

# Derivation of Distributed Generation Impact Factor in a Networked System in Case of Simultaneous Outputs of Multiple Generation Sites

Jung-Uk Lim\* · Thordur Runolfsson

## **Abstract**

A new measure, the distributed generation impact factor (DGIF), is used for evaluating the impact of newly introduced distributed generators on a networked distribution or a transmission system. Distribution systems are normally operated in a radial structure. But the introduction of distributed generation needs load flow calculation to analyze the networked system. In the developed framework, the potential share of every generation bus in each line flow of a networked system can be directly evaluated. The developed index does not require the solution of power flow equations to evaluate the effect of the distributed generation. The main advantage of the developed method lies in its capability of considering simultaneous outputs of multiple generation sites.

Key Words: DGIF(Distributed Generation Impact Factor), Networked Systems, Analytic Method, Load Flow Analysis

#### 1. Introduction

The recent introduction of competitive electric markets as well new generation technologies, such as fuel cells and micro-turbines, has sparked interest in distributed generation (DG). Properly planned and operated, DG can provide a wide variety of benefits, including economic savings, improved environmental performance, and greater reliability [1, 2]. Recently, a variety of methods to maximize the benefits brought by DG are being

researched [3].

The impact of such smaller-scale generation on the radial distribution system has been discussed in the literature [4]. Although distribution systems are mostly radially structured, the introduction of distributed generation requires load flow analysis to consider reverse flows by DG and eventually to analyze the networked system. In addition, a study on the static and dynamic behaviors of a transmission system with significant distributed generation resources is required as well.

Due to the need for improved analysis methods for networked distribution systems or transmission systems with significant DG installations, a new analysis measure for the impact of distributed generators on the networked system is developed.

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In order to evaluate the impact of newly introduced individual distributed generator on each line flow, the distributed generation impact factor (DGIF) is derived from the power flow equations all nodes. The main advantage of the developed method lies in its capability of considering simultaneous outputs of multiple generation sites. Furthermore, its application does not require the solution of complete power flow equations, and thus it provides substantial savings in computational effort.

## 2. Derivation of the DGIF

In the following, it is assumed that the ratio of each bus power demand to the total system demand is constant and known in advance, i.e.

$$k_i = \frac{P_{L,i}}{P_{L,total}} = constant, \text{ for } i = 1, 2, \dots, n$$
 (A)

where,  $P_{L,i}$ : demand at bus-i,  $P_{L,total}$ : total system demand, and n: number of buses.

The power (line) flow  $P_{ij}$  from bus-i to bus-j is explicitly expressed as

$$P_{ii} = f(\theta_i, \theta_i, V_i, V_i) \tag{1}$$

where,  $\theta_i$ ,  $\theta_j$ : voltage angles of bus-i and bus-j and  $V_i$ ,  $V_j$ : voltage magnitude of bus-i and bus-j, respectively.

If we assume that the voltage magnitudes at bus-i and bus-j are constant ( $\Delta V_i = \Delta V_j = 0$ ) then the incremental value of  $P_{ij}$ ,  $\Delta P_{ij}$  can be expressed as follows:

$$\Delta P_{ij} = \frac{\partial f}{\partial \theta_i} \Delta \theta_i + \frac{\partial f}{\partial \theta_j} \Delta \theta_j \qquad (2)$$

$$= \left[ 0 \cdots 0 \frac{\partial f}{\partial \theta_i} 0 \cdots 0 \frac{\partial f}{\partial \theta_j} 0 \cdots 0 \right] \begin{bmatrix} \Delta \theta_1 \\ \Delta \theta_2 \\ \vdots \\ \Delta \theta_{n-1} \end{bmatrix}$$

Let  $\Delta P = [\Delta P_1 \Delta P_2 \cdots \Delta P_{n-1}]^T$  where  $\Delta P_i$  is the net power injection at bus i and  $\Delta \theta = [\Delta \theta_1 \Delta \theta_2 \cdots \Delta \theta_{n-1}]^T$ . Then, it follows from the networks power flow equations that the relationship between real power injections and bus voltage angles can be described as:

$$\Delta P = J_{P\theta} \Delta \theta \tag{3}$$

where  $J_{P\theta}$  is the submatrix of Jacobian representing  $\frac{\partial P}{\partial \theta}$  evaluated at the nominal steady state value. Assuming that matrix  $J_{P\theta}$  is nonsingular from equation (3), we get

$$\Delta\theta = J_{P\theta}^{-1} \, \Delta P \tag{4}$$

The above formulation does not include the slack bus. In order to incorporate the slack bus in equation (2), the following equation derived from the power flow equations is considered [5].

$$\Delta P_{slack} = J_{slack, \theta} \Delta \theta = J_{slack, \theta} J_{P\theta}^{-1} \Delta P \tag{5}$$

where,  $\Delta P_{sluck}$  is the incremental value of active power at slack bus and  $J_{slack,\theta} = \frac{\partial P_{slack}}{\partial \theta}$  evaluated at the nominal operating conditions.

