Journal of the Korean Institute of Illuminating and Electrical Installation Engineers Vol. 20, No.8, pp. 42~47 September 2006 논문 20-8-7

경계 추정치를 가진 로봇 슬라이딩 모드 제어

(Sliding Mode Control with Bound Estimation for Robot Manipulators)

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요 약

본 논문에서는 로봇의 동력학에 대한 정확한 지식을 요구하지 않는 로봇 머니퓰레이터를 위한 경계 추정기법을 가진 슬라이딩 모드 제어기를 제안한다. 경계 추정을 위해 로봇 동력학의 불확실한 비선형 요소들의 경계치를 제 1종의 Fredholm 적분식을 이용하여 표현하고, 슬라이딩 평면 함수값만을 이용한 적응 기법을 제안한다. 또한 로봇 동력학의 중요한 두가지 특성인 왜대칭성과 양정치성을 이용하여 로봇 시스템의 점근적 안정성을 증명한다.

Abstract

In this paper, we propose a sliding mode control with the bound estimation for robot manipulators without requiring exact knowledge of the robot dynamics. For the bound estimation, the upper bound of the uncertain nonlinearities of robot dynamics is represented as a Fredholm integral equation of the first kind and we propose an adaptive scheme which is only dependent on the sliding surface function. Also, we prove the asymptotic stability for the robot systems using two important properties in the robot dynamics: skew-symmetry and positive-definiteness of robot parameters.

Key Words : Sliding mode, Fredholm integral

1. Introduction

Over the years, much attention has been paid to the problem of the controller design for robotic manipulators. However, due to the mechanical characteristics of robotic manipulators such as high nonlinearity and coupling effects between joints, it is difficult to control robot manipulators precisely by conventional control methods at high speed. To overcome this difficulty, various control algorithms using dynamics of robot manipulators have been proposed. But, since the exact model of robot manipulators cannot be easily obtained and the heavy computational burden is required, it is expensive to implement. To avoid these problems, control algorithms using the theory of variable structure system have been developed[1]. Among developed control algorithms using the theory of

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접수일자 : 2006년 6월 8일
1차심사 : 2006년 6월 14일
심사완료 : 2006년 6월 28일

VSS, there still exist some nontrivial difficulties in the design such as the inversion of the inertia matrix. Yeung and Chen[2] proposed a variable structure controller for the set-point regulation problem without having to take the inverse of the inertia matrix. Su and Leung[3] presented a sliding mode controller with the bound estimation for accomplishing trajectory control of robots based on the linear parameterization approach[4], which decomposes the robot dynamics into the product of a constant known vector and a known nonlinear function called the regressor. In an interesting paper[5], assuming that a nonlinear disturbance function is represented as an integral equation, Messner et al proposed an adaptive learning rule for a class of nonlinear systems. Integral equations [6] provide more generally mathematical basis to describe the upper bound of the uncertainties even though we don't have any information of dynamic characteristics.

In this paper, for the trajectory control we propose a sliding mode control with the bound estimation of the uncertain nonlinear parameters without exact knowledge of the robot dynamics. For the bound estimation, we assume that the upper bound of the uncertain nonlinearities is represented as a Fredholm integral equation of the first kind, i.e, an integral of the product of a predefine kernel with an unknown influence function. We also provide a sufficient condition for the existence of such a representation. The construction of an adaptation law is only dependent on the sliding surface function. Using the estimated bound, the sliding mode control is constructed in such a way that the prescribed sliding surface will attract every system's trajectory and upon the intersection with the sliding surface the trajectory will remain there for all subsequent time. To accomplish the given task, we take advantage of two important properties, skew-symmetry and positive-definiteness of robot parameters[4]. Using the above properties, we easily prove the asymptotic stability of the robot manipulator in the sense of Lyapunov stability.

2. Robot Dynamics

It is well known that the dynamic equation for a robot manipulator with n degrees of freedom based on Lagrangian formulation can be described as follows:

$$M(q(t))\ddot{q}(t) + C(q(t),\dot{q}(t))\dot{q}(t) + G(q(t)) = u(t) \qquad (1)$$

where $q(t) \in R^n$ is the joint angle vector, $M(q(t)) \in R^{n \times n}$ is the inertia matrix, $C(q(t),\dot{q}(t)) \in R^n$ is the centrifugal and coriolis matrix, $G(q(t)) \in R^n$ is the gravitational torque, and $u(t) \in R^n$ is the applied joint torque. In the above dynamics, we can find two important properties for our main work.

Property 1: The inertia matrix $M(q)$ is symmetric and positive-definite for all q.

Using this property, it is not necessary to take the inverse of the inertia matrix in the conventional controller design.

