Optimal Design of Nonlinear Squeeze Film Damper Using Hybrid Global Optimization Technique

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The optimal design of the squeeze film damper (SFD) for rotor system has been studied in previous researches. However, these researches have not been considering jumping or nonlinear phenomena of a rotor system with SFD. This paper represents an optimization technique for linear and nonlinear response of a simple rotor system with SFDs by using a hybrid GA- SA algorithm which combined enhanced genetic algorithm (GA) with simulated annealing algorithm (SA). The damper design parameters are the radius, length and radial clearance of the damper. The objective function is to minimize the transmitted load between SFD and foundation at the operating and critical speeds of the rotor system with SFD which has linear and nonlinear unbalance responses. The numerical results show that the transmitted load of the SFD is greatly reduced in linear and nonlinear responses for the rotor system.

Key Words : Squeeze Film Damper, Nonlinear Response, Optimal Design, Simulated Annealing Algorithm, Genetic Algorithm

1. Introduction

The squeeze film dampers (SFDs) are well known as dampers of aero-engine rotors to provide additional damping to rolling element bear-

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ings which themselves have little or no damping. SFDs are essential components of high-speed turbomachinery since they offer the unique advantages of dissipation of vibration energy and isolation of structural components, as well as the capability to improve the dynamic stability characteristics of inherently unstable rotor-bearing systems.

In the design of modern aircraft engines, there is an increasing requirement for high-speed, lightweight and high performance. The required rotors exhibit a trend towards increased flexibility lead-

ing to a high sensitivity to imbalance with large vibration levels and reduced reliability. This means that the performance of SFD Should be improved and be optimized.

When designing the SFD, its shape and oil viscosity used in the SFD are determined to obtain the optimum support damping at the operational speed which designers want to minimize vibration amplitude. Several researchers pointed out that the optimum support damping introduced by the SFD depends on whirling modes of the rotor (Thomsen and Andersen, 1974; Cunningham et al., 1975; Barrett et al., 1978; Satio and Kobayashi, 1982; Ahn et al., 1998). Designers expect a rotor system with SFD to have just linear response, but the rotor system have practically nonlinear response with large vibration ampitude on some condition unexpected in design stage (Mohan and Hahn, 1974; Gunter et al., 1977; Taylor and Kumar, 1980; Holmes and Dogan, 1982; Li and Taylor, 1987; Fen and Hahn, 1989; Zeidan and Vance, 1990; Jung et al., 1992; Yakoub and El-Shafei, 2001). Therefore, some researches have investigated in the bifurcation of the nonlinear response of the unbalanced rotor (Zhao et al., 1994; Zhang et al., 1998; Zhu et al., 2002; Inayat-Hussain et al., 2003).

To reduce vibration amplitude of a rotor system, optimal design of the SFD for rotor system has been studied in previous researches (Rabinowitz and Hahn, 1983; Chen et al., 1988; Nataraj and Ashrafiuon, 1993; Lin et al., 1998; El-Shafei, 2002; Ahn et al., 2003). These works have contributed a lot to the understanding of the dynamic behavior of SFD and provided very useful tools for engineering design practice. However, these researches reported in the literature have been related to just linear response but not to jumping or nonlinear phenomenon of a rotor system with SFD. When the rotor system has nonlinear response, vibration amplitude of the rotor becomes very large. Therefore, to completely stabilize the rotor system, designer should consider the linear and nonlinear response from practical problems of the unbalanced rotor system in the design stage of the SFD.

The objective of the optimum design problem is to find the optimum (maximum or minimum)

value of the objective function in a given domain and the values of the design variables of a shape and performance, which the optimum is reached in this domain for the performance and efficiency of system. Today there exist very efficiently working local optimization methods for parameter optimization called the gradient-based methods (for examples, Nelder and Mead, 1965; Box, 1965; Powell, 1978; Rao, 1996). However, these conventional algorithms need the gradient information of the objective function to design variables. All the more, they may lose a global optimum solution because they are dependent on the starting point of searching and converge on the optimum solution, which is the nearest solution to the starting point, and cannot find all global optimum solutions. In order to overcome these disadvantages, many search algorithms have been developed for global optimization such as genetic algorithm (GA) (Goldberg, 1989) and simulated annealing (SA) (Kirpatrick et al., 1983). They do not involve any gradient information and mathematical formulation but only forward analysis procedure. Unfortunately global optimization algorithms are highly time consuming, because they are based on the iterative strategy which updates unknown parameters gradually. Therefore, a fast and more efficient search algorithm has been strongly required for optimization in spite of rapid progress of computer technology.

An optimization problem is generally said to be difficult if it satisfies some or all of the following criteria: high dimension, many local minima, highly non-linear, non-smooth, noisy and discrete. Traditional gradient-based optimization techniques become stuck in local minima and fail to converge on a global minimum. In certain cases, particularly when facing complex optimization problems with numerous local optima, where traditional optimization methods fail to provide efficiently reliable results, GA can constitute an interesting alternative. Nevertheless, GA can suffer from excessively slow convergence before providing an accurate solution because of its fundamental requirement of using minimal a priori knowledge and not exploiting local information. Since genetic algorithms had been intro-

duced in engineering applications, many modified versions of GA have been reported to reduce the searching time and to raise the global search ability. Many researchers had proposed modified versions of GA in order to set GA operator works adaptively. A local search or meta-heuristic algorithm has been incorporated into GA to make it a more powerful algorithm (Renders and Flasse, 1996 ; Lee et al., 2001 ; Hsiao, et al., 2001 ; Hageman, et al., 2003). GA-SA algorithm has been also a popular combination for improving the efficiency of the global search (Roach and Nagi, 1996 ; Yu et al., 2000 ; Ong et al., 2002). Recently, enhanced genetic algorithm was proposed (Kim, 2003), which can speed up the computation time considerably and find the global and local optimum solutions at the same time.

