, 051)410-4181
** Shalom@mail.hhu.ac.kr, 051)410-4181

for the past five years.

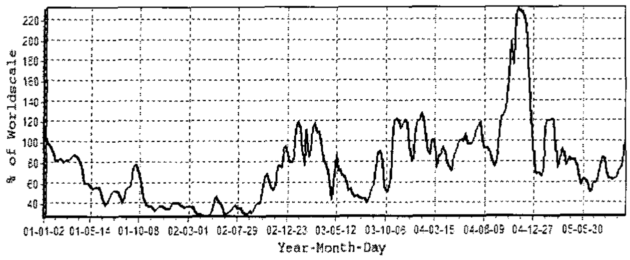

Fig. 1 Spot Rates (VLCC-AG/WEST), Weekly: Jan.2001~Oct.2005 (Hanbada Corporation, 2003).

The importance of spot rate costs is addressed in an example of Fisher and Rosenwein (1989). However, traditional ship scheduling problems ignore these issues, and as a result the solutions are lacking in managing the risk or variance. The new set packing model provides the way to manage risk in ship scheduling by using variance as the measure of variability. For uncertainty associated with price and cost forecasts, this variability is a function of the covariance matrix of the forecast errors and restricted by a quadratic constraint. To the best of our knowledge, this paper provides the first stochastic optimization model in ship scheduling problems, which combines set packing model and a quadratic constraint.

In section 2, we present a new set packing models in ship scheduling problems considering limited risk. Section 3 discusses solution approach to this new model. We describe Kelley's cutting plane method to handle the quadratic constraint and branch-and-bound method to find integer feasible solutions. Section 4 provides computational results with realistic ship scheduling problem. Finally summary and future research topics follow.

2. A New Set Packing Model Considering Limited Risk

Industrial and tramp operators must transport contracted cargoes from origin to destination. They may charter out some of their ships that have no feasible schedule or will benefit by delivering cargo from the spot market. Conversely, they also may ship contracted cargo in the spot market. Most industrial and tramp ship scheduling problems are modelled as set packing problems, which can be solved by generating feasible schedules for each ship and solving a set packing problem. See (Christiansen and Fagerholt, 2002; Fagerholt, 2001; Fagerholt and Christiansen, 2000a; Fagerholt and Christiansen 2000b; Bausch et al., 1998; Kim and Lee, 1997).

2.1 Set packing model for ship scheduling problems

Let set V be the set of ships, and for each ship $v \in V$, let F_v be a set of candidate schedules available for each ship v, let d_v be the spot rate profit when ship v becomes a spot charter, and let binary variable u_v indicate whether ship v becomes a spot charter. For each ship $v \in V$ and each schedule $f \in F_v$, let c_{vf} be the cost of covering schedule f, and let binary variable x_{vf} indicate whether ship v covers schedule f. Let K be the set of cargoes, and for each ship $v \in V$, each schedule $f \in F_v$, and each cargo $k \in K$, let constant a_{kvf} indicate whether schedule f delivers cargo k. The set of cargoes K is further divided into the set of contracted cargoes, K_1, and the set of cargoes in the spot market, K_2. For each cargo $k \in K$, let r_k be the profit from delivering cargo k, let e_k be the spot rate cost of cargo k, and let binary variable s_k indicate whether cargo k is serviced by the spot market. The general formulation (SP) is given by:

$$\max \sum_{v \in V} \sum_{f \in F_v} \Big(\sum_{k \in K} r_k a_{kvf} - c_{vf} \Big) x_{vf} - \sum_{k \in K_1} e_k s_k + \sum_{v \in V} d_v u_v$$

$$s.t. \sum_{v \in V} \sum_{f \in F_v} a_{kvf} x_{vf} + s_k = 1, \ \forall k \in K_1 \qquad (1)$$

$$\sum_{v \in V} \sum_{f \in F_v} a_{kvf} x_{vf} \le 1, \ \forall k \in K_2 \qquad (2)$$

$$\sum_{f \in F_v} x_{vf} + u_v = 1, \ \forall v \in V \qquad (3)$$

$$x_{vf} = \{0,1\}, \ \forall v \in V, \forall f \in F_v \qquad (4)$$

$$u_v = \{0,1\}, \ \forall v \in V, s_k = \{0,1\}, \ \forall k \in K_1 \qquad (5)$$

The objective function (SP) maximizes profit from lifting cargoes and making some of own fleet spot charters; contracted cargo constraints in set (1) ensure that every contracted cargo will be lifted by own fleet or spot charter, and spot cargo constraints in set (2) select any profitable cargoes in the market; ship constraints in set (3) dispatch every ship in own fleet to a schedule or spot market; binary constraints in sets (4) and (5) ensure that each ship can only be assigned to a single schedule or become a spot charter; binary constraints in set (5) is not necessary because of constraints (1) and (3).