Property 2: The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew symmetric. That is, $X^T(\dot{M}(q) - 2C(q,\dot{q}))X = 0$ for arbitrary vector $X \in R^n$ [4].

These two properties are utilized to prove the stability of the robot manipulator under consideration.

3. Controller Design

We now propose a sliding mode control for the position tracking problem of robot manipulators. Let $q_d(t) \in R^n$ and be the desired joint angle vectors. We define the position tracking error by

$$e(t) = q(t) - q_d(t) \tag{2}$$

The sliding surface s=0 is chosen as hyperplane

$$s(t) = \dot{e}(t) + \Lambda e(t) \tag{3}$$

where $\Lambda \in R^{n \times n}$ is a positive definite matrix whose eigenvalues are strictly in the right-hand complex plane. To simplify the analysis, we define the reference velocity error vector, $\dot{e}_r(t) \in R^n$ by the linear combination

$$\dot{e}_r(t) = \dot{q}_d(t) - \Lambda e(t) \tag{4}$$

and we can obtain the fact that $s(t) = \dot{q}(t) - \dot{e}_r(t)$.

We now discuss the desired compensation control law which consists of a linear feedback controller, a nonlinear feedforward compensator, and a nonlinear feedback controller as follows:

$$\begin{aligned} u(t) = {} & \hat{M}(q_o)\ddot{e}_r(t) + \hat{C}(q_0, \dot{q}_0)\dot{e}_r(t) + \hat{G}(q_0) \\ & - K_S s(t) + u_N(t) \end{aligned} \tag{5}$$

where ^ means the estimation of robot parameters and $K_S \in R^{n \times n}$ is a positive definite matrix and $u_N(t)$ represents the nonlinear control which will be described later.

Employing the control law (5) on the robot dynamics described by (1), the following sliding mode equation is obtained:

$$M(q)\dot{s}(t) = -(C(q,\dot{q}) + K_S)s(t) + u_N(t) + H(q,\dot{q}) \tag{6}$$

where

$$\begin{aligned} H(q,\dot{q}) = {} & (\hat{M}(q_0) - M(q))\ddot{e}_r(t) \\ & + (\hat{C}(q_0,\dot{q}_0) - C(q,\dot{q}))\dot{e}_r(t) + (G(q_0) - G(q)) \end{aligned} \tag{7}$$

Thus, the uncertain nonlinear parameters on the robot dynamics can be represented as $H(q,\dot{q})$.

In the design of a class of nonlinear feedback controls, finding of the upper bound for $H(q,\dot{q})$ is very important for the asymptotic stability or uniform ultimate boundedness for robot manipulators. To describe the upper bound of the nonlinearity/uncertainties, we will state the following assumption using a Fredholm integral equation of the first kind.

Assumption 1: There exists a known continuous positive scalar-valued function $\rho(\cdot): R \to R_+$ such that $\|H(q,\dot{q})\| \le \rho(t)$ for all $(q,\dot{q}) \in R^n \times R^n$.

Assumption 2: The function $\rho(\cdot)$ can be represented as a Fredholm integral equation of the first kind, i.e., there exists a predefined kernel $\Psi(\cdot,\cdot): R \times R \to R^l$ and an unknown influence function $\beta(\cdot): R \to R^l$ such that, for an integral interval [a b]⊂R,

$$\rho(t) = \int_a^b \Psi(t,\tau)^T \beta(\tau) d\tau \tag{8}$$

where

$$\sup_{t \in [0\ \infty]} \int_a^b \Psi(t,\tau)^T \Psi(t,\tau) d\tau = \kappa < \infty \tag{9}$$

Remark 1: In general, the purpose of such an integral equation is to determine the unknown function $\beta(\cdot)$ for known functions $\rho(\cdot)$ and $\Psi(\cdot,\cdot)$ [6]. In reverse, given a predefined kernel $\Psi(\cdot,\cdot)$, if we estimate the unknown function $\beta(\cdot)$ using an appropriate method, we may also estimate $\rho(\cdot)$.

Remark 2: Assumption 2 allows the designers to choose any arbitrary kernel regardless of the structure of the nonlinearities and uncertainties. That is, even though the designers don't have any information of the nonlinearities and uncertainties except that they are bounded, one can describe their upper bound using a designer-chosen kernel and an unknown influence function.

Remark 3: The constraint on the predefined kernel given by (9) is the sufficient condition which guarantees that the estimate of $\rho(t)$ which will de defined later is bound.

