Further Results Involving the NBU_{mgf} Class of Life Distributions

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Abstract. A new class of life distributions is studied. This class is defined based on comparing the residual life time to the whole life in the moment generating function order giving "the new better than used in the moment generating function order ageing class (NBU_{mgf})". Fundamental properties of this class are given including some closure properties and characterizations. Finally, we consider new results about comparisons of age and block replacement policies when the underlying distribution belongs to NBU_{mgf} aging classes.

Key Words : *Life distributions, NBU_{mgf} ageing class, characterization of life distributions, block replacement policies.*

1. INTRODUCTION AND MOTIVATION

Because the accurate distribution of the life of an element or a system is often unavailable in practical situation, nonparametric ageing properties have been found to be quite useful in modeling ageing or wear-out process and to conduct maintenance policy in reliability. In particular, new better than used aging classes are commonly used in reliability theory to model situations in which the lifetime of a new unit is better than the lifetime of a used one. Such classes are defined by stochastic comparisons of the residual live of a used unit with the lifetime of a new one. Such ageing classes are derived via several notions of comparison between random variables. Of the most commonly used comparison we find, cf. *Muller* and *Stoyan* (2002) and *Shaked* and *Shanthikumar* (1994), the stochastic comparison and the increasing concave comparison. Formally, if X and Y are two random variables with distributions F and G (survivals $\overline{F}$ and $\overline{G}$), respectively, then we say that X is smaller than Y in the:

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(i) stochastic order (denoted by $X \leq_{st} Y$), if

$$E\left[\phi(X)\right] \leq E\left[\phi(Y)\right], for all increasing functions \phi;$$

(ii) increasing concave order (denoted by $X \leq_{icv} Y$), if

$$E[\phi(X)] \leq E[\phi(Y)] \quad for all increasing concave functions \phi$$

In the context of lifetime distributions, some of the above orderings of distributions have been used to give characterizations and new definitions of ageing classes. By ageing we mean the phenomenon whereby an older system has a shorter remaining lifetime, in some statistical sense than a younger one (*Bryson* and *Siddiqui*, 1969). One of the most important approaches to the study of ageing is based on the concept of the residual life. For any random variable X, let

$$X_t = [X - t | X > t], \quad t \in \{x : F(x) < 1\},$$

denote a random variable whose distribution is the same as the conditional distribution of $X - t$ given that $X > t$. When X is the lifetime of a device, X_t can be regarded as the residual lifetime of the device at time t, given that the device has survived up to time t. We say that X is

(i) new better than used (denoted by $X \in NBU$) if

$$X_t \leq_{st} X \quad for all \quad t \geq 0;$$

(ii) new better than used in the increasing concave order (denoted by $X \in NBU(2)$) if

$$X_t \leq_{icv} X \quad for all \quad t \geq 0.$$

For a recent literature on some new ageing classes as well as others we refer the readers to *Kayid* and *Ahmad* (2004), *Ahmad* and *Kayid* (2005), *Ahmad, Kayid* and *Pellerey* (2005) and *Ahmad, Kayid* and *Li* (2005). Much of the earlier literature is cited in those papers where definitions, inter-relations and discussion of above classes are presented.

Recently, the NBU_{mgf} aging class introduced by *Li* (2004) while *Zhang* and *Li* (2004), studied the preservation properties of the class under reliability operations and shock models. *Ahmad* and *Kayid* (2004) studied some other preservations and characterizations properties and introduced a new test of such class. Before proceeding to state the results, we give an overview of the aging class that will be considered in the paper.