From equation (5), we obtain an augmented linear combination as follows:

$$\left[ -J_{slack,\theta}J_{P\theta}^{-1} \mid 1 \right] \begin{bmatrix} \Delta P \\ - \\ \Delta P_{slack} \end{bmatrix} = 0 \tag{6}$$

Let  $[-J_{slack,\theta}J_{P\theta}^{-1}|1]$  be a vector of the linear combination factors  $[q_1,q_2,\cdots,q_{n-1},q_n]$ , then

$$\begin{bmatrix} q_1, q_2, \cdots, q_{n-1}, q_n \end{bmatrix} \begin{bmatrix} \Delta P \\ - \\ \Delta P_{stack} \end{bmatrix} = 0$$
 (7)

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By replacing 
$$\Delta P = \begin{bmatrix} \Delta P \\ - \\ \Delta P_{stack} \end{bmatrix}$$
 with  $\Delta P = \frac{1}{2} \frac{$ 

 $\Delta P_G - \Delta P_L$  in equation (7), where  $\Delta P_G$  is a vector which denotes incremental of real power in all generation buses and  $\Delta P_L$  is a vector which denotes incremental amounts of real power in all load buses, we get the following,

$$[q_{1}, q_{2}, \cdots, q_{n-1}, q_{n}]\begin{bmatrix} \Delta P_{G,1} \\ \Delta P_{G,2} \\ \vdots \\ \Delta P_{G,n-1} \\ \Delta P_{G,n} \end{bmatrix}$$

$$= [q_{1}, q_{2}, \cdots, q_{n-1}, q_{n}]\begin{bmatrix} \Delta P_{L,1} \\ \Delta P_{L,2} \\ \vdots \\ \Delta P_{L,n-1} \\ \Delta P_{L,n} \end{bmatrix}$$
(8)

By applying the assumption (A) that the ratio of each bus power demand to the total system demand is constant we get

$$[\Delta P_{L,1}\Delta P_{L,2}\cdots\Delta P_{L,n-1}\Delta P_{L,n}]^{T}$$

$$=[k_{1}k_{2}\cdots k_{n-1}k_{n}]^{T}\Delta P_{L,total}$$
(9)

Hence the relationship between incremental generation and incremental total demand, including the slack bus, becomes

$$\Delta P_{L, total} = \frac{1}{r} [q_1, q_2, \cdots, q_{n-1}, q_n] \begin{bmatrix} \Delta P_{G,1} \\ \Delta P_{G,2} \\ \vdots \\ \Delta P_{G,n-1} \\ \Delta P_{G,n} \end{bmatrix}$$
(10)

where,  $r = [q_1, q_2, \dots, q_{n-1}, q_n][k_1 k_2 \dots k_{n-1} k_n]^T$ . By combining equation (2) and (4), we get

$$\Delta P_{ij} = \begin{bmatrix} 0 & \cdots & 0 & \frac{\partial f}{\partial \theta_i} & 0 & \cdots & 0 & \frac{\partial f}{\partial \theta_j} & 0 & \cdots & 0 \end{bmatrix} J_{R_{\theta}}^{-1} \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_{n-1} \end{bmatrix}$$
(11)

and from equation(9) and  $\Delta P = \Delta P_G - \Delta P_L$ , not

including the slack bus, we obtain

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_{n-1} \end{bmatrix} = \begin{bmatrix} \Delta P_{G,1} \\ \Delta P_{G,2} \\ \vdots \\ \Delta P_{G,n-1} \end{bmatrix} - \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_{n-1} \end{bmatrix} \Delta P_{L, total}$$
(12)

By combining equations (10), (11) and (12) we finally obtain for all buses

$$\Delta P_{ij} = \begin{bmatrix} 0 & \cdots & 0 & \frac{\partial f}{\partial \theta_{i}} & 0 & \cdots & 0 & \frac{\partial f}{\partial \theta_{j}} & 0 & \cdots & 0 \end{bmatrix} *$$

$$\begin{bmatrix}
v_{11} & v_{12} & \cdots & v_{1n} \\
v_{21} & v_{22} & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
v_{n-1} & v_{n-1} & z & \cdots & v_{n-1} & n
\end{bmatrix} * \begin{bmatrix} \Delta P_{G,1} \\ \Delta P_{G,2} \\ \vdots \\ \Delta P_{G,n} \end{bmatrix}$$

$$= \begin{bmatrix} d_{1}^{ij} & d_{2}^{ij} & \cdots & d_{n}^{ij} \end{bmatrix} \begin{bmatrix} \Delta P_{G,2} \\ \Delta P_{G,2} \\ \vdots \\ \Delta P_{G,n} \end{bmatrix}$$