Based on the above assumption, we propose the following adaptation law for estimating the upper bound of the nonlinearity/uncertainty $\rho(t)$:

$$\frac{\partial}{\partial t}\bar{\beta}(t,\tau) = \|s\| Q\Psi(t,\tau) \tag{10}$$

where $\bar{\beta}(\cdot,\cdot): R\times R \rightarrow R^l$ is the estimate of the unknown influence function $\beta(\cdot)$ in (8) and $Q \in R^{l\times l}$ is a positive definite diagonal matrix. And $\bar{\rho}(t)$, the function estimate of $\rho(t)$, is defined by

$$\bar{\rho}(t) = \int_a^b \Psi(t,\tau)^T \bar{\beta}(t,\tau)d\tau \tag{11}$$

Defining the influence error by $\tilde{\beta}(t,\tau) = \bar{\beta}(t,\tau) - \beta(\tau)$ and using (8) and (10), we obtain

$$\frac{\partial}{\partial t}\tilde{\beta}(t,\tau) = \|s\| Q\Psi(t,\tau) \tag{12}$$

and

$$\tilde{\rho}(t) = \int_a^b \Psi(t,\tau)^T \tilde{\beta}(t,\tau)d\tau = \bar{\rho}(t) - \rho(t) \tag{13}$$

Now, we consider the nonlinear control law:

$$u_N(t) = \begin{cases} -\dfrac{s}{\|s\|}\bar{\rho}(t) & \text{if } s(t)\neq 0 \\ 0 & \text{if } s(t)=0 \end{cases} \tag{14}$$

which represents the nonlinear feedback control for suppression of the effect of the uncertainty and drives the system trajectories toward the switching surface until intersection occurs. Then, we may state the following theorem.

Theorem 1: Consider the robot manipulator (1) with the sliding surface (3) and control laws (5), (10) and (14). If assumptions 1 and 2 are valid, then $s(t)=0$ is asymptotically stable.

Proof: Consider the following Lyapunov function

$$V(s,\tilde{\beta},t) = \frac{1}{2}s^T M s + \frac{1}{2}\int_a^b \tilde{\beta}(t,\tau)^T Q^{-1}\tilde{\beta}(t,\tau)d\tau \tag{15}$$

Differentiating (15) with respect to the time yields and using the Property 1

$$\dot{V} = s^T(-Cs - K_S s + u_N + H) + \frac{1}{2}s^T\dot{M}s + \int_a^b \tilde{\beta}^T Q^{-1}\frac{\partial}{\partial t}\tilde{\beta}d\tau \tag{16}$$

Using the Property 2 and Assumption 1 and 2, (16) becomes

$$\begin{aligned}\dot{V} &= -s^T K_S s + s^T u_N + s^T H + \|s\|\int_a^b \Psi^T\tilde{\beta}d\tau \\ &\le -s^T K_S s - \|s\|\bar{\rho} + \|s\|\rho + \|s\|\tilde{\rho} \\ &= -s^T K_S s < 0\end{aligned} \tag{17}$$

This implies that $s(t)$ is stable and bounded and $\int_a^b \tilde{\beta}^T\tilde{\beta}d\tau$ is bounded. We will now show that $\tilde{\rho}(t)$ is bounded. Taking absolute value of (13) and using the Schwarz inequality and (9) in Assumption 2, we obtain

$$|\tilde{\rho}| = \left|\int_a^b \Psi^T\tilde{\beta}\,d\tau\right| \le \left[\int_a^b \Psi^T\Psi\,d\tau\right]^{\frac{1}{2}}\left[\int_a^b \tilde{\beta}^T\tilde{\beta}\,d\tau\right]^{\frac{1}{2}} < \infty.$$

Since $\tilde{\rho}(t)$ and $s(t)$ are bounded, $\dot{s}(t)$ is also bounded in (6). To complete the proof of asymptotic stability, if we define $W(t) = s^T K_S s$ then, $W(t) \le -\dot{V}(t,s,\tilde{\beta})$.

By Barbalat's lemma, we can prove $\lim_{t\rightarrow\infty} W(t) = 0$. Consequently, since K_S is a positive-definite matrix, $s(t)$ is uniformly continuous and $s(t)\rightarrow 0$ as $t\rightarrow\infty$. Therefore, $s(t)=0$ is asymptotically stable.

Remark 4: As shown in (10), the proposed adaptation scheme uses only the sliding surface function and the designer-defined kernel function. It does not require any information on robot dynamics.

4. Illustrative Example

We consider a two-link robot manipulator model as shown in Fig. 1.