Definition 1.1 A non-negative random variable X is said to be new better than used in moment generating function order (denoted by $F \in NBU_{mgf}$) iff

$$X_t \leq_{mgf} X \quad for all \quad t \geq 0.$$

Equivalently, $F \in NBU_{mgf}$ iff

$$\int_0^\infty e^{sx} \overline{F}(x+t) dx \leq \overline{F}(t) \int_0^\infty e^{sx} \overline{F}(x) dx, \ for alls \geq 0.1.1 \qquad (1.1)$$

Also, (1.1) can be written as

$$\int_0^\infty e^{sx} \overline{F}(x) dx \leq \frac{\int_0^t e^{sx} \overline{F}(x) dx}{1 - e^{st} \overline{F}(t)}, \ for alls \geq 0, \ t \geq 0.1.2 \qquad (1.2)$$

Several applications and properties of the moment generating function order, when the random variable represents the lifetime of a system or a unit, can be found in *Shaked* and *Shanthikumar* (1994), *Li* (2004) and *Zhang* and *Li* (2004)).

Note that the following distributions belong to the NBU_{mgf} class since they are in the new better than used in expectation class (*Ahmad* and *Kayid*, 2004):

(*i*) the Weibull Family:

$$\overline{F}_1(x) = e^{-X^\theta}, \ x \geq 0, \ \theta \geq 0;$$

(*ii*) the Linear Failure Rate Family:

$$\overline{F}_2(x) = e^{-x - \frac{\theta}{2} x^2}, \quad x \geq 0, \ \theta \geq 0;$$

(*iii*) the Makeham Family:

$$\overline{F}_3(x) = e^{-x - \theta(x + e^{-x} - 1)}, \quad x \geq 0, \ \theta \geq 0.$$

On the other hand, the concept of aging is very useful in comparisons between various replacement policies. Several authors, see for example *Barlow* and *Proschan* (1981), *Brown* (1980), *Shaked* and *Zhu* (1992) *Yue* and *Cao* (2001) and *Belzunce et al.* (2005), give some results which involve stochastic comparisons of several replacement policies and renewal processes. In this paper the comparisons are made between theses policies when the underlying distribution belong to the new better than used in moment generating function order aging class (abbreviated to the NBU_{mgf} class).

The construction of this paper is as follows: some preservation and characterization properties are discussed in *Section* 2. In *Section* 3 we present some results on

stochastic comparisons of excess life times of renewal process leading to NBU_{mgf} life distributions. Finally, in *Section* 4 we make stochastic comparisons between replacement policies when the lifetimes have the NBU_{mgf} property.

Throughout the paper it is always assumed in the sequel that the non-negative random variables X and Y have F and G as their respect common distribution functions and $\overline{F} = 1 - F$ and $\overline{G} = 1 - G$ denote their corresponding survival functions.

2. PRESERVATION AND CHARACTERIZATIONS RESULTS

In this section we give some new preservation results in the NBU_{mgf} class. First, we establish preservation under transformation. Before giving the results we define the notion of star-shaped functions (see *Marshll* and *Olkin*, 1979).

Definition 2.1 A function $g : [0, \infty) \longrightarrow [0, \infty)$ is said to be star-shaped if $g(0) = 0$ and $g(x)/x$ is increasing in $x \geq 0$.

Proposition 2.1 Let X be a random variable, and let $Y = h(X)$ where h is increasing and let us denote by g the inverse of h. If X is NBU_{mgf} and $g(x)$ is a star-shaped function then Y is NBU_{mgf}.

Proof. Let F_X (F_Y) denote the distribution function of X (Y). Consider

$$e^{st}\overline{F}_Y(t) \int_0^\infty e^{sx}\overline{F}_Y(x)dx - \int_t^\infty e^{sx}\overline{F}_Y(x)dx$$

$$= e^{st}\overline{F}_Y(t) \int_0^t e^{sx}\overline{F}_Y(x)dx - \left(1 - e^{st}\overline{F}_Y(t)\right) \int_t^\infty e^{sx}\overline{F}_Y(x)dx$$

$$= e^{st}\overline{F}_X(g(t)) \int_0^t e^{sx}\overline{F}_X(g(x))dx - \left[1 - e^{st}\overline{F}_X(g(t))\right] \int_t^\infty e^{sx}\overline{F}_X(g(x))dx,$$