$$= \begin{bmatrix} d_{1}^{ij} & d_{2}^{ij} & \cdots & d_{n}^{ij} \end{bmatrix} \begin{bmatrix} \Delta P_{G,2} \\ \Delta P_{G,2} \\ \vdots \\ \Delta P_{G,n} \end{bmatrix}$$

$$= \begin{bmatrix} d_{1}^{ij} & d_{2}^{ij} & \cdots & d_{n}^{ij} \end{bmatrix} \begin{bmatrix} \Delta P_{G,2} \\ \Delta P_{G,2} \\ \vdots \\ \Delta P_{G,n} \end{bmatrix}$$

where the  $(n-1) \times n$  matrix

$$\begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ v_{n-1} & v_{n-1} & z & \cdots & v_{n-1n} \end{bmatrix}$$
 is derived from

equations (10), (11) and (12) [6].

The coefficient  $d_k^{ij}$  in equation (13), which is the sensitivity of the flow on line ij with respect to the power generation at bus-k characterizes the incremental power flow in the transmission line from bus-i to bus-j when a unit power is increased at bus-k. This coefficient is defined as the Distributed Generation Impact Factor (DGIF). Therefore, one can directly obtain the contribution of each bus of distributed generators on all lines by applying the DGIF. The relationship between the incremental line flow and the incremental power generation at any bus via DGIFs is given by equation (13), i.e.

$$\Delta P_{ij} = d_{1}^{ij} \Delta P_{G,1} + d_{2}^{ij} \Delta P_{G,2} + \cdots$$

$$\cdots + d_{n-1}^{ij} \Delta P_{G,n-1} + d_{n}^{ij} \Delta P_{G,n}$$
(B)

## 3. Case Studies

## A. 5-bus Networked System

The developed DGIF method was tested on a small-scale 5-bus networked system. As shown in Fig. 1, DG G1 produces 5 MW to meet load increase at bus 5. In order to compare the power flows in each line obtained from the DGIFs and equation (B), another load flow calculation considering a generation increase G1 and a load increase D1 is performed. The results, as shown in Table I, agree quite well with those of the load flow calculation.

Practically, the incorporation of new DG cutputs will not severely change the existing power flow because a portion of the power increase by DG is relatively small compared to that of the existing power flows.

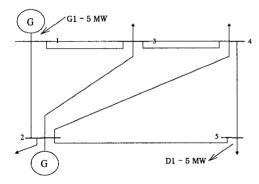


Fig. 1. A 5-bus Networked System

Table 1. Comparison of Line Flows Using Load Flow Calculation and DGIF Application

Line	Base Case Load Flow	Load Flow Calculation	DGIF Application
1-2	89.33	93.56	93.55
1-3	41.79	42.95	42.94
2-3	24.47	24.61	24.58
2-4	27.71	28.05	28.01
2-5	54.66	58.30	58.29
3-4	19.39	20.60	20.58
4-5	6.60	8.14	8.15

## B. IEEE 14-bus System

The developed method has also been applied to the IEEE 14-bus system to show its capability in an application to a large-scale networked system [7]. Furthermore, we will consider simultaneous outputs of multiple generation sites. The analysis of the cases considered is actually difficult through the conventional load flow analysis. The following three cases are considered.

- Case 1) D1-Load increase at bus-12 and G1-DG generation of 20 MW at bus-1
- Case 2) D2-Load increase at bus-1 and G2-DG generation of 20 MW at bus-12
- Case 3) Simultaneous occurrences of D1, G1, D2, G2

As shown in Fig. 2, Case 1 is in the same direction as the existing main flow while Case 2 is in the opposite direction. Line flows obtained from the DGIFs will be compared with those obtained from another load flow calculation in Case 1 and Case 2. Using a conventional load flow analysis it is seen that the net result in Case 3 is an almost no change in line flows before and after load–generation changes, but each generator plays its own role. When individual contribution of each generator needs to be evaluated, the proposed method using DGIFs is certainly effective because it shows individual change of each line flow even though simultaneous outputs of multiple generation sites happen.

Table II shows simulation results by the conventional load flow calculation and the DGIF method and comparisons between both methods. The values in the third and fourth column represent error rates of the power flows in each line obtained from the DGIFs, as compared with those from another load flow calculation in Case 1 and Case 2, respectively. The error rates in Table II are considered as not significant because they

are of the order of 1[%].

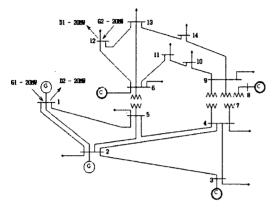


Fig. 2. IEEE 14-bus System with Demand-Generation Changes.

