

# Numerical Analysis of Supersonic Axisymmetric Screech Tone Noise Using Optimized High-Order, High-Resolution Compact Scheme

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## Abstract

The screech tone of underexpanded jet is numerically calculated without any specific modeling for the screech tone itself. Fourth-order optimized compact scheme and fourth-order Runge-Kutta method are used to solve the 2D axisymmetric Euler equation. Adaptive nonlinear artificial dissipation model and generalized characteristic boundary condition are also used. The screech tone, generated by a closed loop between instability waves and quasi-periodic shock cells at the near field, is reasonably analyzed with present numerical methods for the underexpanded jet having Mach number 1.13. First of all, the centerline mean pressure distribution is calculated and compared with experimental and other numerical results. The instantaneous density contour plot shows Mach waves due to mixing layer convecting supersonically, which propagate downstream. The pressure signal and its Fourier transform at upstream and downstream shows the directivity pattern of screech tone very clearly. Most of all, we can simulate the axisymmetric mode change of screech tone very precisely with present method. It can be concluded that the basic phenomenon of screech tone including the frequency can be calculated by using high-order and high-resolution schemes without any specific numerical modeling for screech tone feedback loop.

**Keywords:** *Computational Aeroacoustics, Screech Tone, Jet Noise, Compact Scheme, Feedback*

## 1. Introduction

Supersonic jet noise consists of three principal components: the turbulent mixing noise, the broadband-shock-associated noise, and the screech tones[1]. Turbulent mixing noise is produced when a large scale turbulence structure propagates downstream as supersonic Mach wave. Shock-associated noise and screech tone propagate upstream. Shock-associated noise is produced by interaction between shock cell and vortex, however screech tone is produced by feedback loop near the nozzle exit.

When a launcher takes off, there exists some kind of noise

generated by jet flow. Especially a jet noise which propagates upstream can give serious damage to equipments in the launcher and launcher itself. So, we can know that an analysis of screech tone is very important to make a safe launch. For a long time, researches on screech tone have been made with experiments and theoretical equations. Since the nineties, numerical simulations have been widely used. But without the help of numerical modeling, it has been hardly possible to simulate screech tones numerically. In this work, the screech tone of underexpanded jet is numerically calculated not with any specific modeling for screech tone itself but with optimized high order high resolution compact scheme.

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## II. Governing equations and numerical schemes

The governing equations of motion for the axisymmetric inviscid flow are:

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \mathbf{H} = 0 \quad (1)$$

where

$$\mathbf{Q} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho e \end{bmatrix}, \mathbf{E} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (\rho e + p)u \end{bmatrix}, \mathbf{F} = \begin{bmatrix} \rho v \\ \rho vu \\ \rho v^2 + p \\ (\rho e + p)v \end{bmatrix}, \mathbf{H} = \frac{1}{y} \begin{bmatrix} \rho v \\ \rho vu \\ \rho v^2 \\ (\rho e + p)v \end{bmatrix}$$

To be a correct simulation of screech tone, the flow near the nozzle exit should be simulated correctly and the wave, which propagates upstream with relatively small amplitude, should be analyzed precisely. So to make a successful analysis of screech tone, we have to use a method of CAA (computational aeroacoustics) which has high-resolution and high-order. In this work, we use fourth-order OHOC (optimized high-order compact) [2-3] schemes for the evaluation of spatial derivatives and the fourth-order Runge-Kutta scheme for the integration in time.

And to prevent unwanted non-physical reflections around computational boundaries, we use generalized characteristic boundary conditions[4] as the time-dependent boundary conditions. Also, nonreflecting inflow / outflow and inviscid wall conditions are imposed at the boundaries of the computational domain and along the nozzle inner and outer walls. Generalized characteristic boundary condition protect computational domain from non-physical computational errors. Boundary conditions are based on the local one-dimensional characteristics in the direction normal to the boundary. The characteristics are analyzed by transforming the governing equations to the quasi-one-dimensional characteristic wave convection equations in the normal direction, considering the other terms in the transverse directions as a source term. In the case of supersonic inflow, all primitive variables are should be specified.

Fig.1 shows the entire computation domain schematically. The

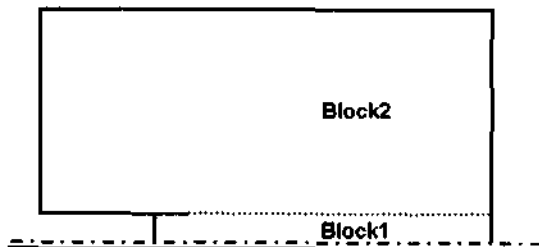


Figure 1, Computational domain.

domain consists of two sub-domains. In the upper region, the domain extends 10 diameters back from the nozzle exit. It is 20 diameters long in x direction and 17 diameters in the y direction. The total number of grid point is about 220,000 and grid points are condensed around the nozzle exit both in the x and y direction.