since g is star-shaped then the previous expression is greater than

$$e^{st}\overline{F}_X(g(t)) \int_0^t e^{sx}\overline{F}_X\left(\frac{g(t)}{t}x\right) dx - \left[1 - e^{st}\overline{F}_X(g(t))\right] \int_t^\infty e^{sx}\overline{F}_X\left(\frac{g(t)}{t}x\right) dx,$$

now taking $y = g(t)x/t$ in the integral and $\alpha = st/g(t)$, the previous expression has the same sign that

$$e^{\alpha g(t)}\overline{F}_X(g(t)) \int_0^{g(t)} e^{\alpha y}\overline{F}_X(y) dy - \left[1 - e^{\alpha g(t)}\overline{F}_X(g(t))\right] \int_{g(t)}^\infty e^{\alpha y}\overline{F}_X(y) dy,$$

$$= e^{\alpha g(t)}\overline{F}_X(g(t)) \int_0^\infty e^{\alpha y}\overline{F}_X(y) dy - \int_{g(t)}^\infty e^{\alpha y}\overline{F}_X(y) dy;$$

which is positive since X is NBU_{mgf}, so Y is NBU_{mgf}.

Next, we show that a random sum of exponentials is indeed NBU_{mgf}.

Theorem 2.1 Let $\{X_i\}_{i=1}^{\infty}$ be independent and exponentially random variables with mean $E[X_i] = 1/\lambda_{i-1}$, let N be a positive integer-valued random variable, with survival probability $\overline{P}_k = P[N > k]$, which is independent of the X_i, let for $j : 0, 1, 2, ...$

$$\pi_{j,j}(s) = \frac{1}{\lambda_j + s},$$

$$\pi_{j,k}(s) = \frac{1}{\lambda_j + s} \prod_{i=k}^{j-1} \frac{\lambda_i}{\lambda_i + s} \qquad for k = 0, 1, ..., j-1$$

and let for $k : 0, 1, 2, ...$

$$T_k(s) = \sum_{j=k}^{\infty} \frac{\pi_{j,k}(s)\overline{P}_j}{\overline{P}_k}.$$

If for any $s > 0$,

$$T_0(s) \geq T_k(s) \quad for every \ k = 0, 1, 2, ...; 2.1 \qquad (2.1)$$

then T is NBU_{mgf}, where $T = \sum_{i=1}^{N} X_i$.

Proof. In a first place observe that the survival function of $T = \sum_{i=1}^{N} X_i$ is given by

$$\overline{H}(t) = \sum_{k=0}^{\infty} z_k(t)\overline{P}_k, \ where \ \overline{P}_k = P[N > k],$$

and
$$z_k(t) = P\left[\sum_{i=1}^{k} X_i \leq t < \sum_{i=1}^{k+1} X_i\right] \ for \ k \geq 1 \ with \ z_0(t) = P[X_1 > t].$$

Let $s > 0$, then

$$
\begin{aligned}
e^{st}\overline{H}(t) \int_0^{\infty} e^{sx}\overline{H}(x)dx &= \sum_{k=0}^{\infty} e^{st} z_k(t)\overline{P}_k \sum_{k=0}^{\infty} \overline{P}_k \pi_{k,0}(s) \\
&\geq \sum_{k=0}^{\infty} e^{st} z_k(t) \sum_{j=k}^{\infty} \overline{P}_j \pi_{j,k}(s) \\
&= \sum_{k=0}^{\infty} e^{st}\overline{P}_k \sum_{j=0}^{k} z_j(t)\pi_{j,k}(s) \\
&= \sum_{k=0}^{\infty} \overline{P}_k \int_t^{\infty} e^{sx} z_k(x)dx \\
&= \int_t^{\infty} e^{sx}\overline{H}(x)dx,
\end{aligned}
$$

where the inequality follows by condition (2.1), so T is NBU_{mgf}.