2.2 Quadratic set packing model considering limited risk

Operating ships involves many unknown factors, e.g., spot rate fluctuation, fuel costs, weather change, and machine breakdown. Among them, operating costs and market fluctuation directly affect overall profit. Operating costs of the fleet are relatively constant and controllable compared with the randomness of the spot rate costs, so we focus on the variance of the spot market. The quadratic set packing (QSP) model for ship scheduling problems can be formulated as:

$$\max \sum_{v \in V} \sum_{f \in F_v} (-c_{vf} + E[d_v] + E[r_{vf}] - E[e_{vf}]) x_{vf}$$

$$s.t. \quad \sum_{v \in V} \sum_{f \in F_v} a_{kvf} x_{vf} + s_k = 1, \ \forall k \in K_1$$

$$\sum_{v \in V} \sum_{f \in F_v} a_{kvf} x_{vf} \le 1, \qquad \forall k \in K_2 \tag{6}$$

$$\sum_{f \in F_v} x_{vf} \le 1, \ \forall v \in V \tag{7}$$

$$var\left[\sum_{v \in V} \sum_{f \in F_v} (d_v + r_{vf} - e_{vf}) x_{vf}\right] \le \epsilon \tag{8}$$

$$x_{vf} = \{0,1\}, \forall v \in V, \forall f \in F_v, s_k = \{0,1\}, \forall k \in K_1 \tag{9}$$

where,

$E[*] = the\ expected\ value\ of\ *$, $r_{vf} = \sum_{k \in K} r_k a_{kvf}$, and $e_{vf} = \sum_{k \in K_1} e_k a_{kvf}$.

The objective function of (QSP) maximizes the expected profit of the shipper. Constraints in set (6) ensure that all cargoes are serviced either by a ship in the fleet or by a spot charter, while some cargoes in the spot market do not need to be carried if they are not profitable. Constraint set (7) imply that each ship in the fleet is assigned exactly one schedule or sent to the spot charter market. Constraint (8) restricts the variance of the profit. The covariant matrix must be positive semi-definite because variance is always nonnegative. Constraints in set (9) impose binary requirements on the variables.

Market shortages and surpluses may cause large increases and decrease on all chartering rates. Consequently, the spot rate cost e_k will be given by

$$e_k = \alpha_k^e M + \beta_k^e + \gamma_k^e \tag{10}$$

where M is an independent random variable representing the fluctuation of spot market prices, α_k^e is a constant rate for how the market random variable M changes the spot rate, β_k^e is the expected cost of chartering ship on the spot market, and γ_k^e is an independent random variable for the fluctuation from β_k^e. We assume $E[M] = E[\gamma_k^e]$. The random variables r_k and d_v defined analogously; that is,

$$r_k = \alpha_k^r M + \beta_k^r + \gamma_k^r \tag{11}$$

$$d_v = \alpha_v^d M + \beta_v^d + \gamma_v^d \ . \tag{12}$$

This type of random profit modeling can often be found in calculating the return on a portfolio (Sharpe, 1970)

3. Solution approach

General form of the continuous relaxation of (QSP) can be written as:

$$\begin{aligned} &\max\ cx \\ &s.t.\ Ax \le b \\ &x^TQx \le \varepsilon \\ &x \ge 0 \end{aligned}$$

where c is objective coefficient row vector, A is left-hand side coefficient matrix of cargo and ship constraints, b is right-hand side vector of cargo and ship constraints, and Q is a covariance matrix of (QSP) respectively. Q is semidefinite. By using Kelley's cutting plane method (Kelley 1960), we can replace the quadratic constraint, $x^TQx \le \varepsilon$, by generating linear constraints, $2x^TQw \le \varepsilon + w^TQw, \forall w \in R^n$.

