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Testing Whether a Survival Distribution is Better Mean Residual Life at Age t_0

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Abstract. The better mean residual life at t_0 (BMRL- t_0) class of life distribution is introduced by Kulasekara and Park (1987). They proved that the BMRL- t_0 class contains the DMRL class, but it is a proper subclass of the NBUE class. In this paper we develop a new family of tests for testing exponentiality against the BMRL- t_0 (WMRL- t_0) alternatives based on the goodness of fit approach. It is shown that the suggested test is better than the one introduced by Kulasekara and Park (1987) in the sense of Pitman asymptotic efficiency values.

Key Words : *Decreasing mean residual life, life distributions, goodness of fit, hypothesis testing, asymptotic normality, efficiency, power, Monte Carlo methods*

1. INTRODUCTION

In reliability theory various classes of life distributions for ageing have been studied extensively by many authors. Among the well known classes of life distributions are IFR, IFRA, NBU, NBUE AND HNBUE. Dual classes are defined by reversing the directions of monotonicity or of inequality.

Recently Ahmad et al. (2001) introduced a new approach for testing exponentiality against the classes of lifetime distributions. That is called a goodness of fit approach. Alwasel and El-Bassiouny (2001)used this approach for testing whether a survival function is new better than used of a specified age. El-Bassiouny and Alwasel (2003) used this approach also for testing mean residual times. The classes of decreasing mean residual life (DMRL) and new better than used in expectation (NBUE) are defined in terms of the mean residual life.

Let F be a life distribution (*i.e.* $F(t)=0,\ for\, t<0$) with a finite first moment.

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Then the mean residual life at time t is defined as:

$$e_F(t) = \int_t^\infty \overline{F}(u)du \Big/ \overline{F}(t) \quad for\ 0 \le t \le F^{-1}(1) \tag{1.1}$$

where $\overline{F} = 1 - F$.

Kulasekara and Park (1987) introduced the following definition for the BMRL- t_0.

Definition 1.1 Let $t_0 > 0$, a life distribution F (*i.e.* $F(x) = 0,\ for\ x < 0$) with finite mean is better mean residual life at t_0 (BMRL- t_0) if

$$e_F(t+x) \le e_F(t) \quad for\,all\ \ 0 \le t \le t_0\ and\ for\,all\,x \ge 0, \tag{1.2}$$

where $e_F(t)$ is the mean residual life at time t.

The dual class of worse mean residual life at t_0 (WMRL- t_0) is defined analogously by reversing the direction of the inequality (1.2). The BMRL- t_0 property can be stated as that any item of age t_0 or less has a greater mean residual life than does an older item.

In equality (1.2), let us fix t_0, i.e.,

$$e_F(t_0 + x) \le e_F(t_0) \quad for\,all\,x \ge 0\ and\,a\ fixed\ 0 \le t_0 \le \infty, \tag{1.3}$$

where $e_F(t)$ is defined in (1.1).

Remark. If $t_0 \to 0$ in (1.2), then we get $e_F(x) \le e_F(0) = \mu$, which gives the definition of the NBUE class. (i.e. BMRL- t_0 becomes NBUE).

In this paper we focus on testing H_0: F is exponential against H_a F is BMRL- t_0 and not exponential.

This paper is organized as follows: in section 2, we present a test statistic based on the goodness of fit approach, which is simpler to compute and has higher asymptotic Pitman relative efficiency for several alternatives, Including two alternatives given by Kulasekara and Park (1987). In section 3, an example from Hendi and Abouammoh (2001) in the medical science, is presented as an application of the proposed test on a real life data.

2. TESTING AGAINST BMRL- t_0 CLASS

From (1.3), we propose

$$\delta = \int_0^\infty \{e_F(t_0) - e_F(t_0 + x)\} dF(x) \tag{2.1}$$

as a measure of departure from the null hypothesis H_0. Based on the goodness of fit approach, we can take in place of (2.1) the following measure of departure,

$$\delta_{t_0} = \int_0^\infty \{e_F(t_0) - e_F(t_0 + x)\} dF_0(x) \tag{2.2}$$

for testing H_0 against H_a. Since this measure is scale invariant, then without loss of generality we can take $F_0(x) = 1 - e^{-x}$. In order to derive an expression for δ_{t_0} we need the following lemma.