## III. Numerical results

We have been able, using the numerical algorithm described earlier, to simulate the screech tone of supersonic jet numerically. Fig.2 shows the instantaneous pressure contour of Mach number 1.13 cold jet. From the contour, we can observe not only Mach wave which propagates downstream but also screech tone which propagates upstream. As well known, we can know that sound waves of the screech tones are radiated out in a region around the fourth to fifth shock cells downstream of nozzle exit.

One important component of screech tone generation is the shock cell structure inside the jet plume. To ensure that the simulated shock cells are just same as those in an actual experiment, we compare the time-averaged pressure distribution

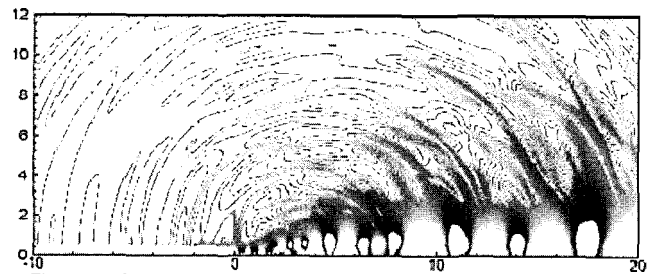


Figure 2, Pressure contour (0.91 <math>\langle p/p\_a \rangle</math> (1.09).

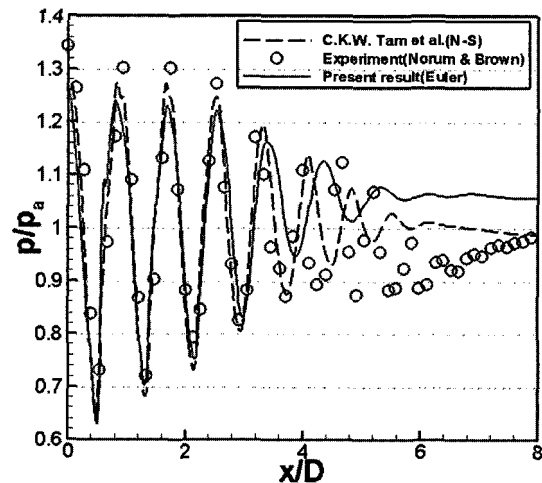


Figure 3, Time-averaged pressure distribution along the centerline of a Mach 1.2 jet.

along the centerline of the simulated jet at Mach 1.2 with both the experimental result by Norum and Brown[5] and other numerical result by C.K.W. Tam et al[6]. The time-averaged pressure distribution along the centerline is shown at Fig.3. From the figure, it is clear that the first five shock cells are in good agreement with experiment data in terms of the both shock cell spacing and amplitude.

In the region of  $x/D > 5$ , the present result shows the over-predicted time-averaged pressure. Since the present method solves Euler equation, and Euler equation can not consider the viscous effect and three-dimensional effect, which are under investigation. And at near field region ( $x/D < 4$ ), the viscous and three-dimensional effects are not dominant, but as the flow goes downstream ( $x/D > 5$ ) the viscous and three-dimensional effects become larger. Other numerical results and experimental data contain those effects but present result does not take account of those effects, so the simulated flow is more concentrated at center line. This leads the result of over-predicted time-averaged pressure. Although present result shows over-predicted pressure value in downstream region ( $x/D > 5$ ), the result shows good agreement near the nozzle exit, where the screech tone noise is generated, the present method can simulate supersonic jet screech tone correctly.

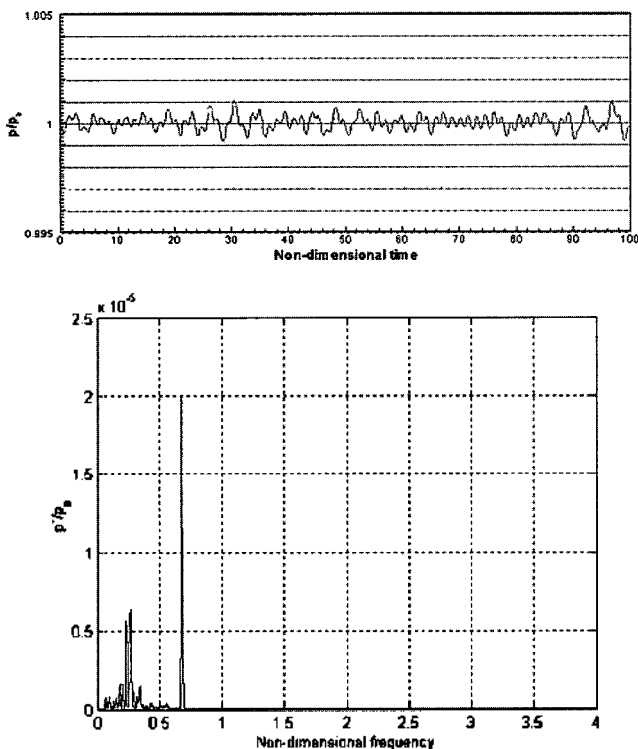


Figure 4. A pressure signal and its Fourier Transform in the upstream.