3. STOCHASTIC COMPARISONS OF EXCESS LIFETIMES OF RENEWAL PROCESS

Let us consider a renewal process with independent and identically distributed non-negative inter arrival times X_i with common distribution F and $F(0) = 0$. Let $S_0 = 0$ and $S_k = \sum_{i=1}^{k} X_i$ and consider the renewal counting process N(t)=Sup{n: $S_n \leq$ t}. Several papers have investigated some characteristics of the renewal process related to ageing properties of F (see, for instance, *Barlow* and *Proschan* (1981), *Shaked* and *Zhu* (1992) and *Brown* (1980). *Barlow* and *Proschan* (1981), *Shaked* and *Zhu* (1992) and *Chen* (1994) investigated the relationship between the behavior of the renewal function $M(t) = E(N(t))$ and ageing property of F. Some other results are given for the excess lifetime at time $t \geq 0$, that is, $\gamma(t) = S_{n(t)+1} - t$, which is the time of the next event at time t. Recall that $\gamma(0)$ has distribution function F, that is, $\gamma(0) =_{st} X_i$. Some examples of such results are the following:

(*i*) *Chen* (1994) showed that if $\gamma(t)$ is stochastically decreasing in $t \geq 0$, then $F \in NBU$;

(*ii*) *Li et al.* (2000) showed that, if $\gamma(t)$ is decreasing in $t \geq 0$ in the increasing concave
order then $F \in NBU(2)$.

Next we show a similar result for the moment generating function order and NBU_{mgf} ageing class.

Theorem 3.1 If $\gamma(t)$ is decreasing in t for all $t \geq 0$ in the moment generating function order, then $F \in NBU_{mgf}$.

Proof. First we observe the following equality

$$P(\gamma(t) \geq x) = \overline{F}(x + t) + \int_0^t P(\gamma(t - y) \geq x) dF(y).$$

The transformation $M_{\gamma(t)}(s)$ can then be written as

$$M_{\gamma(t)}(s) = \frac{\int_t^\infty e^{sx} \overline{F}_X(x) dx}{e^{st}} + \int_0^t \int_0^\infty e^{sx} P(\gamma(t - y) \geq x) dx dF(y),$$

and given that $\gamma(t)$ is decreasing in the moment generating function order, we have the following inequality

$$
\begin{aligned}
M_{\gamma(t)}(s) &\geq \frac{\int_t^\infty e^{sx} \overline{F}_X(x) dx}{e^{st}} + \int_0^t \int_0^\infty e^{sx} P(\gamma(t) \geq x) dx dF(y) \\
&= \frac{\int_t^\infty e^{sx} \overline{F}_X(x) dx}{e^{st}} + F(t) M_{\gamma(t)}(s).
\end{aligned}
$$

Therefore,

$$M_{\gamma(t)}(s) \geq \frac{\int_t^\infty e^{sx} \overline{F}_X(x)dx}{e^{st}\overline{F}(t)}.$$

Now, given that $\gamma(0) \geq_{mgf} \gamma(t)$ and by previous inequality, we have that

$$\int_0^\infty e^{sx} \overline{F}_X(x)dx = M_{\gamma(0)}(s) \geq M_{\gamma(t)}(s) \geq \frac{\int_t^\infty e^{sx} \overline{F}_X(x)dx}{e^{st}\overline{F}(t)},$$

and therefore $F \in NBU_{mgf}$.

Whereas these results give sufficient conditions for the ageing property of F, in practical situations it would be more interesting to derive some properties for $\gamma(t)$ from the ageing property of F. In fact, given a renewal process it is more feasible to study it if F has some ageing property than if $\gamma(t)$ has some of the previous properties. Some examples of such results in this direction is the following:

(*i*) *Barlow* and *Proschan* (1981, p. 169) showed that, if $F \in NBU$, then

$$\gamma(t) \leq_{st} \gamma(0), \quad forall t \geq 0;$$

(*ii*) *Belzunce et al.*(2001) showed that if $F \in NBU(2)$, then

$$\gamma(t) \leq_{icv} \gamma(0), \quad forall t \geq 0.$$

Next we give a new result for the NBU_{mgf} class.