This paper considers a hybrid optimization algorithm, named HGASA, which is a combined genetic algorithm with simulated annealing algorithm and local search method. This algorithm is constructed in three steps : the first step is initial global search by the genetic algorithm, which allows fast convergence and weak dependence on initial parameters ; next step, the global search performance is much improved by introducing the SA, which is entirely different global search from GA; and finally, a local search algorithm is adapted to improve the accuracy of final results.

In this paper, the optimization methodology of SFD considering linear and jumping or nonlinear phenomenon of a simple rotor system with the SFD is studied by using HGASA. The aims of this study are to minimize the transmitted load between SFD and foundation at the operating and critical speeds of the rotor system which has nonlinear unbalance responses. The numerical results show that transmitted load of the SFD greatly reduced in linear and nonlinear responses for the rotor system.

2. Motion Equation of Squeeze Film Damper

Figure 1 shows a typical SFD which consists of a non-rotating inner damper and a stationary ball bearing supporting shaft. The inner damper is
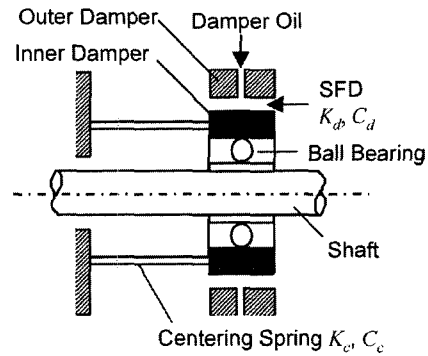

Fig. 1 Typical configuration of a squeeze film damper

mounted on the outer race of a ball bearing and prevented from spinning by employing a soft centering spring which can be preloaded to offset gravity or other constant side loads. As a result, the motion of the shaft center is nearly circular, and this centered circular orbit response is synchronous with the unbalance excitation at shaft speed. The SFD provides rotor damping at the bearing location as a result of pure squeezing motion between the inner and outer journals.

When we used the short bearing approximation to model the SFD, assumed cavitation in the oil film (π film) and assumed circular synchronous motion for the rotor, the equivalent stiffness K_d and damping coefficients C_d of SFD were represented by (Chen et al., 1988)

$$K_d = \frac{2\mu R\Omega\varepsilon}{(1-\varepsilon^2)^2}\left(\frac{L}{C}\right)^3,$$
$$C_d = \frac{\pi\mu R}{2(1-\varepsilon^2)^{3/2}}\left(\frac{L}{C}\right)^3 \tag{1}$$

where R, L and C are radius, length and radial clearance of the SFD, respectively. μ is viscosity of lubricant, $\varepsilon(=e/C)$ is the damper eccentricity ratio and e is the eccentricity of the inner damper journal. Ω is equal to the synchronous whirl velocity of shaft. Characteristics of K_d and C_d are depended on damper eccentricity ratio ε. Furthermore, the dimensionless stiffness and damping coefficients of the SFD obtained from Eq. (1) is as follows,

$$K_d^* = \frac{K_d}{\mu R\Omega}\left(\frac{L}{C}\right)^3, \quad C_d^* = \frac{C_d}{\mu R}\left(\frac{L}{C}\right)^3 \tag{2}$$

When the unbalanced mass eccentricity of rotor is increased, the damper eccentricity ratio ε is increased. Furthermore, stiffness and damping coefficients of the SFD are increased and their nonlinear properties as shown in Fig. 2 also increased and the lines of the two coefficients cross each other. When the eccentricity of rotor increased, the nonlinear responses of jumping phenomena will occur as shown in Fig. 3 and the rotor system with the nonlinear response will have multi-solutions. HGASA is very convenient for optimization of a system with multi-solutions.

A simple rotor system shown in Fig. 4 is used in the study of optimization to minimize the linear and nonlinear responses of the rotor system with SFD and its geometry is shown in Fig. 5. In Figs. 4 and 5, ω is the shaft rotational speed, O is static equilibrium position, E is axis of the shaft at disk and G is center of gravity (c.g.) of disk. This rotor system consists of a massless elastic shaft on which single disk mass M is mounted at midspan. The rotor is assumed to operate in two end SFDs with K_c and C_c which are stiffness and

Fig. 2 Characteristics of stiffness and damping coefficients according to the variation of eccentricity ratio ε

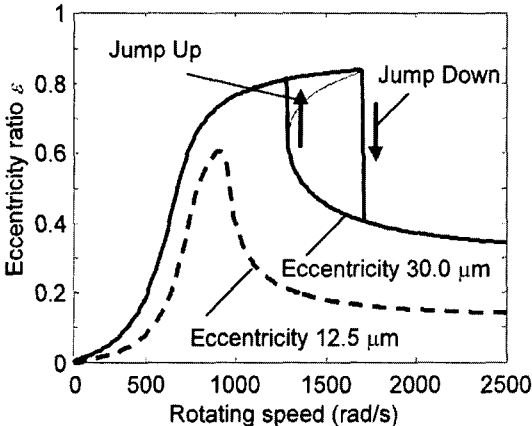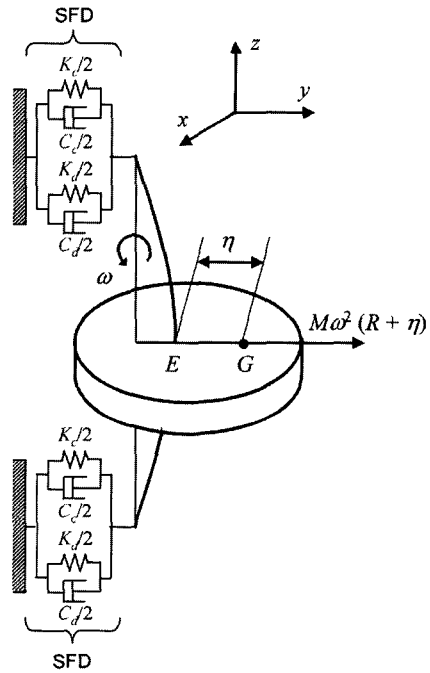**Fig. 4** Whirling of a flexible rotor system with squeeze film damper