The data in fifth column and sixth column which come from Case 3 are very interesting. In the conventional power flow method, simultaneous outputs of multiple generation sites are treated as an association of DG generations. In Case 3, an association of counter-flow DG generations can dramatically reduce line flows. The simultaneous occurrences of D1, G1, D2, G2 become nearly zero as shown in fifth column of Table II.

However, in the proposed method, we can get line flow changes by each DG generation respectively as shown in sixth column of Table II. This is one of the main advantages of the method using the DGIF. In other words, when simultaneous outputs of multiple generation sites take place, individual contribution of each generator to each line can be evaluated by the proposed method using DGIFs. Note that the conventional load flow analysis method produces one value per line in fifth column of Table II whereas the proposed method has two values per line in sixth column of Table II. In particular, the sixth column of Table II shows how line flows are individually influenced by each generation site.

Table 2. Simulation Results from The Conventional Method and the DGIF application

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	Base			Case3	Case3		
	Case	Casel	Case2	Flow	Flow		
Line	Line	Error		Change	Changes		
Line			Error	(MW)	(MW)		
	Flow	([%])	([%])	Load	Using		
	(MW)			Flow	DGIF		
1-2	197.54	0.37[%]	0.35[%]	-1.44	18.44 / -17.15		
2-3	46.39	0.30[%]	0.28[%]	-0.64	3.48 / -3.22		
2-4	90.21	0.30[%]	0.28[%]	-0.50	6.72 / -6.31		
1-5	117.11	0.33[%]	0.32[%]	0.94	9.78 / -9.27		
2-5	82.39	0.33[%]	0.31[%]	-0.20	6.89 / -6.47		
3-4	38.23	0.35[%]	0.32[%]	-0.62	3.33 / -3.10		
4-5	-33.58	0.12[%]	0.11[%]	-1.30	1.03 / -0.95		
5-6	127.64	0.50[%]	0.47[%]	-0.37	15.80 / -15.06		
4-7	69.40	0.30[%]	0.28[%]	0.14	5.23 / -4.94		
7-8	0.00	0.00[%]	0.00[%]	0.00	0.00 / 0.00		
4-9	39.45	0.30[%]	0.28[%]	0.16	2.91 / -2.76		
7-9	69.40	0.30[%]	0.28[%]	0.14	5.23 / -4.94		
9-10	49.25	0.32[%]	0.30[%]	0.15	4.00 / -3.75		
6-11	17.44	0.86[%]	0.82[%]	-0.19	-3.74 / 3.58		
6-12	44.99	1.18[%]	1.12[%]	-0.07	13.24 / -12.58		
6-13	43.01	0.59[%]	0.56[%]	-0.11	6.30 / -6.07		
9-14	30.10	0.55[%]	0.52[%]	0.15	4.14 / -3.94		
10-11	19.54	0.79[%]	0.75[%]	0.14	3.87 / -3.66		
12-13	-21.11	1.61[%]	1.64[%]	-0.05	-8.48 / 8.67		
13-14	-3.99	3.71[%]	3.64[%]	-0.16	-3.70 / 3.63		

#### 4. Conclusion

The distribution generation impact factor (DGIF) of newly introduced distributed generators on a networked system is analytically derived. Using the developed DGIF, we can directly evaluate the impact of individual distributed generator on each line flow of a networked system. The DGIF can be applied to the cases of simultaneous outputs of multiple generation sites which the conventional method is not feasible for.

Simulation results on a 5-bus networked system and a IEEE 14-bus system show that the results using the DGIF agree quite well with those of a

conventional load flow calculation. The explanation of this property lies in the fact that the portion of power change by DG is relatively small compared to that of the existing power flows. Simulation results also show the main advantage of the developed method lies in its capability of considering simultaneous outputs of multiple generation sites. The DGIF can be utilized for operation or planning of the networked distribution systems or the transmission systems including significant DG applications.

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# **Biography**

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Jung-Uk Lim was born in Korea, on September 27, 1970. He received the B.S. degree from Hanyang University, Korea in 1996 and M.S. and Ph.D. degrees from Seoul National University, Korea in 1998 and 2002, respectively. He is a research professor at Myongji University and currently, he works with Dr. Runolfsson as a postdoctoral research associate at the University of Oklahoma, United States. His research field includes FACTS, distributed generation, power quality and system protection.

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Thordur Runolfsson is a professor in the School of Electrical and Computer Engineering at the University of Oklahoma, United States. He received the BS in Electrical and Computer Engineering and Mathematics from the University of Wisconsin in 1983 and his Ph.D. in EE-Systems from the University of Michigan, Ann Arbor in 1988. His research interests include systems and control, uncertainty analysis, modeling and simulation and power systems and energy efficiency.