Theorem 3.2 If $F \in NBU_{mgf}$, then $\gamma(t) \leq_{mgf} \gamma(0)$, for all $t \geq 0$.

Proof. Let us suppose that $F \in NBU_{mgf}$. From the equality

$$P(\gamma(t) \geq u) = \overline{F}(t+u) + \int_0^t \overline{F}(t-s+u)dM(s),$$

the transformation $M_{\gamma(t)}(s)$ can be written as

$$M_{\gamma(t)}(s) = \frac{\int_t^\infty e^{su}\overline{F}(u)du}{e^{st}} + \frac{\int_0^t \int_{t-x}^\infty e^{su}\overline{F}(u)dudM(x)}{e^{s(t+x)}}.$$

If $F \in NBU_{mgf}$, we have

$$\begin{aligned}
M_{\gamma(t)}(s) &\leq \overline{F}(t)\int_0^\infty e^{su}\overline{F}_X(u)du \\
&\quad + \int_0^t \overline{F}(t-x)\int_0^\infty e^{su}\overline{F}(u)dudM(x) \\
&= \int_0^\infty e^{su}\overline{F}(u)du \left(\overline{F}(t) + \int_0^t \overline{F}(t-x)dM(x)\right) \\
&= \int_0^\infty e^{su}\overline{F}(u)du,
\end{aligned}$$

so $\gamma(t) \leq_{mgf} \gamma(0)$.

4. REPLACEMENT POLICIES COMPARISONS

In this section we compare several basic replacement policies under the assumption that the lifetime have NBU_{mgf} distribution. Under the age replacement policy, a unit is replaced upon failure or at age T, whichever comes first. Under a block replacement policy, the unit in operation is replaced upon failure times T, $2T$, $3T$,.... Let $N_A(t, T)$ and τ_{nA} denote the number of failures in $[0, t]$ and the interval between the $(n - 1)th$ and nth failure under an age replacement policy, respectively. Define $N_B(t, T)$ and τ_{nB} in a similar way for a block replacement policy. Let $N(t)$ and τ_n denote the number of renewals in $[0, t]$ and the interval between the $(n - 1)th$ and nth renewal for an ordinary process, respectively. Next, *Theorem* 1 and 2 state that the age replacement diminishes, in the sense of moment generating function order, the number of failures experienced in any particular time interval $[0, t]$, $0 < t < \infty$, if and only if F is in NBU_{mgf} class.

Theorem 4.1 $F \in NBU_{mgf}$ if, and only if,

$$\tau_n \leq_{mgf} \tau_{nA}, \quad n \geq 0.$$

Proof. It is easy to see that

$$P(\tau_n > x) = F(x) \quad for x \geq 0,$$

and for $jT \leq x < (j + 1)T$, $j \geq 0$,

$$P(\tau_{nA} > x) = [\overline{F}(T)]^j \overline{F}(x - jT).$$

Hence

$$
\begin{aligned}
\int_0^\infty e^{sx} P(\tau_{nA} > x)dx &= \sum_{j=0}^\infty \int_j^{(j+1)T} e^{sx} \left[\overline{F}(T)\right]^j \overline{F}(x - jT)\, dx \\
&= \sum_{j=0}^\infty \left[\overline{F}(T)e^{sT}\right]^j \int_0^T e^{sx}\overline{F}(x)dx \\
&= \int_0^T \frac{e^{sx}\overline{F}(x)dx}{\left[1 - e^{sT}\overline{F}(T)\right]} 4.1
\end{aligned}
\tag{4.1}
$$

By definition of moment generating function order, $\tau_n \leq_{mgf} \tau_{nA}$ if, and only, if

$$\int_0^\infty e^{sx}\overline{F}(x)dx \leq \int_0^\infty e^{sx} P(\tau_{nA} > x)dx. 4.2 \tag{4.2}$$

It follows from (4.1) that (4.2) is equivalent to

$$\int_0^\infty e^{sx}\overline{F}(x)dx \le \int_0^T \frac{e^{sx}\overline{F}(x)dx}{\left[1 - e^{sT}\overline{F}(T)\right]}.$$

Appealing to (1.2), the result follow.