Fig. 3 Unbalanced response of a rotor system with SFD and single mass
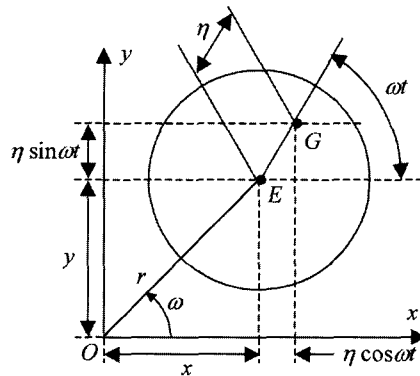

Fig. 5 Geometry of unbalance whirl at the disk

damping coefficients of the centering spring shown in Fig. 1. The rotor mass is concentrated at rotor c.g., at a distance η from the point where a disk attaches to the shaft. The motion equation of Fig. 4 is found from the free-body diagram to be

$$M\ddot{x}+D\dot{x}+Kx=M\omega^2\eta\cos\omega t$$
$$M\ddot{y}+D\dot{y}+Ky=M\omega^2\eta\sin\omega t \tag{3}$$

where $K=K_c+K_d$ and $D=C_d$ because value of C_c is assumed $C_c=0$. In these expressions x and y are the coordinates of the shaft center. Writing the whirl radius as $z=x+iy$ where $i=\sqrt{-1}$, these equations may be combined into the single expression as follow:

$$M\ddot{z}+D\dot{z}+Kz=M\omega^2\eta e^{i\omega t} \tag{4}$$

Eq. (4) can be solved for the shaft whirl radius z by substituting:

$$z=Ze^{i\omega t} \tag{5}$$

where Z is the whirl amplitude. Substituting Eq. (5) in Eq. (4) the whirl amplitude at the disk gives:

$$Z=\left|\frac{M\omega^2\eta}{(K-M\omega^2)+i\omega D}\right|=\frac{\eta\lambda^2}{\sqrt{(1-\lambda^2)^2+(2\zeta\lambda)^2}} \tag{6}$$

where critical speed $\omega_n^2=K/D$, damping ratio $\zeta=D/(2(KM)^{1/2})$, frequency ratio $\lambda=\omega/\omega_n$. The value of these parameters depend on the shaft rotational speed ω.

The maximum value of transmitted force F_{tr} on both SFDs for an unbalanced rotor is given by:

$$M_{tr}=|D\dot{z}+Kz|=|(K+i\omega D)Z|$$
$$=\left|\frac{M\omega^2\eta(K+i\omega D)}{(K-M\omega^2)+i\omega D}\right|=M\eta\omega^2\sqrt{\frac{1+(2\zeta\lambda)^2}{(1-\lambda^2)^2+(2\zeta\lambda)^2}} \tag{7}$$

3. Hybrid Optimization Algorithm : HGASA

It has been verified that GA and SA are both probabilistic search algorithms capable of finding the global minimum amongst many local minima. Empirically, GA lacks a hill-climbing capability, and it does not work well when the objective function is a multi-modal function or a highly

coupled function (Kim, 2003). SA has a statistical hill-climbing capability and the solution state does not stay at a fixed point for a long time. So, no matter what kind the shape of the solution space of the objective function is, if GA is applied in the first step to get a relatively best solution and it is taken as the initial guess of SA, the final solution must be better than that obtained from a single GA or SA. A new hybrid algorithm which is a combination of GA and SA (HGASA) is proposed. In this algorithm, the initial global search is done by the GA because it allows fast convergence and does not depend on initial parameters. The global search performance is greatly improved by introducing the SA which is entirely a different global search method. That is to say, using hybrid optimization techniques, the probability of convergence to local minima is reduced by the complementary global search. Comparing with individual GA and SA, the HGASA has advantages in at least two aspects. Firstly, it can overcome the demerit of GA and secondly it increases the probability of finding the global optimum (Kim et al., 2003).

3.1 Procedure of HGASA
The proposed algorithm consists of three main parts as follows:

(1) GA for initial global search
(2) SA for supplement global search
(3) Simplex method for local concentration search

The main structure of HGASA is shown in Fig. 6. Traditional GA is executed until satisfying termination condition by function assurance criterion FAC to determine the convergence of GA. When the FAC equals to 1, this means all individuals of GA couldn't improve themselves anymore, GA is terminated, and the best individual is transferred to SA as the initial search point. After SA task, simplex method is carried out for the local concentration search.

