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Modeling and Analysis of Warranty Cost for 2D-Policies Associated with Sale of Second-hand Products

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Abstract. The market for second hand products has been growing for a variety of reasons (e.g., new products appearing at a faster rate and the expected life of products increasing due to rapid advances in technology). The demand for warranties for second-hand products has been growing along with the growth of the market for second-hand products. Warranty for new products (consumer durables, industrial and commercial, and specialized defense products) has received a lot of attention. In contrast, warranties for second-hand product have received very little attention. Often, dealers of second-hand product such as cars offer 2D-warranties (Year and Kilometers). The expected warranty cost associated with a second-hand product for 2D-Policies is a function of the age of the item and its usage (as it affects failures over the warranty period), the warranty terms and the servicing strategy used by the dealer. This paper deals with development of models for warranty cost analysis along with the decision on sale price and warranty cover for 2D-Warranty policies associated with sale of second-hand products.

Key Words : *warranty cost, 2D-policies.*

1. INTRODUCTION

A warranty is a contractual obligation incurred by a manufacturer (vendor /seller) in connection with the sale of a product. The purpose of warranty is to establish liability in the event of a premature failure of an item or the inability of the item to perform its intended function. The contract specifies the promised product performance and, when it

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is not met then, the redress available to the buyer as compensation. Consumers have started demanding protection against poor quality second-hand products and the high cost of product failures occurring shortly after purchase. In response to this consumer demand for protection, public policy makers have begun enacting laws requiring dealers to offer warranties and effectively service warranty claims. This is now apparent throughout the market place for cars, consumer durables, electrical & electronic goods.

Researchers from various disciplines have carried out studies of several warranty policies along with different aspects of warranties. A taxonomy for warranty policies for new products can be found in Blischke and Murthy (1992). Iskandar, B. P. (1993) gives modeling and analysis of two-dimensional warranty policies. Glickman and Berger (1976) developed model for optimal price and warranty protection. Offering warranty results in additional cost to the manufacturer due to the servicing of claims under warranty. Review of the literature on studies of warranty aspects can be found in Blischke and Murthy (1996). A review of mathematical models can be found in Murthy and Blischke (1992). Further details of these models and analysis of warranty costs associated with one and two-dimensional policies can be found in Blischke and Murthy (1994). Cost analysis of two-attribute warranty policies based on the product usage rate can be found in Chun and Tang (1999). In contrast, warranties for second-hand products have received very little attention. Chattopadhyay and Murthy (1996) is the earliest paper to develop a model for the cost analysis of a warranty policy for second-hand products. Chattopadhyay (1999) proposes A warranty is a contractual obligation between manufacturer (or dealer) and buyer (or consumer) with the sale of a product defining redress action against product failure or unsatisfactory product performance.

From the dealer's point of view, offering a warranty not only provides assurance against product performance but also can be used as an important marketing tool to increase the probability of selling an item. However, this is achieved at an additional cost -- the cost of servicing claims over the warranty period. The dealer needs to be able to estimate this cost so as to compare different warranty options in deciding on a warranty policy. The warranty terms must be selected in a manner that takes into account the complex interactions between different variables. Better warranty terms have a positive impact (from promotional and protectional points of view) whereas the increased cost has a negative impact on the likelihood of the sale of a second-hand item. The cost analysis is more complex than that for new products as each second-hand item is unique in terms of its age and condition. Another complication is that dealers often have only partial knowledge about the condition of an item, that is, it's past usage and maintenance history.

Dealers of second-hand products need to do a proper cost analysis of alternative warranty policies and warranty terms to be offered with the sale of a particular item. Often the dealer of second-hand products such as cars offers 2D-warranties (Year and Kilometers). This paper deals with modeling warranty cost for two-dimensional FRW policy along with the decision on sale price and warranty cover associated with sale of second-hand products.

The outline of the paper is as follows. In Section 2 we develop models for the expected warranty cost and dealers profit for selling second-hand products with a 2-Dimensional FRW policy. In Section 3 we carry out the analysis with a numerical

example for illustration. Finally, we conclude with a brief discussion on topics for future research.

2. MODELING WARRANTY COST AND DECISION ON SALE PRICE AND WARRANTY COVER

In this section we develop model for 2-D FRW policy.