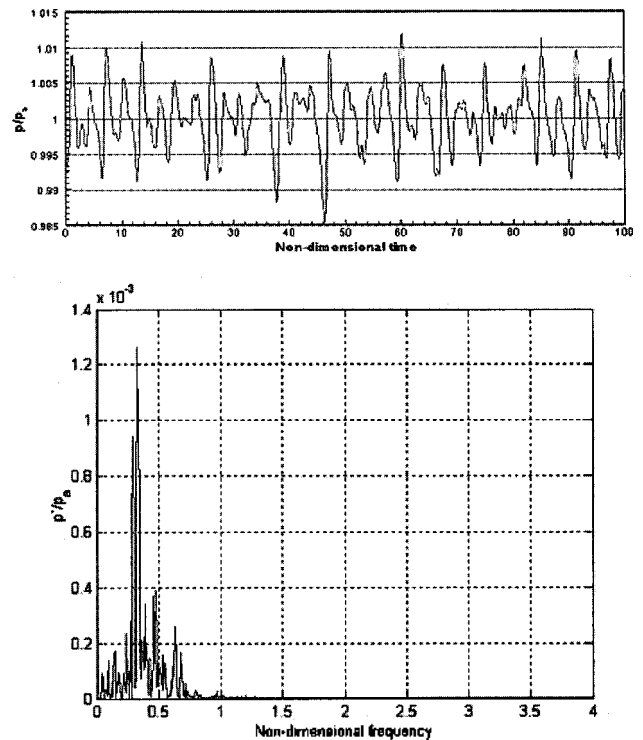


Figure 5. A pressure signal and its Fourier Transform in the downstream.

One of the most important characteristics of screech tone is that it mainly propagates upstream. We can confirm this from Fig.4 and Fig.5. Fig.4 shows a pressure signal in the upstream and its Fourier transform of a 1.13 jet. Two dominant peaks are presented here. The non-dimensional frequency of bigger peak is about 0.68, and that of smaller one is about 0.27.

From Fig.5, however, we observe that the lower non-dimensional frequency has bigger amplitude than higher one. The higher frequency represents screech tone frequency and lower one does Mach wave radiation. So, as well known, we can confirm that screech tone mainly propagates upstream, on the other hand, the Mach wave mainly radiates downstream.

It is well known that at low supersonic jet Mach numbers there are two axisymmetric screech modes: the A1 and the A2 modes. The frequencies of A1 and A2 modes are varies and suddenly shifted with the increase of jet Mach number. In the present simulation, both the A1 and the A2 modes are simulated with various jet Mach number. Fig.6 shows the variation of  $\lambda/D$ , where  $\lambda$  is the acoustic wavelength of the tone, with jet Mach number. We can obtain the value of  $\lambda/D$  from the relationship written below:

$$\frac{\lambda}{D} = \frac{a_\infty}{fD} \quad (2)$$

where  $f$  is the screech tone frequency.

And the numerical results are compared with both the experimental results by Ponton and Seiner[7] and other numerical results by C.K.W. Tam et al.[6]. The numerical results show a good agreement with other results, especially experimental results in Fig.6.

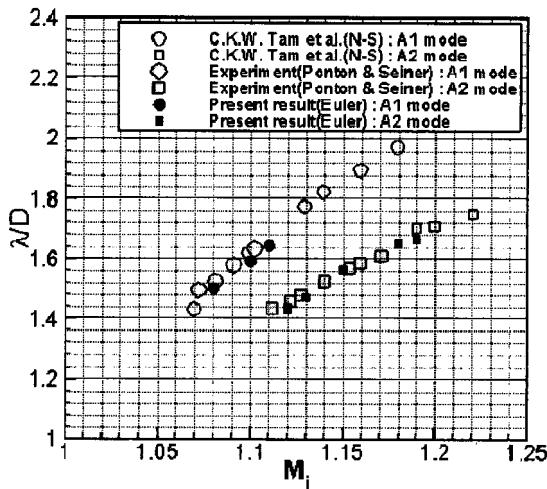


Figure 6. Wavelengths of screech tone.

From experimental data, we can observe two axisymmetric screech tone modes and the mode change occur in the region of jet Mach number between 1.10 and 1.11. Present numerical simulations show that the axisymmetric mode changes between jet Mach number 1.11 and 1.12. We can conclude that present method has a merit of predicting the mode change of axisymmetric screech tones, and show better results compared with other numerical results.

#### IV. Conclusions

In the present research, we simulated the axisymmetric screech tones with the help of high-order, high-resolution computational aeroacoustics (CAA) scheme. This method shows a good result of reproducing screech tones computationally and analyzes a successful mode change of axisymmetric modes. We can conclude that in the range of low supersonic jet Mach number, the axisymmetric screech tones can be simulated without any special modeling for screech tone itself and its feedback loop.

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