3.2 Global candidate search : GA
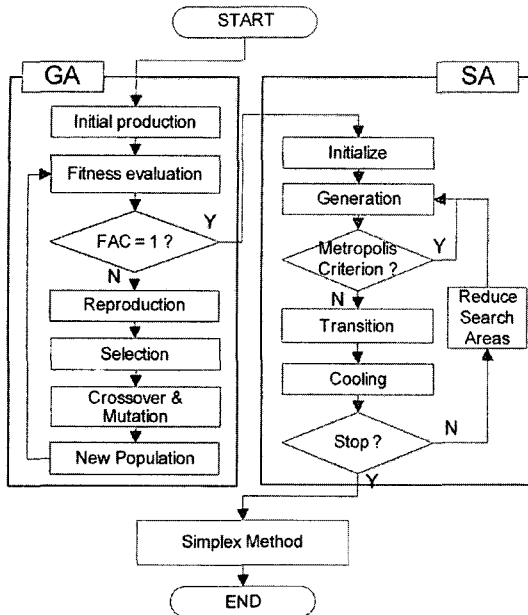Each candidate for optimum solutions is decided by the function assurance criterion (FAC)

Fig. 6   Flow chart of HGASA

(Friswell and Mottershead, 1996). The FAC defined by Eq. (8) is a standard value to estimate the convergence of the initial candidate.

$$FAC = \frac{|\mathbf{f}_{i-1}^T \mathbf{f}_i|^2}{(\mathbf{f}_{i-1}^T \mathbf{f}_{i-1})(\mathbf{f}_i^T \mathbf{f}_i)} \tag{8}$$

where $\mathbf{f}_i$ is the row vector, formed by the fitness values of the individuals at the i-th generation and $\mathbf{f}^T$ is the transpose of $\mathbf{f}$.

The row size depends on the number of optimum solutions according to a designer's requirement. The range of FAC theoretically is from 0 to 1.

3.3 Supplement global search : SA

SA was derived from an analogy with the annealing process of material physics (Kirkpatrick et al., 1983). It is well known that certain materials have multiple stable states which have differing molecular distributions and energy levels. The annealing process consists of heating the substance until it is molten, then slowly and discretely lowering the temperature. The substance is allowed to reach thermal equilibrium at each temperature. Eventually the temperature is lowered until the material freezes. If the temperature is lowered sufficiently slowly, the annealing

process can always pick out the global minimum energy state from the almost unlimited number of possible states (Kirkpatrick et al., 1983).

Generally, in order to apply the simulation of annealing to optimization problems, the following preparatory steps are needed. Firstly, the analogues of the physics concepts in the optimization problem itself should be identified. Note that the energy function corresponds to objective function and the configurations of particles represent the configurations of parameter values. To find a low-energy configuration becomes to find a near optimal solution, and the temperature represents the control parameter for the simulation. And then a cooling schedule (annealing schedule) should be selected, consisting of a set of decreasing temperatures, together with the amount of time to spend at each temperature. The last step is to supply a way of generating and selecting new solutions.

Various researchers have suggested different implementations of SA algorithm. In particular, the most thoroughly investigated SA software is the adaptive SA (ASA) code introduced at the web site (http://www.ingber.com/). ASA was developed to statistically find the best global solution for a nonlinear constrained non-convex cost-function over a D-dimensional space. This algorithm permits an annealing schedule for temperature decreasing exponentially in annealing time, which is faster than Cauchy annealing and Boltzmann annealing (Ingber and Rosen, 1992). The introduction of re-annealing also permits adaptation to changing sensitivities in the multi-dimensional parameter space.

3.4 Local concentration search : simplex method

The simplex method is a local search technique that uses the evaluation of the current set of data to determine the promising search direction. Nelder and Mead (1965) developed a modification to the basic simplex method that allows the procedure to adjust its search step according to the evaluation result of the new point generated. This is achieved through three ways. Firstly, if the reflected point is very promising (i.e., better

than the best point in the current simplex), a new point, further along the reflection direction, is generated using the equation:

$$X_e = \bar{X} + \gamma(\bar{X} - X_w) \qquad (9)$$

where γ is the expansion coefficient $(\gamma > 1)$, $\bar{X}$ and X_w are the original point and the worst point, respectively.

Secondly, if the reflected point X_e is worse than the worst point X_w in the initial simplex, a new point, close to the centroid on the same side of X_w is generated using the following equation:

$$X_c = \bar{X} - \beta(\bar{X} - X_w) \qquad (10)$$

where β is called the contraction coefficient $(0 < \beta < 1)$ because the resulted simplex is contracted.

Finally, if the reflected point X_e is not worse than X_w, but is worse than the second worst point in the original simple, a new point, close to the centroid on the opposite side of X_w is generated using the contraction coefficient β.

$$X_c = \bar{X} + \beta(\bar{X} - X_w) \qquad (11)$$

4. Optimization Procedure and Results

The rotor system with SFD shown in Fig. 4 was employed to investigate optimization of SFD by HGASA. Its system parameters used in nonlinear response analysis by Taylor and Kumar (1980) are also used in this study and given in Table 1. The operational speed range considered from 100 rad/s to 2500 rad/s. The nonlinear phenomenon of the rotor with SFD has been pointed out under high unbalance and cavitation condition of SFDs.

Table 1 Parameters of SFD system

Parameters	Values
Rotor mass, M (kg)	33.45
SFD radius, R (mm)	64.80
SFD width, L (mm)	22.7
SFD clearance, C (μm)	100
Viscosity of lubricant, μ (N·s/m^2)	2.66×10^{-3}
Stiffness of centering spring, K_c (N/m)	2.154×10^7

To obtain high-performance and high-stability of rotor system against nonlinear response of the rotor, designer should consider linear and nonlinear responses of the rotor system with SFD in the design stage of the SFD. As shown in Fig. 7, when the mass eccentricity of the rotor is relatively small $(\eta = 12.5 \ \mu m)$, the frequency response of the transmitted force is almost linear. However, when the mass eccentricity of the rotor is large $(\eta = 30.0 \ \mu m)$, the nonlinear responses of jumping phenomena occur and whirl amplitudes of rotor are changed according to increasing and decreasing the rotational speed of the rotor. It means that optimization results depend on the rotational speed of the rotor.