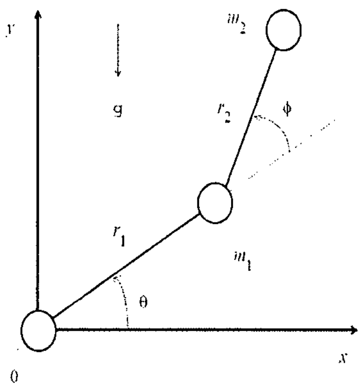

Fig. 1. Two link robot manipulator model

$$\begin{bmatrix} M_{11}(\phi) & M_{12}(\phi) \\ M_{12}(\phi) & M_{22}(\phi) \end{bmatrix}\begin{bmatrix} \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} C_{12}\dot{\phi} & C_{12}\dot{\theta} + C_{12}\dot{\phi} \\ -C_{12}\dot{\theta} & 0 \end{bmatrix} + \begin{bmatrix} G_1(\theta,\phi)g \\ G_2(\theta,\phi)g \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where

$$M_{11}(\phi) = (m_1 + m_2)r_1^2 + m_2 r_2^2 + 2m_2 r_1 r_2 \cos\phi$$
$$M_{12}(\phi) = m_2 r_2^2 + m_2 r_1 r_2 \cos\phi,\ M_{22}(\phi) = m_2 r_2^2$$
$$C_{12}(\phi) = -m_2 r_1 r_2 \sin\phi$$
$$G_1(\theta,\phi) = (m_1 + m_2) r_1 \cos\phi + m_2 r_2 \cos(\theta+\phi)$$
$$G_2(\theta,\phi) = m_2 r_2 \cos(\theta+\phi).$$

Parameter values are $r_1 = 1.0m$, $r_2 = 0.8m$, $m_1 = 0.5kg$, and $m_2 = 0.5kg$. Let $q \equiv [\theta,\phi]^T$. The desired trajectory chosen to describe the transition from a given initial value $q_d(0)$ to a desired final value $q_d(3)$ in the time interval 3 sec, with $\dot{q}_d(0) = \dot{q}_d(3) = 0$ is a simple function given by

$$q_d(t) = q_d(0) + \frac{q_d(3) - q_0(0)}{2\pi}\left(\frac{2\pi}{3}t - \sin\frac{2\pi}{3}t\right).$$

In order that the sliding surfaces are decoupled, we choose the sliding surface as

$$s_1 = \lambda_1(\dot{\theta} - \dot{\theta}_d) + (\theta - \theta_d),\ s_2 = \lambda_2(\dot{\phi} - \dot{\phi}_d) + (\phi - \phi_d)$$

where $\lambda_1 = \lambda_2 = 7$. In the simulation, the control parameters are chosen as

$$K_s = diag[10\ \ 5],$$
$$\hat{M} = \begin{bmatrix} m_1 r_1^2 & 0 \\ 0 & m_2 r_2^2 \end{bmatrix},\ \hat{C} = 0,\ \hat{G} = 0$$
$$\Psi(t,\tau) = [1\ f(t-\tau)],\ f(t) = \frac{1}{0.04\sqrt{2\pi}} e^{-0.5(t/0.04)^2}$$
$$Q = diag[20\ \ 20].$$

And the integral interval is $[a\ b] = [-0.5\ \ 0.5]$. We take the initial and final positions to be

$$\theta_d(0) = \theta(0) = -1.57,\ \phi_d(0) = \phi(0) = 0$$
$$\theta_d(3) = \phi_d(3) = 1.$$

Fig. 2 shows the control inputs and the tracking errors. From this figure, the proposed controller effectively controls the robot manipulators under the nonlinearities and uncertainties.

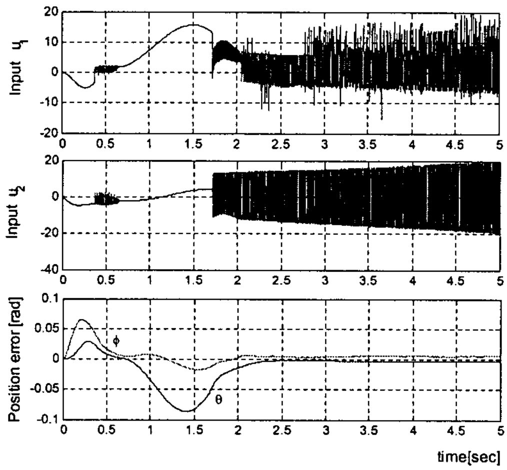

Fig. 2. Control inputs and tracking errors

5. Conclusion

In this paper, a sliding mode control scheme for the trajectory tracking problem of the robot manipulator is presented using two important properties in the robot dynamics, i.e, skew-symmetry and positive-definiteness of robot parameters. To describe the bound of uncertain robot parameters, we assumed that it is formulated as an integral of the product of a predefined kernel with an unknown influence function. Based-on the formula, a simple adaptation law is proposed for estimating the bound. Thus, the main feature of the proposed sliding mode is that the derivation of the control law does not require exact knowledge of the nonlinear robot system. The simulation result shows that the proposed method effectively controls the robot manipulator.

This work was supported by a research grant from Academic Research Foundation of Hankyong National University for a scholarly exchange program in 2004.

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