Two-Dimensional FRW policy: In this policy the manufacturer/ dealer agrees to repair or provide a replacement for failed items (age A and usage M at sale) free of charge up to a time W or up to a usage (M+U) which ever occurs first from the time of the second-had purchase. W is called the warranty period and U is called the usage limit. The warranty region is a rectangle as shown in Figure 1.

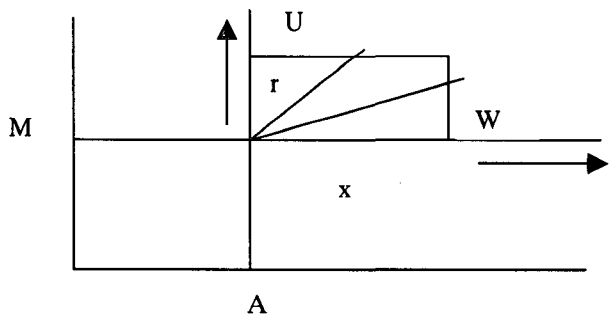

Figure 1. 2-D Warranty region for a second-hand product of age A and usage M

Assumptions

- Products are repairable.
- Item failures are statistically independent.
- Item failure, in a probabilistic sense, is only a function of its age and usage.
- Dealer uses minimal repair by which the failed components are repaired or replaced and therefore the failure rate of the product does not change after rectifications [Barlow and Hunter (1960)].
- The time to carry out a rectification action by repair or replacement is negligible compared to the mean time between failures and this time can be ignored.
- An item failure results in an immediate claim and all claims are valid.
- Warranty expires when the total usage reaches to U or period covered W whichever occurs first.
- The age and usage of second hand items are known.

Notation

A:	age of second hand product
M:	usage of second hand product
W:	calendar years for warranty of second hand product
U:	usage limit for warranty of second hand product
R:	usage per unit time (usage rate), a random variable
$G(r)$:	distribution function of R
P:	price of second hand product
k_1, k_2, a, b, c :	parameters of function of total sales
c_r :	repair cost
x:	time in the warranty period
$h(x/r)$:	failure rate at time x condition of usage rate = r

Model

We use one-dimensional approach to analyze the two dimensional warranty. First, conditional on $R = r$, let $h(x/r)$ denote failure rate of working unit at time x. Then, the failure process under minimal repair follows a Non-homogeneous Poisson Process and the number of failure of a second hand product for given R in the warranty region follows Poisson distribution. The mean value is given by

$$\int_A^{A+WorA+U/r} h(x/r)dx$$

if $r < U/W$, the upper limit is $A+W$, otherwise it is $A + U/r$. Therefore, the expected number of failures over the warranty region for second hand product is given by

$$E[N(A,M,W,U)] = \int_0^{U/W}\int_A^{A+W} h(x/r)dxdG(r) + \int_{U/W}^{\infty}\int_A^{A+U/r} h(x/r)dxdG(r), \quad (1)$$

and the expected warranty cost is

$$E[C(A,M,W,U)] = c_r\int_0^{U/W}\int_A^{A+W} h(x/r)dxdG(r) + \int_{U/W}^{\infty}\int_A^{A+U/r} h(x/r)dxdG(r). \quad (2)$$

When the model in existing studies are applied and extended, we can assume the total sales as a function of price and warranty region,

$$S(p,A,M,W,U) = k_1 p^{-a}(k_2 + W + cU)^b \quad (3)$$

where $k_1 > 0$, $k_2 \geq 0$, $a > 0$, $c > 0$ and $0 < b < 1$. Here k_1 is an amplitude factor, and k_2 is a constant that allows for demand without warranty. Let $c(A, M)$ be the total cost of second hand product with age A and usage M. Then expected profit per unit sale is

$$\pi(A, M, p, W, U) = p - c(A, M) - c_r \ \mathrm{E}[N(A, M, W, U)] \tag{4}$$

and the total expected profit is

$$\Pi(p, A, M, W, U) = k_1 p^{-a}(k_2 + W + cU)^b \ \{p - c(A, M) - c_r \{\int_0^{U/W}\int_A^{A+W} h(x/r)dxdG(r) + \int_{U/W}^{\infty}\int_A^{A+U/r} h(x/r)dxdG(r)\}\}. \tag{5}$$

We use this total expected profit as a deciding criterion. The price and warranty regions are decision variables and the price p and warranty region, W and U for the best total expected profit are obtained.