To investigate optimization performance of the rotor according to various mass eccentricities, case studies were conducted in this paper. First of all, disk mass eccentricity η was assumed as 30 μm to provide the nonlinear response of the rotor. The design variables are the radius R, length L and clearance C of SFD. The problems of the optimum design for the rotor system with SFD can be stated as:

Find design variable $\mathbf{X} = (L \ R \ C)^T$
Minimize

$$f(\mathbf{X}) = \alpha F_{tr}(\mathbf{X})_{max}|_{\omega_1}^{\omega_2} + \delta \left\{ \sum_{\omega=\omega_1}^{\omega_2} F_{tr}(\mathbf{X}) \right\} \qquad (12)$$

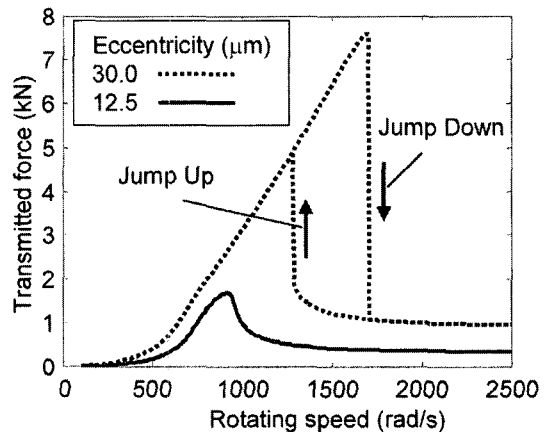Subject to side constaints of design variables,

Fig. 7 Linear and nonlinear frequency responses according to disk mass eccentricity

Table 2 Different design objectives $(\eta = 30 \ \mu m)$

Case No.	Objective function	Operation conditions $\omega_1 = 100$ rad/s, $\omega_2 = 2500$ rad/s $\omega_3 = 800$ rad/s, $\omega_4 = 2000$ rad/s
Case 1	$f(\mathbf{X}) = F_{tr}(\mathbf{X})_{max}\|_{\omega_1}^{\omega_2}$	Start-up $(\omega_1 \rightarrow \omega_2)$
Case 2	$f(\mathbf{X}) = F_{tr}(\mathbf{X})_{max}\|_{\omega_2}^{\omega_1}$	Slowdown $(\omega_2 \rightarrow \omega_1)$
Case 3	$f(\mathbf{X}) = F_{tr}(\mathbf{X})\|_{\omega_1}^{\omega_2} + F_{tr}(\mathbf{X})_{max}\|_{\omega_2}^{\omega_1}$	Start-up and Slowdown
Case 4	$f(\mathbf{X}) = \left\{ \sum\limits_{\omega=\omega_3}^{\omega_4} F_{tr}(\mathbf{X}) \right\}_{avg}$	Start-up $(\omega_3 \rightarrow \omega_4)$
Case 5	$f(\mathbf{X}) = \left\{ \sum\limits_{\omega=\omega_4}^{\omega_3} F_{tr}(\mathbf{X}) \right\}_{avg}$	Slowdown $(\omega_4 \rightarrow \omega_3)$
Case 6	$f(\mathbf{X}) = \left\{ \sum\limits_{\omega=\omega_3}^{\omega_4} F_{tr}(\mathbf{X}) + \sum\limits_{\omega=\omega_4}^{\omega_3} F_{tr}(\mathbf{X}) \right\}_{avg}$	Start-up and Slowdown
Case 7	$f(\mathbf{X}) = \alpha F_{tr}(\mathbf{X})_{max}\|_{\omega_1}^{\omega_2} + \delta \left\{ \sum\limits_{\omega=\omega_3}^{\omega_4} F_{tr}(\mathbf{X}) \right\}_{avg}$ $\alpha = 0.7, \ \delta = 0.3$	

For side constraint A :

25 mm $\leq R \leq$ 105 mm, 7.5 mm $\leq L \leq$ 52.5 mm, 25 μm $\leq C \leq$ 180 μm

For side constraint B :

50 mm $\leq R \leq$ 70.0 mm, 15 mm $\leq L \leq$ 35.0 mm, 50 μm $\leq C \leq$ 120 μm

The first term in Eq. (12) is intended to minimize the peak value of transmitted force within the speed range from ω_1 to ω_2 while the second term is intended to minimize overall transmitted force. α and δ are weight factors for the first and second term, respectively. They are selected from 0 to 1 and can be adjusted to provide the proper compromise between reduction of the maximum transmitted force and the forces in an operating frequency range. Since the whirl amplitudes of rotor by jumping phenomena are changed according to start up and solwdown of the rotational speed of the rotor, the optimization case study should be investigated according to the rotitional speed. Furthermore, the optimization should be considered according to the speed conditions of a rotor. From these optimization conditions, the case study procedures are arranged as shown in Table 2. The optimum parameters obtained from each case study are listed in Table 3. To investigate optimization results according to variation of side constraints, two side constraints are used and the side constraint A was