Lemma 2.1 Let X be a variable with distribution F. Then

$$\begin{aligned}\delta_{t_0} &= \upsilon(t_0)\left(\overline{F}(t_0) - EI(X > t_0)e^{-(X-t_0)}\right) - \overline{F}(t_0) EX\, I(X > t_0)\left(1 - e^{-(X-t_0)}\right) \\ &\quad + e^{t_0}\, \overline{F}(t_0) EI(X > t_0)\left(-Xe^{-X} + e^{-t_0} - e^{-X} + t_0 e^{-t_0}\right)\end{aligned} \tag{2.3}$$

where $\upsilon(t) = \int_t^\infty \overline{F}(u) du$.

Proof. From (2.2) and using (1.1), we get

$$\delta_{t_0} = \int_0^\infty \{\overline{F}(t_0 + x)\upsilon(t_0) - \overline{F}(t_0)\upsilon(t_0 + x)\} dF_0(x).$$

Since $\overline{F}(x) = EI(X > x)$ *and* $\upsilon(x) = E(X - x)I(X > x)$, then

$$\delta_{t_0} = \upsilon(t_0) E \int_0^\infty I(X > t_0 + x) e^{-x} dx \; - \overline{F}(t_0) E \int_0^\infty (X - (t_0 + x)) I(X > t_0 + x) e^{-x}\, dx .$$

After some operations, we easily get the statement of the lemma.

Based on a random sample $X_1, \ldots, X_n$ from a distribution F. We wish to test H_0 against H_a. We note that, under $H_0, \delta = 0$, while it is positive under H_a. Thus, we may be testing on its estimate. A direct empirical estimate of δ_{t_0} is

$$\begin{aligned}\hat{\delta}_{t_0} &= \frac{2}{n(n-1)} \sum \sum_{i<j} \left\{X_i - X_i e^{-(X_j - t_0)} + t_0\, e^{-(X_j - t_0)} - X_j + 1 - e^{-(X_j - t_0)}\right\} \\ &\quad \times I(X_i > t_0) I(X_j > t_0)\end{aligned} \tag{2.4}$$

Note that with

$$\varphi_{t_0}(X_1,X_2)=\{X_1-X_1e^{-(X_2-t_0)}+t_0e^{-(X_2-t_0)}-X_2+1-e^{-(X_2-t_0)}\}I(X_1>t_0)I(X_2>t_0) \tag{2.5}$$

then $\hat{\delta}_{t_0}$ is a classical U-statistic, c.f. Lee (1989). We state and prove the following theorem.

Theorem 2.1. As $n\to\infty, \sqrt{n}(\hat{\delta}_{t_0}-\delta_{t_0})$ is asymptotically normal with mean 0 and variance σ^2 where σ^2 is given in (2.6). Under $H_0:\delta_{t_0}=0$ the variance is given by (2.7).

Proof. Using standard U-statistic theory, c.f. Lee (1989), we can evaluate the asymptotic variance, which is equal to

$$\sigma^2=Var\{E[\varphi_{t_0}(X_1,X_2)\mid X_1]+E[\varphi_{t_0}(X_1,X_2)\mid X_2]\}$$

where,

$$\begin{aligned}E[\varphi_{t_0}(X_1,X_2)\mid X_1]=\{&X_1EI(X_2>t_0)-X_1Ee^{-(X_2-t_0)}I(X_2>t_0)+t_0Ee^{-(X_2-t_0)}I(X_2>t_0)\\&-EX_2I(X_2>t_0)+EI(X_2>t_0)-E\,e^{-(X_2-t_0)}I(X_2>t_0)\ \}I(X_1>t_0)\end{aligned}$$

$$\begin{aligned}E[\varphi_{t_0}(X_1,X_2)\mid X_2]=\{&EX_1I(X_1>t_0)-e^{-(X_2-t_0)}EX_1I(X_1>t_0)+t_0Ee^{-(X_2-t_0)}I(X_1>t_0)\\&-X_2EI(X_1>t_0)+EI(X_1>t_0)-e^{-(X_2-t_0)}I(X_2>t_0)EI(X_1>t_0)\ \}I(X_2>t_0)\ .\end{aligned}$$