The following theorem is given in terms of stochastic comparisons of the nth arrival times τ_n^k, $k = 1, 2$, and therefore, if τ_n^k denotes the time in which failure occurs, these comparisons yield us an interpretation of the comparisons of the counting processes in terms of the intervals of time in which the failures occur and it allows us to select the best process by taking the process in which failures take a more long time than the other process. Before to give the result we need to give the following notion.

Consider now two counting processes $\{N_1(t), \ t \ge 0\}$ and $\{N_2(t), \ t \ge 0\}$, with arrival times $\{\tau_n^1, n \ge 1\}$ and $\{\tau_n^2, n \ge 1\}$. Next we define new criteria for comparing such processes.

Definition 4.1 Let $\{N_1(t), \ t \ge 0\}$ and $\{N_2(t), \ t \ge 0\}$, be two counting processes as above we say that, $N_1(t) \le_{mgf} N_2(t)$ if, and only if, for all $s > 0$, $n = 0, 1, 2, ...,$

$$\int_0^\infty e^{st}P\{N_1(t) \ge n\}\,dt \le \int_0^\infty e^{st}P\{N_2(t) \ge n\}\,dt.$$

Under this definition, we have obtained the following result.

Theorem 4.2 Let $\{N_1(t), \ t \ge 0\}$ and $\{N_2(t), \ t \ge 0\}$, be two counting processes. Let τ_n^1 and τ_n^2 denote the interval between the $(n-1)$th and nth renewal for $\{N_1(t), t \ge 0\}$ and $\{N_2(t), t \ge 0\}$, respectively. Then $N_2(t) \le_{mgf} N_1(t)$ if, and only if, $\tau_n^1 \le_{mgf} \tau_n^2$, $\quad n \ge 0$.

Proof. *Necessity*: Suppose that $N_2(t) \le_{mgf} N_1(t)$, then we have from *Definition 2.1* that

$$\int_0^\infty e^{st}P\{N_2(t) \ge n\}\,dt \le \int_0^\infty e^{st}P\{N_1(t) \ge n\}\,dt,$$

or equivalently

$$\int_0^\infty e^{st}P\{N_1(t) = 0\}\,dt \le \int_0^\infty e^{st}P\{N_2(t) = 0\}\,dt.$$

Observing that

$$P\{N_k(t) = 0\} = P\{\tau_n^k > t\}, \quad k = 1, 2, \ n \ge 1,$$

we have

$$\int_0^\infty e^{st}P\left\{\tau_n^1 > t\right\}\,dt \le \int_0^\infty e^{st}P\left\{\tau_n^2 > t\right\}\,dt, \quad n \ge 1.$$

Thus, $\tau_n^1 \le_{mgf} \tau_n^2$.

Sufficiency: Suppose that $\tau_n^1 \leq_{mgf} \tau_n^2$, $n \geq 1$. Then we have (see *Shaked* and *Shanthikumar* (1994))

$$\sum\nolimits_{n=1}^{m} \tau_n^1 \leq_{mgf} \sum\nolimits_{n=1}^{m} \tau_n^2,$$

or equivalently

$$\int_0^\infty e^{st} P\left\{\sum\nolimits_{n=1}^{m} \tau_n^1 > t\right\} dt \leq \int_0^\infty e^{st} P\left\{\sum\nolimits_{n=1}^{m} \tau_n^2 > t\right\} dt.$$

Hence

$$\int_0^\infty e^{st} P\left\{\sum\nolimits_{n=1}^{m} \tau_n^1 \leq t\right\} dt \geq \int_0^\infty e^{st} P\left\{\sum\nolimits_{n=1}^{m} \tau_n^2 > t\right\} dt.$$

It follows that

$$\int_0^\infty e^{st} P\left\{N_2(t) \geq n\right\} dt \leq \int_0^\infty e^{st} P\left\{N_1(t) \geq n\right\} dt,$$

for all $s > 0$, $n = 0, 1, 2, \dots$. Thus, $N_2(t) \leq_{mgf} N_1(t)$.