Table 3 Optimum parameters for nonlinear response $(\eta = 30 \ \mu m)$

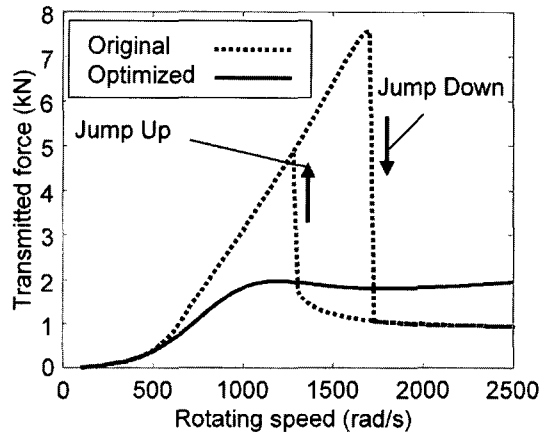|	Cases 1, 2 and 3	Cases 4 and 6	Case 5	Case 7
R	105	56.07	25	105
L	52.5	48.64	7.5	52.5
C	156.6	180	125.9	179.39

Fig. 8 Transmitted force for Cases 1, 2 and 3

used for Cases 1-7.

As shown in Fig. 8 and Table 3, optimization results obtained from Case 1 to minimize the maximum transmitted force are the same those from Cases 2 and 3. It means that optimization of SFD to minimize the maximum transmitted force do not depend on the rotational speed of the rotor.
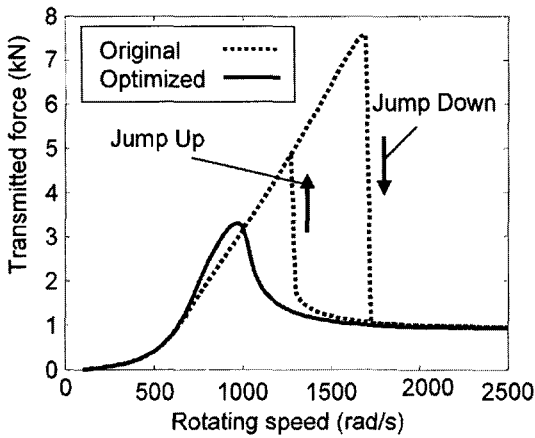
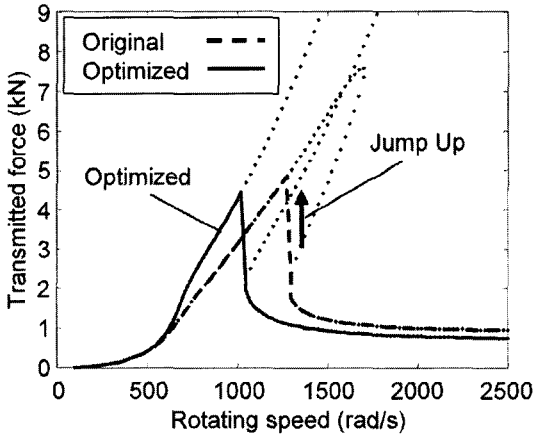**Fig. 9** Transmitted load for Cases 4 and 6

Fig. 10 Transmitted load for Case 5

Table 4 Optimum parameters for linear response ($\eta = 12.5 \ \mu m$)

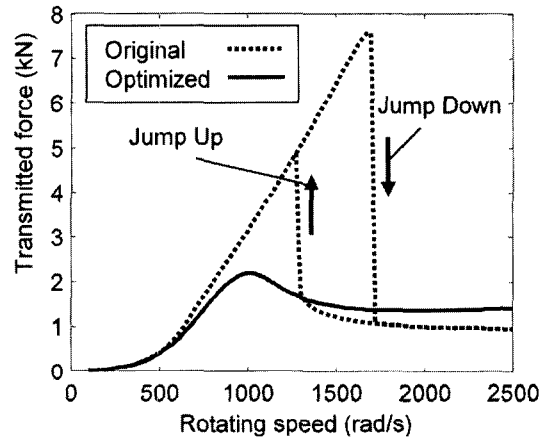|	Optimum results for Case 1	
	Case 8 (side constraint A)	Case 9 (side constraint B)
R	105	70
L	52.5	35
C	159.2	91.8

Fig. 11 Transmitted load for Case 7
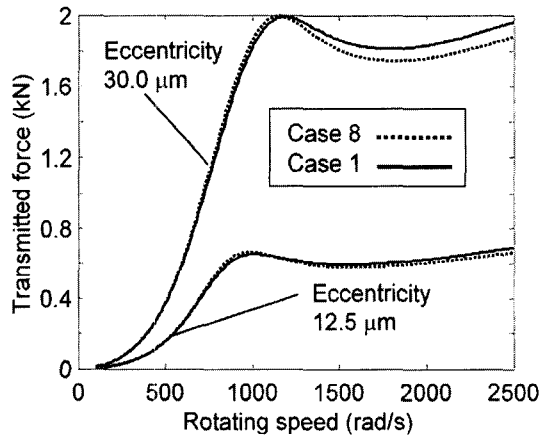
Fig. 12 Comparison transmitted load of optimized SFDs for nonlinear and linear responses

Fig. 9 shows that the optimization results of Case 4 is the same those of Case 6. Since the maximum transmitted force for Case 5 is higher than that by Cases 4 and Case 5 also has nonlinear response with large vibration amplitude, it can be said that optimization results obtained from Case 5 shown in Fig. 10 are not better than those from Case 4 they depend on the rotational speed. Figure 11 obtained from the Case 7 shows that the maximum force and the forces in the operating frequency range greatly decreased than that of the original design.

The optimum parameters listed in Table 4 are obtained from Case 1 with disk mass eccentricity $\eta = 12.5 \ \mu m$ and the side constraints A and B for linear response of the rotor system.