Even if it converges slow because of numerous possible planes with respect to w, Kelley's cutting planes are linear and easy to manipulate. We can solve the continuous relaxation of (QSP) by using Kelley's cutting plane algorithm.

Kelley's cutting plane algorithm.

Let $w \in R^n$ and $W \subset R^n$.

Step. 1 Solve following system to get $x = w$.

$\max \{cx \mid Ax \le b, 2x^TQw \le \varepsilon + w^TQw,, \forall w \in W, x \ge 0\}$.

If $x^TQx \le \varepsilon$, then the solution is optimal.

Step. 2 $W \leftarrow W \cup \{w\}$, go to Step. 1.

To get feasible integral solutions, we generate Kelly's cutting planes within a branch-and-bound tree. After finding the optimal solution for the continuous relaxation of (QSP) at a node, we compare the solution value with the current lower bound (LB). If the solution is worse than LB, we prune the node, if not, continue search at the node. If the solution is integral and better than current LB, then we update current solution, if not, we prune the node. If the solution is infeasible, we also prune the node. We continue this until we find the optimal solution.

4. Computational Results

Realistic ship scheduling problems (Kim, 1999) have been tested on a Dual 3.06-GHz Intel Xeon Workstation. We implemented Kelley's cutting plane method in COIN/BCP structure(http://www.coin-or.org/).

We could get optimal solutions for all the problems that operate 10 ships within three minutes, however for the most instances that operate 20 and 30 ships, we could not finish branch and bound procedures in five hours to check if current best solutions are optimal. Table 1 and Table 2 show the results for the problem instances that operate 20 and 30 ships, respectively.

Table 1 Computational results (20 ships)

Var.(%)	Profit(%)	Gap(%)	Solution CPU(sec)	Cuts	Total CPU(sec)
20 ships, 20 cargoes, 563 vars x 40 constraints					
88.4	97.7	1.65	1056	58210	> 5 hrs
79.5	95.9	1.73	1	26	> 5 hrs
71.5	93.1	-	324	25706	> 5 hrs
63.1	91.0	0.01	9	790	> 5 hrs
56.0	87.9	0.00	29	2902	152
47.9	84.3	0.48	30	1842	> 5 hrs
20 ships, 30 cargoes, 985 vars x 50 constraints					
88.6	98.8	0.85	2	41	> 5 hrs
79.5	98.7	0.00	32	853	32
72.1	97.8	0.00	11	379	11
62.5	96.2	0.16	30	957	> 5 hrs
56.0	94.7	0.00	10	274	17
48.9	92.2	0.23	35	1296	> 5 hrs
20 ships, 40 cargoes, 1739 vars x 60 constraints					
88.7	97.7	1.71	20	131	> 5 hrs
80.7	98.4	0.00	1210	9564	1256
71.9	97.5	0.00	21	191	24
63.4	93.0	-	51	423	> 5 hrs
54.8	93.4	0.57	16	107	> 5 hrs
48.3	90.8	0.50	101	822	> 5 hrs
20 ships, 50 cargoes, 2655 vars x 70 constraints					
88.8	98.1	-	32	49	> 5 hrs
80.9	98.4	-	76	170	> 5 hrs
71.0	96.8	-	92	195	> 5 hrs
63.8	95.7	0.21	469	1146	> 5 hrs
55.8	94.1	-	312	706	> 5 hrs
48.8	92.3	0.00	576	1380	1051
20 ships, 60 cargoes, 3182 vars x 79 constraints					
90.2	99.4	0.00	24	14	34
80.9	98.6	0.00	165	145	165
72.0	97.4	-	467	470	> 5 hrs
64.0	94.7	1.41	115	121	> 5 hrs
55.7	94.2	-	104	101	> 5 hrs
49.0	92.2	-	1113	1435	> 5 hrs

To set the amount of variance restriction, i.e., the value of ε, we solved QSP without quadratic constraint and calculated the variance (var) of the optimal solution (sol), then we set 6 different levels of ε, which are 95% of var, 90% of var, 80% of var, 75% of var, and 70% var, respectively. The first column shows how much variance can be restricted by the best solution, which is represented by the percentage of var. The second column shows the profit ratio to the value of sol. The third column indicates the gap between current upper bound and the value of the best solution found. The fourth column shows CPU time in seconds to find the best solution within 5 hours, and fifth column is the number of cuts generated. The last column tells wether the time limit (5 hours) exceeds.