Let planned replacement occur at fixed time points $\{0 < t_1 < t_2 < \dots\}$ under policy 1, and at time points $\{0 < t_1 < t_2 < \dots\} \cup \{t_0\}$ under policy 2. Let $N_i(t)$ be the number of failures in $[0, t]$ under policy $i = 1, 2$. Then $F \in NBU_{mgf}$ if and only if

$$N_2(t) \leq_{mgf} N_1(t) \quad for each t \geq 0. \tag{4.3}$$

The following result state that the block replacement diminishes increases, in the sense of moment generating function order, the number of failures experienced in any particular time interval $[0, t]$, $0 < t < \infty$, if and only if $F \in NBU_{mgf}$ class.

Theorem 4.3 $F \in NBU_{mgf}$ if, and only if, $N_B(t, T) \leq N(t)$, $t \geq 0$, $T \geq 0$.

Proof. Suppose that $F \in NBU_{mgf}$. Let planned replacement occur at fixed time points $\{0, T, \dots, (i-1)T\}$ under policy i, $i = 1, 2, \dots$. Let $N_i(t)$ be the number of failures in $[0, t]$ under policy i, $i = 1, 2, \dots$. It follows from (4.3) that

$$N(t) \geq_{mgf} N_1(t) \geq_{mgf} \dots \geq_{mgf} N_i(t) \geq \dots.$$

Let $i \to \infty$; then we have

$$N(t) \geq_{mgf} N_\infty(t) = N_B(t, T).$$

Conversely, suppose that $N(t) \geq_{mgf} N_B(t, T)$, $t \geq 0$, $T \geq 0$. Then

$$\int_0^\infty e^{st} P\left\{N(t) \geq 1\right\} dt \geq \int_0^\infty e^{st} P\left\{N_B(t, T) \geq 1\right\} dt.$$

Hence,

$$\int_0^\infty e^{st} P\left\{N(t) = 0\right\} dt \leq \int_0^\infty e^{st} P\left\{N_B(t, T) = 0\right\} dt,$$

or equivalently,

$$\int_0^\infty e^{st}\overline{F}(t)dt \le \int_0^\infty e^{st}P\left\{N_B(t,T)=0\right\}dt. \tag{4.4}$$

Observing that

$$P\left\{N_B(t,T)=0\right\} = P\left\{X_1 > T, ..., X_k > kT, X_{k+1} > t - kT\right\}$$

$$= \left[\overline{F}(T)\right]^k \overline{F}(t-kT),$$

for $kT \le t < (k+1)T$, $k = 0,1,2,...$. Then we have

$$\int_0^\infty e^{st}P\left\{N_B(t,T)=0\right\}dt = \sum_{k=0}^\infty \int_{kT}^{(k+1)T} e^{st}\left[\overline{F}(T)\right]^k \overline{F}(t-kT)dt$$

$$= \sum_{k=0}^\infty \left[\overline{F}(T)e^{sT}\right]^k \int_0^T e^{st}\overline{F}(t)dt$$

$$= \frac{\int_0^T e^{st}\overline{F}(t)dt}{1 - e^{sT}\overline{F}(T)}. \tag{4.5}$$

We obtain from (4.4) and (4.5) that

$$\int_0^\infty e^{st}\overline{F}(t)dt \le \frac{\int_0^T e^{st}\overline{F}(t)dt}{1 - e^{sT}\overline{F}(T)},$$

and the result follow from (1.2).

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