To compare with unbalanced responses of the rotor optimized for linear ($\eta = 12.5 \ \mu m$) and nonlinear ($\eta = 30.0 \ \mu m$) conditions, the response obtained by using design parameters of Case 1 listed in Table 3 and the parameters listed of the side constraint A in Table 4 are shown in Fig. 12.

Even though these results should be investigated for multi-degree of freedom rotor system with SFDs, responses obtained from the simple rotor system with SFDs shown in Fig. 4 have almost the same for the case studies.

Table 4, Figs. 13 and 14 represent that unbalanced response of the rotor system optimized by the optimization algorithm depend on side constraints of design variables. Therefore, a designer should be careful on the limitation of the side constraint in the optimization process to obtain simply global optimization results. Figure 15 shows relationship between linear and nonlinear responses obtained from optimized parameters
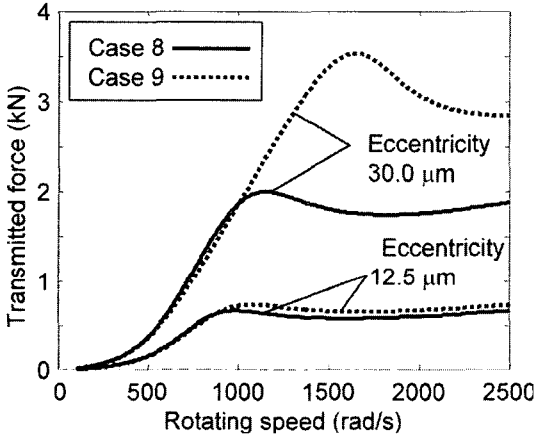

Fig. 13   Comparison transmitted load of optimized SFDs between side constraints A and B

Fig. 14   Comparison eccentricity ratios of optimized SFDs between side constraints A and B

listed in Tables 3 and 4. In Fig. 15, left side of each radius line represents linear boundary and right side of that does nonlinear boundary. Figure 15(a) shows that the rotor system with SFD for the original design variables and $\eta = 12.5$ μm has linear response while it has nonlinear response for $\eta = 30.0$ μm. However, the rotor systems using SFD with the optimized desgn variable by the case studies except Case 5 have linear responses. Even though, the tendency is different under about clearance of 110 μm as shown Fig. 15(a). When the SFD width L is increased, the boundary of linear response is increased. Furthermore, when the clearance is decreased, the boundary of linear response is increased.

Damping ratio of the rotor system with SFD is very useful parameter to evaluate stability of the system with linear response. That is, for linear SFD, it is generally known that higher damping ratio represents lower whirl amplitude. Furthermore, less damping results in larger transmitted forces for $\lambda < \sqrt{2}$, but less damping results in smaller transmitted forces for $\lambda > \sqrt{2}$. Therefore, responses of the rotor system with nonlinear SFD were investigated by using the parameters optimized by case studies. Figure 16(a) shows the transmitted force with $\eta = 30$ μm according to the rotating speed and whirl amplitude with $\eta = 30$ μm and damping of the system is shown in Fig. 16(b) and (c). Table 5 shows damping ratio without eccentricity obtained from the parameters optimized by case studies. When assumed eccentricity ratio ε is zero, the term of the equivalent stiffness K_d is disappeared and the damping ratio of

Table 5   Damping ratio of linearized SFD ($\varepsilon = 0$)

Parameters obtained from case studies and Table 1	Damping ratio $\left(\dfrac{\pi \mu R (L/C)^3}{4\sqrt{K_c M}} \right)$
Cases 1, 2 and 3	0.3080
Case 9	0.3020
Case 8	0.2932
Case 7	0.2049
Cases 4 and 6	0.0861
Table 1	0.0590
Case 5	0.0004

SFD does not depend on the rotational speed. The order of damping ratio magnitude is different between Table 5 and Fig. 16(c). Figure 16 shows that the common theory for the rotor system with linear SFD is not valid in the rotor system with nonlinear SFD. Namely, higher damping ratio does not always represent lower whirl amplitude, less damping does not always results in larger transmitted forces for $\lambda > \sqrt{2}$ and less damping also does not always results in smaller transmitted forces for $\lambda > \sqrt{2}$. It means that the stability of the rotor system with SFD can not be evaluated by damping ratio. Furthermore, a optimization procedure is necessary to obtain stability of the system in the design stage of the SFD.