Table 2 Computational results (30 ships)

Var.(%)	Profit(%)	Gap(%)	Solution CPU(sec)	Cuts	Total CPU(sec)
30 ships, 20 cargoes, 804 vars x 50 constraints					
89.2	98.4	0.94	2	45	> 5 hrs
80.9	96.6	0.51	8	354	> 5 hrs
70.3	93.2	0.70	176	7635	> 5 hrs
63.5	87.1	-	6	270	> 5 hrs
56.0	86.4	1.15	16	963	> 5 hrs
48.7	83.3	0.27	1674	54831	> 5 hrs
30 ships, 30 cargoes, 1409 vars x 60 constraints					
No feasible solution found					> 5 hrs
80.82	97.60	0.83	5	59	> 5 hrs
71.80	96.64	0.00	110	1369	109
63.39	94.11	0.62	42	703	> 5 hrs
56.24	92.47	0.00	3	25	3
48.66	88.71	-	754	11721	> 5 hrs
30 ships, 40 cargoes, 2474 vars x 70 constraints					
89.92	99.65	-	88	331	> 5 hrs
80.93	98.92	-	85	308	> 5 hrs
72.16	97.91	-	22	84	> 5 hrs
63.97	96.79	0.00	66	287	83
56.18	94.49	-	19	54	> 5 hrs
No feasible solution found					> 5 hrs
30 ships, 50 cargoes, 3788 vars x 80 constraints					
89.94	99.42	0.04	441	421	> 5 hrs
80.99	98.68	-	1390	1173	> 5 hrs
71.69	93.92	-	133	137	> 5 hrs
63.65	95.72	0.86	439	477	> 5 hrs
56.16	94.98	-	446	471	> 5 hrs
No feasible solution found					> 5 hrs
30 ships, 60 cargoes, 4561 vars x 89 constraints					
89.96	99.76	0.00	387	187	386
80.88	98.99	-	89	39	> 5 hrs
71.06	98.25	-	1400	813	> 5 hrs
63.63	97.35	-	144	57	> 5 hrs
56.23	96.16	-	458	242	> 5 hrs
48.78	94.33	0.02	12820	3624	> 5 hrs

We can reduce the variance about twenty percent with only three percent profit reduction, which can be interpreted as cost, for almost all the cases. As what we have expected, the more cargoes we can have, the more chances we can get to reduce the variance with small cost.