It is easy to show that

$$\begin{aligned}\sigma^2=E[\{&(-X_1+t_0+1)Ee^{-(X_1-t_0)}I(X_1>t_0)+2EI(X_1>t_0)-e^{-(X_1-t_0)}EX_1I(X_1>t_0)\\&+(t_0-1)e^{-(X_1-t_0)}EI(X_1>t_0)\}I(X_1>t_0)]^2\ ,\end{aligned} \tag{2.6}$$

which under H_0, becomes

$$\begin{aligned}\sigma_0^2&=E\left\{\left[\frac{e^{-t_0}}{2}(-X_1+t_0+3)-2e^{-X_1}\right]I(X_1>t_0)\right\}^2\\&=\frac{31}{12}\overline{F}^3(t_0)-\frac{5}{2}\overline{F}^4(t_0).\end{aligned} \tag{2.7}$$

Thus the proof of theorem (2.1) is completed.

Remark. Note that, under H_0 and for $t_0 = 0$ in (2.7), we get $\sigma_0^2 = \frac{1}{12}$, which is the asymptotic null variance of the NBUE test statistic of Hollander and Proschan (1975).

To perform the proposed test in this paper, calculate $\sqrt{\frac{n}{\sigma_0^2}}\hat{\delta}_{t_0}$ and reject H_0, if this value exceeds Z_α the upper α – percentile of the standard normal variate.

To compare our proposed test statistic with others in the literature we use the concept of Pitman asymptotic efficacy defined by, see Lehmann (1983) or Pitman (1979):

$$PAE(\hat{\delta}_{t_0}) = \frac{1}{\sigma_0^2}\left(\frac{\partial}{\partial\theta}\delta_{t_0}\,|_{\theta=\theta_0}\right)^2 . \tag{2.8}$$

From (2.3), we get

$$\begin{aligned}\frac{\partial}{\partial\theta}\delta_{t_0} &= \int_{t_0}^{\infty}\overline{F}_\theta(u)\,du\left(\overline{F}_\theta(t_0) - \int_{t_0}^{\infty} e^{-(X-t_0)} f_\theta'(x)\,dx\right) + \int_{t_0}^{\infty}\overline{F}_\theta(u)\,du\left(\overline{F}_\theta(t_0) - \int_{t_0}^{\infty} e^{-(X-t_0)} f_\theta(x)\,dx\right)\\
&- \overline{F}_\theta(t_0)\int_{t_0}^{\infty} x\, f_\theta'(x)\,dx - \overline{F}_\theta'(t_0)\int_{t_0}^{\infty} x\, f_\theta(x)\,dx + \overline{F}_\theta'(t_0)\int_{t_0}^{\infty} x\, e^{-(x-t_0)} f_\theta(x)\,dx\\
&+ \overline{F}_\theta(t_0)\int_{t_0}^{\infty} x\, e^{-(x-t_0)} f_\theta'(x)\,dx + 2\overline{F}_\theta'(t_0)\overline{F}_\theta'(t_0)(t_0+1)\\
&- e^{-t_0}\overline{F}_\theta'(t_0)\int_{t_0}^{\infty}\left(x e^{-x} + e^{-x}\right) f_\theta(x)\,dx\\
&- e^{-t_0}\overline{F}_\theta(t_0)\int_{t_0}^{\infty}\left(x e^{-x} + e^{-x}\right) f_\theta'(x)\,dx .\end{aligned} \tag{2.9}$$

We consider the following two families of alternatives which are often used for efficiency comparisons by many authors:

(i) The Linear Failure Rate Distribution:

$$f_\theta(x) = (1+\theta x)\exp\{-x - \theta x^2/2\}, \quad x \ge 0, \theta \ge 0.$$

(ii) The Makeham Distribution:

$$f_\theta(x) = [1 + \theta(1 - e^{-x})]\exp\{-x - \theta[x + e^{-x} - 1]\}, \quad x \ge 0, \theta \ge 0.$$

The null exponential is attained at $\theta = 0$.