5. Conclusions

Requirements of modern aircraft engines for high-speed, lightweight and high performance have been increased. A flexible rotor system with SFD installed in the enginos to isolate its vibration has practically nonlinear response with large vibration amplitude by some conditions unexpected in a design stage. Therefore, the nonlinear response of an unbalanced rotor system should be considered in the design stage. In this study, the optimization procedures for the linear and nonlinear responses of a simple rotor system with SFDs was investigated through case studies. The obtained
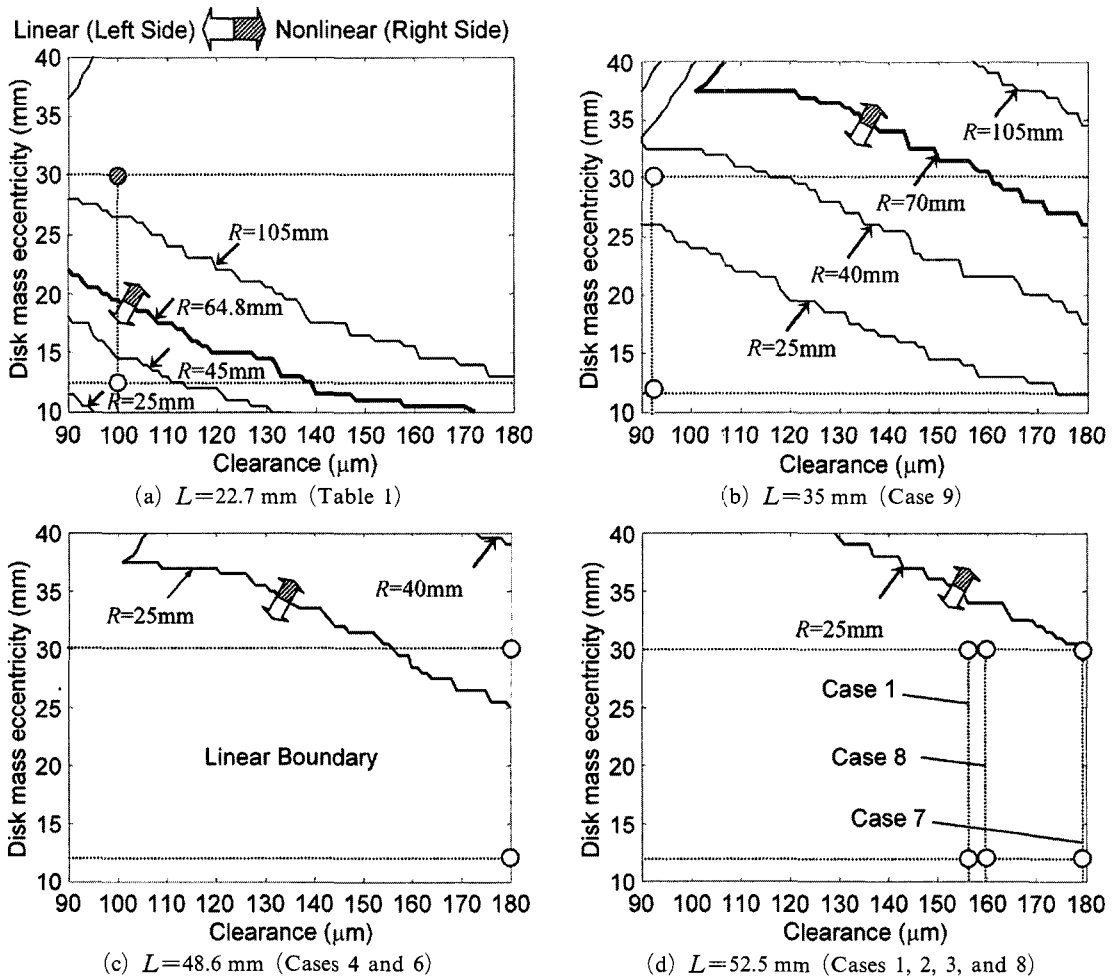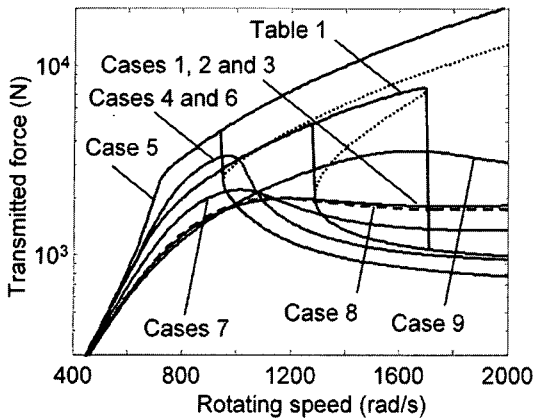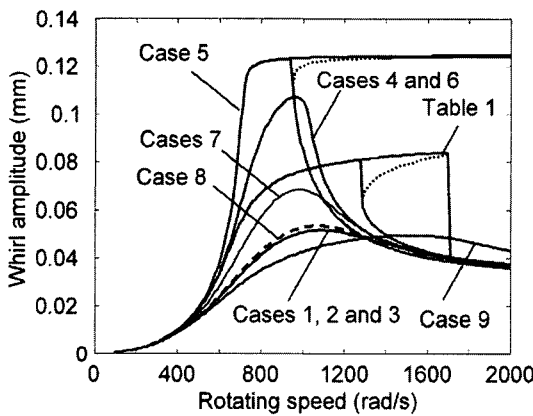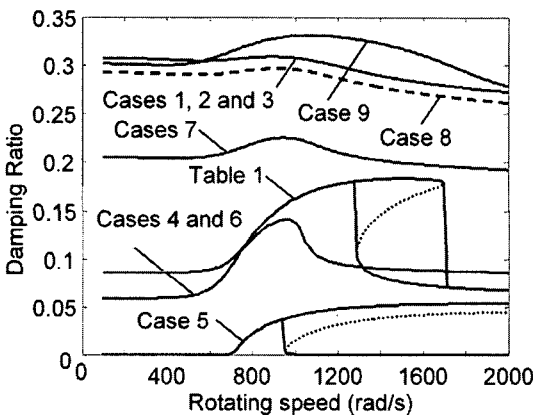

Fig. 15 Relation between linear and nonlinear responses for design parameters of SFD

(a) Transmitted forces

(b) Whirl amplitude

(b) Damping ratio

Fig. 16 Responses of the rotor system with the optimized parameters of SFD

results point out that designers should be careful to the boundary of side constraints of design parameters when they plan to optimize linear and

nonlinear responses of the rotor system, and the proposed optimization techniques for nonlinear response of the rotor system will be useful in the design stage of aero-engine rotor system as fundamental knowledge. Furthermore, the stability of the rotor system with SFD can not be evaluated by damping ratio.

Acknowledgments

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