3. NUMERICAL EXAMPLE

We consider trading of second-hand cars such as Toyota Corolla, which is popular in Australia and use proper estimators about model parameters: A= 2 years and M = 5(× 10,000 miles), $c(2,5)$= 10(10,000$), c_r =0.5($500). And failure rate is a linear function of usage and age, then

$$h(t/r) = \theta_0 + \theta_1 r + (\theta_2 + \theta_3 r)x\,.$$

Let $\theta_0 = 0.03, \theta_1 = 0.07, \theta_2 = \theta_3 = 0.03, k_1 = 10{,}000, k_2 = 2\ years, c = 0.5, b = 0.5.$ R is uniformly distributed over [0,4] and this implies that the mean usage per year is 20,000 miles. Then the total profit is given by

$$\Pi(p, 2, 5, W, U) = 10000p^{-2}(2 + W + 0.5U)^{0.5} \ \{p - 10 - 0.5(1/4) \times \{\int_0^{U/W}\int_2^{2+W} \{0.03 + 0.07r + (0.03 + 0.03r)x\}dxdr + \int_{U/W}^{\infty}\int_2^{2+U/r} \{0.03 + 0.07r + (0.03 + 0.03r)x\}dxdr\}\}.$$

From the above expression the expected warranty cost for various combinations of U and W and total profits for various price warranty combinations are given in Table 1.

Table 1. Expected warranty cost and total profit for 2D FRW policy

Price (× $1,000)	(*W*, *U*) (year, × 1,000km)	Number of Sale	Expected Warranty cost/car	Total profit (× $1,000)
12	1, 2	147	0.45	228.0
	2, 4	184	0.92	198.3
	3, 6	214	1.41	127.3
13	1, 2	126	0.45	319.0
	2, 4	157	0.92	325.5
	3, 6	182	1.41	290.8
14	1, 2	108	0.45	384.0
	2, 4	134	0.92	415.6
	3, 6	157	1.41	408.0

From Table 1, we see that for this second-hand car, the best decision is to sell at a price of $14,000, with a warranty cover of 2 years and 40,000 km for maximum total profit.

4. CONCLUSION

In this paper we have developed models for estimation of warranty cost and best decision on price and warranty terms from dealer's profit point of view for 2-D FRW policy associated with sale of second-hand products. There is scope for further research in this area by considering two-dimensional approach, developing models for other complex 2-D policies and combining upgrade action of second-hand products by dealers before sale

REFERENCES

Barlow, R. E. and Hunter, L. (1960). Optimum preventive maintenance policies. *Operations Research*, **8**, 90-100.

Blischke, W.R. and Murthy, D.N.P. (1992). Product warranty management-I: A taxonomy for warranty policies. *European Journal of Operations Research*, **62**, 127-148.

Blischke, W.R. and Murthy, D.N.P. (1994). *Warranty cost analysis*, Marcel Decker, New York.

Blischke, W.R. and Murthy, D.N.P. (1996). *Product warranty handbook.* Marcel Decker, New York.

Chattopadhyay, G.N. (1999). *Modeling and Analysis of Warranty Costs for Second-hand Products.* Unpublished doctoral thesis, The University of Queensland, Brisbane.

Chattopadhyay, G.N. and Murthy, D.N.P. (1996). Modeling warranties for second-hand products. *Proceedings of second Australia Japan workshop on stochastic models in engineering*, technology and management, Gold Coast, Australia, July 1996, 91-99.

Chun, Y. H. and Tang, K (1999). Cost analysis of two-attribute warranty policies based on the product usage rate. *IEEE transactions on Engineering Management*, **46. 2**, 201-209.

Glickman, J.S and Berger, P. D. (1976). Optimal price and protection period for a product warranty. *Management Science*, **22**, 1381-1396.

Iskandar, B. P. (1993). *Modeling and analysis of two-dimensional warranty policies*. Ph.D Dissertation, The University of Queensland, Australia.

Murthy, D.N.P. and Blischke, W.R. (1992). Product warranty management-III: A review of mathematical models. *European Journal of Operations Research*, **63**, 1-34.

Murthy, D.N.P. and Chattopadhyay, G.N. (1999). Warranties for second-hand products. *Proceedings of the Ninth International Conference of Flexible Automation and Intelligent Manufacturing (FAIM)*, Tilburg, Netherlands, June 1999, 1145-1159.