Carrying out the Pitman asymptotic efficiency calculations for the above two alternatives, we get,

$$PAE(\delta_{t_0}, LFR)$$
$$= T_1^2(t_0)$$
$$= \frac{\left\{e^{-2t_0}\left[-\frac{3}{2}t_0^3 - \frac{3}{2}t_0^2 - \frac{13}{8}t_0 - \frac{17}{16}\right] + e^{-4t_0}\left[\frac{t_0^3}{2} + \frac{t_0^2}{2} - \frac{3t_0}{8} - \frac{3}{16}\right]\right\}^2}{\frac{31}{12}e^{-3t_0} - \frac{5}{2}e^{-4t_0}} \tag{2.10}$$

and,

$$PAE(\delta_{t_0}, Makeham)$$
$$= T_2^2(t_0)$$
$$= \frac{\left\{e^{-2t_0}\left[-t_0^2 + \frac{7t_0}{4} - \frac{1}{4}\right] + e^{-3t_0}\left[\frac{-7t_0}{6} - \frac{7}{9}\right] + e^{-4t_0}\left[t_0^2 + \frac{t_0}{4} - \frac{7}{4}\right] + e^{-5t_0}\left[\frac{7t_0}{6} - \frac{59}{36}\right]\right\}^2}{\frac{31}{12}e^{-3t_0} - \frac{5}{2}e^{-4t_0}} \tag{2.11}$$

Table 2.1 contains the PAE values of $\hat{\delta}_{t_0}$ for various values of t_0 with the two alternatives LFR and Makeham and the asymptotic relative efficiencies of $\hat{\delta}_{t_0}$ with the DMRL and NBUE tests of Hollander and Proschan (1975), referred to as V* and k*.

Table 2.1 Pitman asymptotic efficiency and ARE

	Alternative : LFR			Alternative : Makeham		
t_0	$\hat{\delta}_{t_0}$	$\hat{\delta}_{t_0}/k^*$	$\hat{\delta}_{t_0}/V^*$	$\hat{\delta}_{t_0}$	$\hat{\delta}_{t_0}/k^*$	$\hat{\delta}_{t_0}/V^*$
0.00	18.75	25.00	22.85	23.15	277.78	396.65
0.25	3.68	4.90	4.48	0.748	8.97	12.81
0.50	3.57	4.76	4.35	0.121	1.45	2.06
0.70	4.42	5.89	5.39	0.028	0.33	0.47
1.0	7.01	9.34	8.54	0.005	0.054	0.08

From table 2.1, it is shown that the PAE values of our proposed test $\hat{\delta}_{t_0}$ are very high when $t_0 = 0$ (which gives the NBUE class). But the trend of these values for the two alternatives is different, for example in the case of LFR, the minimum of the values is associated to $t_0 = 0.5$, but they are highly increasing when t_0 either 0 or ∞.

To compare our test with that of Kulasekera and Park (1987) test (T*), we present the ARE for some selected values of t_0 and the same values they used for α in table 2.2.

Table 2.2 ARE values of $\hat{\delta}_{t_0}$ compared with T*

t_0 \ α		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	F_1	27.73	30.7	32.9	32.2	27.4	23.4	24.6	25.8	24.5
	F_2	379.9	526.1	969.1	771.4	611.9	414.7	362.8	372.2	387.5
0.25	F_1	5.4	6.01	6.4	6.3	5.4	4.58	4.8	5.1	4.8
	F_2	12.3	16.9	22.5	24.9	19.8	13.4	11.7	12.0	12.5
0.5	F_1	5.3	5.8	6.3	6.1	5.2	4.5	4.7	4.9	4.7
	F_2	1.98	2.74	3.62	4.02	3.19	2.16	1.89	1.94	2.02
0.7	F_1	6.53	7.23	7.74	7.59	6.45	5.52	5.79	6.08	5.78
	F_2	0.45	0.63	0.83	0.92	0.73	0.49	0.43	0.44	0.46
1	F_1	10.36	11.46	12.28	12.03	10.24	8.75	9.19	9.64	9.16
	F_2	0.074	0.10	0.14	0.15	0.12	0.08	0.07	0.07	0.08

From table 2.2, we can say the following:

- For all values of t_0, the ARE are always more than one in the case of LFR distribution (F1).
- For t_0 less than or equal to 0.5, the ARE values exceed one in the case of Makeham distribution (F2), but they are less than one for all $t_0 > 0.5$.
- In the case of LFR distribution (F1), the ARE values reach their minimum at $t_0 = 0.5$ and increases as $t_0 \to 0.5$ or 1, i.e. it takes the shape of a bath tub.
- In the case of Makeham distribution (F2), the ARE values decrease as $t_0 \to 1$.