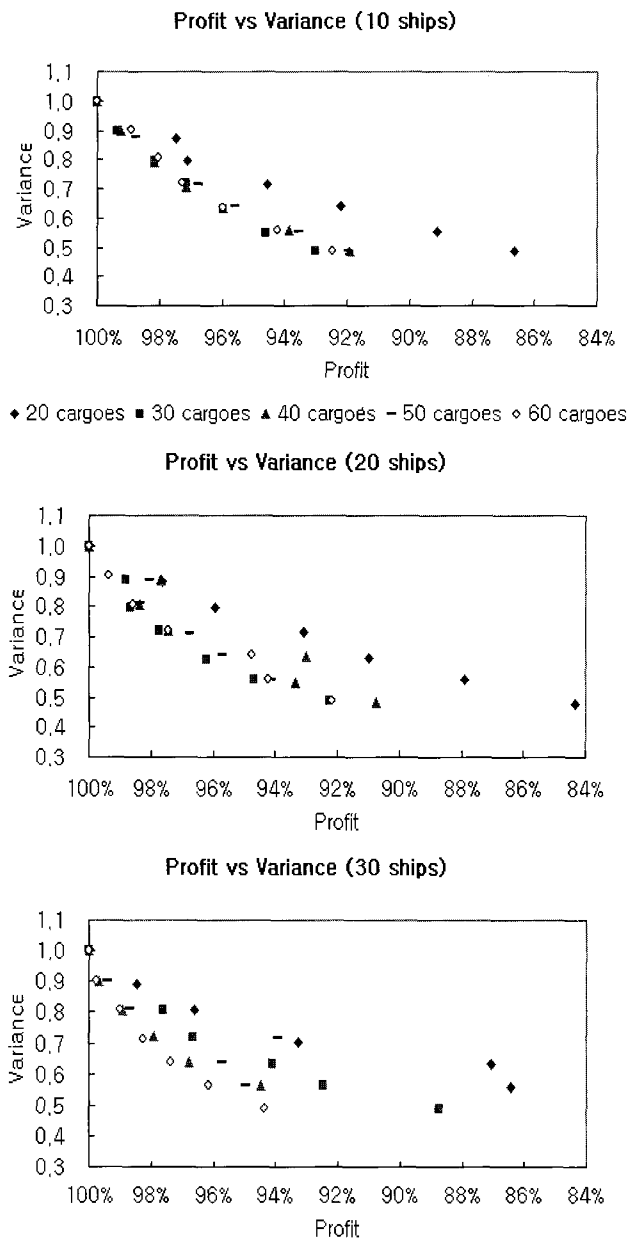

Fig. 1 Profit vs Variance plots.

Fig. 1 shows plots of profit reduction with respect to the reduced variance for all the problem instances. It is well depicted that the more ships and cargos we have, the less cost requires to reduce variance.

5. Summary and future research topics

We presented a new ship scheduling set packing mode that constrains variance as well as a solution approach using Kelley's cutting plane method. Computational results showed that Kelley's cutting plane method fits well for the new ship scheduling set packing model.

However, we could not finish branch-and-bound tree search within the time limit, which requests further study on efficient algorithms to get the optimal solution in shorter time.

Kelley's cutting plane method needs all the columns *a priori* with the covariance matrix. If the problem has many feasible schedules for each ship or has many ship and cargo instances, the number of variables increases rapidly. Then we may not be able to use Kelley's cutting plane method, because of memory issue to hold all the columns and covariance matrix and prolonged time for constructing the problem with totally enumerated columns. Column generation method should be introduced for this kind of large instances.

References

[1] Bausch, D. O., Brown, G. G. and Ronen, D. (1998), "Scheduling Short-Term Maritime Transport of Bulk Products," *Maritime Policy and Management* 25, 335-348.

[2] Christiansen, M., Fagerholt, K. and Ronen, D. (2004), "Ship routing and scheduling: Status and perspectives", *Transportation Science* 38(1), 1-18.

[3] Christiansen, M. and Fagerholt, K. (2002), "Robust ship scheduling with multiple time windows", *Naval Research Logistics* 49(6), 611-625.

[4] Fisher, M. and Rosenwein, M. (1989), "An interactive optimization system for bulk-cargo ship scheduling", *Naval Research Logistics* 36, 27-42.

[5] Fagerholt, K. (2001), "Ship scheduling with soft time windows: an optimisation based approach", *Eur. J. Oper. Res.* 13, 559-571.

[6] Fagerholt, K. and Christiansen, M. (2000a), "A combined ship scheduling and allocation problem", *J. Oper. Res. Soc.* 51(7), 834-842.

[7] Fagerholt, K. and Christiansen, M. (2000b), "A travelling salesman problem with allocation, time window and precedence constraints: an application to ship scheduling", *Int. Trans. Oper. Res.* 7(3), 231-244.

[8] Kelley, J. E., Jr. (1960), "The cutting plane method for solving convex programs", *J. of the society for Industrial and Applied Mathematics*, Vol. 8, No. 4, 703-712.

[9] Kim, S. and Lee, K. (1997), "An optimization-based

decision support system for ship scheduling", *Computers and Industrial Engineering*, 33(3-4), 689-692.

[10] Kim, S. H. (1999), "Optimization-Based Decision Support System For Maritime Transportation Problems In Tramp Shipping", *Ph.D. thesis*, Industrial Engieering, Pusan National University, Pusan, South Korea.

[11] Mulvey, J., Vanderbei, R. and Zenios, S. (1995) "Robust optimization of large-scale system". *Operations Research*, 43(2):264-281.

[12] Sharpe, W. F. (1970), "Portfolio Theory and Capital Markets", McGraw-Hill.

[13] Winston, W. L. (1994) "Operations Research: Applications and Algorithms". *International Thomson Publishing*, 3rd ed.

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