It is also easy to see that our proposed test is consistent and unbiased. For samples 5(1)25(5)50 and $t_0 = 0.2, 0.5, 0.6, 0.8, 1.0$, and using 5,000 replications, the upper percentiles %95 and %99 of the statistic $\hat{\delta}_{t_0}$ is given in table (2.3). Also the power of the statistic $\hat{\delta}_{t_0}$ for samples 5(1)20 for the above two alternatives for %95 percentile and $t_0 = 0.2, 0.5, 0.8$ are presented in tables (2.4 , 2.5 and 2.6), respectively.

Table 2.3 Critical values of $\hat{\delta}_{t_0}$

	$t_0 = 0.2$		$t_0 = 0.5$		$t_0 = 0.6$		$t_0 = 0.8$		$t_0 = 1$	
n	%95	%99	%95	%99	%95	%99	%95	%99	%95	%99
5	0.1203	0.2582	0.1613	0.3475	0.1626	0.3594	0.1376	0.3498	0.0913	0.3078
6	0.1006	0.2220	0.1453	0.2969	0.1400	0.3175	0.1207	0.2872	0.0989	0.2650
7	0.0840	0.1987	0.1316	0.2619	0.1315	0.2777	0.1062	0.2675	0.0918	0.2283
8	0.0719	0.1830	0.1200	0.2417	0.1193	0.2581	0.1051	0.2336	0.0830	0.2207
9	0.0604	0.1593	0.1092	0.2213	0.1125	0.2303	0.0982	0.2181	0.0791	0.1900
10	0.0529	0.1471	0.0987	0.2114	0.1055	0.2086	0.0872	0.1985	0.0743	0.1712
11	0.0421	0.1384	0.0914	0.2016	0.0982	0.2035	0.0830	0.1771	0.0717	0.1505
12	0.0367	0.1297	0.0865	0.1915	0.0931	0.1955	0.0798	0.1757	0.0686	0.1478
13	0.0301	0.1209	0.0820	0.1816	0.0908	0.1904	0.0756	0.1635	0.0662	0.1490
14	0.0228	0.1139	0.0767	0.1699	0.0849	0.1817	0.0746	0.1550	0.0627	0.1352
15	0.0163	0.1071	0.0726	0.1624	0.0827	0.1675	0.0703	0.1446	0.0614	0.1309
16	0.0125	0.0959	0.0686	0.1508	0.0709	0.1609	0.0674	0.1416	0.0582	0.1305
17	0.0087	0.0873	0.0665	0.1446	0.0757	0.1535	0.0655	0.1361	0.0574	0.1224
18	0.0014	0.0862	0.0611	0.1395	0.0730	0.1477	0.0639	0.1307	0.0552	0.1204
19	0.0011	0.0822	0.0586	0.1363	0.0707	0.1421	0.0620	0.1300	0.0529	0.1154
20	0.0011	0.0740	0.0555	0.1317	0.0648	0.1370	0.0604	0.1269	0.0521	0.1129
21	0.0009	0.0680	0.0523	0.1283	0.0643	0.1363	0.0587	0.1227	0.0509	0.1055
22	0.0009	0.0662	0.0476	0.1247	0.0628	0.1337	0.0573	0.1168	0.0496	0.1057
23	0.0009	0.0669	0.0456	0.1265	0.0606	0.1307	0.0557	0.1162	0.0486	0.1012
24	0.0007	0.0626	0.0432	0.1215	0.0582	0.1266	0.0549	0.1142	0.0477	0.0995
25	0.0007	0.0581	0.0406	0.1191	0.0575	0.1210	0.0548	0.1108	0.0471	0.0996
30	0.0007	0.0524	0.0382	0.1105	0.0561	0.1171	0.0529	0.1066	0.0455	0.0944
35	0.0004	0.0505	0.0371	0.1106	0.0545	0.1158	0.0522	0.1065	0.0452	0.0899
40	0.0002	0.0492	0.0360	0.1092	0.0536	0.1157	0.0519	0.1048	0.0445	0.0887
45	0.0002	0.0456	0.0348	0.1028	0.0524	0.1125	0.0508	0.1012	0.0438	0.0856
50	0.0002	0.0451	0.0336	0.1039	0.0507	0.1090	0.0498	0.0997	0.0429	0.0843

Table 2.4 Power values of $\hat{\delta}_{t_0}$ for $t_0 = 0.2$

Distribution	Parameter θ	Sample size 5	10	15	20
Linear failure Rate	2	0.7541	0.8715	0.9345	0.9552
	3	0.8869	0.9032	0.9386	0.9867
	4	0.9672	0.9778	0.9889	0.9966
Makeham	2	0.6137	0.7771	0.7918	0.8526
	3	0.7989	0.8221	0.8602	0.9098
	4	0.8333	0.8749	0.8823	0.9561

Table 2.5 Power values of $\hat{\delta}_{t_0}$ for $t_0 = 0.5$

Distribution	Parameter θ	Sample size 5	10	15	20
Linear failure Rate	2	0.7001	0.7715	0.8374	0.9128
	3	0.8239	0.8735	0.8999	0.9560
	4	0.8772	0.9349	0.9456	0.9886
Makeham	2	0.5922	0.7251	0.7504	0.8471
	3	0.6619	0.8034	0.8333	0.8919
	4	0.7940	0.8660	0.8712	0.9442

Table 2.6 Power values of $\hat{\delta}_{t_0}$ for $t_0 = 0.8$

Distribution	Parameter θ	Sample size 5	10	15	20
Linear failure Rate	2	0.7001	0.7715	0.8374	0.9128
	3	0.8239	0.8735	0.8999	0.9560
	4	0.8772	0.9349	0.9456	0.9886
Makeham	2	0.5922	0.7251	0.7504	0.8471
	3	0.6619	0.8034	0.8333	0.8919
	4	0.7940	0.8660	0.8712	0.9442

Thus we have shown that, based on Monte Carlo methods, the test statistic $\hat{\delta}_{t_0}$ has not only simplicity advantages over earlier ones but also has better asymptotic relative efficiency and power.

3. APPLICATION

In this section data from the medical field is analyzed. We show that the proposed test gives consistent conclusions about the distribution of the data with previous obtained results.

Example. This data set consist of the survival times of 40 patients suffering from blood cancer (Leukemia) from one of the hospitals in Saudi Arabia, Hendi and Abouammoh (2001). The ordered life times (in days) are:

115, 181, 255, 418, 441, 461, 516, 739, 743, 789, 807, 865, 924, 983, 1024, 1062, 1063, 1165, 1191, 1222, 1122, 1251, 1277, 1290, 1357, 1369, 1408, 1455, 1478, 1549, 1578, 1578 ,1599, 1603, 1605, 1696, 1735, 1799, 1815, 1852.

To apply the test we choose $t_0 = 1{,}137$ $(\approx 3\ years)$. We obtain $\hat{\delta}_{t_0} = -0.0476$, $\hat{\sigma}_0^2 = 0.2178$, and $\sqrt{\frac{40}{\hat{\sigma}_0^2}}\hat{\delta}_{t_0} = -0.644$, with a corresponding p-value of 0.259. Hence H_0 is not rejected, agreeing with the conclusion of Hendi and Abouammoh (2001).

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REFERENCES

Alwasel, I. and El-bassiouny, I. (2001). Testing whether a survival function is new better than used of specified age, *Egyptian J. of Stat.*, **45**, no.2, 155-162.

El-bassiouny, I. and Alwasel, I. (2003). A goodness of fit Approach to testing mean residual times. *J. Appl. Math. and Comp.*, **143**, 301-307.

Ahmad, I.A., Alwasel, I. and Mugdadi, A.R. (2001). A Goodness of Fit Approach to Major Lifetesting Problems. *Int. J. of Reliab. and Appl.*, **2**, no. 2, 81-97.

Hendi M. and Abbouammoh A. (2001). Testing new better than renewal used life distributions based on U-test, *Commun. Statist.- Theor. Methods*, **30**, 2135-2147.

Hollander, M. and Proschan, F. (1975). Tests for the mean residual life. *Biometrika*, **62**, 585-593.

Kulasekera, B. and Park D. H. (1987). The class of better mean residual life at age t_0. *Micro. Reliab.*, **27**, no. 4, 725-735.

Lee, A. J. (1989). *U-Statistics*, Marcel-Dekker, New York, N.Y.

Lehmann, E.L. (1983). *Theory of point estimation*. Wiley, New York.

Pitman, E.J.G. (1979). *Some basic theory for statistical inference*. Chapman and Hall, London.