가중치 알파 쉐이프를 기반으로 하는 산포된 자료의 볼륨 모델링

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요 약

본 논문은 주어진 자료 점들에게 가중치를 부여하여 볼륨 자료를 여러 단계별로 상세함을 표현하는 방법을 제시하고자 한다. 단계별로 상세함을 표현하기 위하여 웨이브렛 변환 과 알파쉐이프와의 관계를 얻고자 연구하였다. 산포된 자료란 자료점들 사이에 특별한 상관관계가 없는 자료들의 수집이라 정의할 수 있다. 볼륨 트라이베리에이트 공간상에 보간의 정확도 는 3 차원 공간상에 흩어진 자료들의 위치정보 뿐만 아니라 자료들이 갖고 있는 값 (명암도)에도 영향을 받는다. 자료 점들에게 각각 해당되는 웨이브렛 계수를 가중치로 부여 하여 근사치의 정확 도를 개선할 수 있다.

키워드 : 볼륨자료 가시화, 산포된 자료 보간법, 가중치 알파 쉐이프, 웨이브렛 변환

Volume Modeling of Scattered Data based on Weighted Alpha Shapes

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ABSTRACT

This paper describes a method to achieve different level of detail for the given volumetric data by assigning weight for the given data points. The relation between wavelet transformation and alpha shape was investigated to define the different level of resolution. Scattered data are defined as a collection of data that have little specified connectivity between data points. The quality of interpolant in volumetric trivariate space depends not only on the distribution of the data points in $\mathfrak{R}^3$, but also on the data value (intensity). We can improve the quality of an approximation by using wavelet coefficient as weight for the corresponding data points.

Key Words : Volumetric Data Visualization, Scattered Data Interpolation, Weighted Alpha Shapes, Wavelet Transformation

1. Introduction

Visualization and computer modeling enable us to create a mathematical model of a phenomenon that can be displayed using dynamic computer graphics. The resulting visualization yields new insights for us, and these new ways of looking at the phenomenon permit us to find trends hidden in the original data[1]. Visualization implies creating a pictorial form for data. The geometry composing such a picture can be classified by its dimensionality. To visualize the given data appropriately, its characteristics should be understood. Volumetric data occurs in many areas of science and engineering. For example, temperature readings taken at various locations in a room or density measurements from interior of an object are considered.

Scattered data modeling is concerned with the approximation of mathematical objects by using samples taken at an unorganized set of discrete points, a scattered trivariate data points. Many different phenomena in natural sciences and engineering exhibit multiple levels of details. Among the wide range of examples are transport processes in fluid flow, where finer details of free turbulences may be due to irregular vortex motions or the evolution of shock fronts. To represent the mathematical model at the relevant range of scales, multiresolution methods are required[2, 3].

Effective multiresolution methods are essentially concerned with balancing the two conflicting requirements of

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low data size and high fidelity, where the goal is to keep the required computational costs at a given model resolution as small as possible. Therefore, multiscale modeling is usually concerned with information reduction, model simplification, and data compression. One of good example of multiresolution application area is visualization of medical images.. Specially, visualization of ROI(region of interest) requires high resolution compared to other regions.

From the mathematical point of view, multiresolution modeling relies on multirsolution analysis, which includes the theory of wavelets. Another way to generate multiresolution models is through subdivision. There are basically two different concepts for designing multiresolution methods in scattered data modeling. Firstly, the hierachical decomposition of the model into several levels of detail leads to multilevel methods. Secondly, a different approach for designing multiresolution methods is given by one-level modeling concepts. The weighted alpha shapes method lead us to achieve multiresolution representation, also. The weights were computed based on Euclidean distance in Edelsbrunner's paper [10]. But, his method is not practical in case of large volumetric data due to expensive computation of distance. In this paper, wavelets are used to achieve multilevel representation. The weighted alpha shapes method is applied based on wavelet coefficients.

The quality of a piecewise linear interpolation in space depends not only on the distribution of the data points in $\Re^3$, but also on the data values. The α-shapes consider the positional information only, we would like to use intensity information also. Wavelet transformation allows a function to be described in terms of a coarse overall shape, plus details that range from broad to narrow. We treat a volume as the coefficients corresponding to a three-dimensional piecewise-constant basis and wavelet-transform them. Wavelet transformation allow us to rank the given data points in terms of importance. We assign the weight for each data point based on wavelet coefficients. We provide a method to visualize the volumetric scattered data points in the desired level of detail.

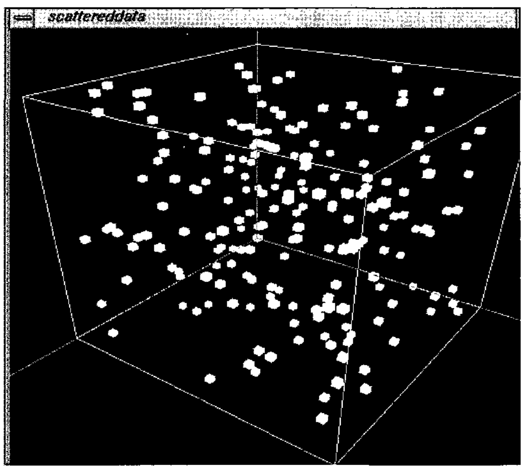
2. Problem Description

The common problem of volumetric data is that the amount of data is too much. It is important to manipulate a large number of data efficiently. If we can select only important data among a large data sets, the speed of processing and the efficiency of data communication can be significantly improved. We have to deal with two conflicting requirements of low data size and high fidelity, where the goal is to keep the required computational costs at a given model resolution as small as possible. Multiresolution can be one of solution for low data size and high fidelity. Wavelet transformation can be applicable to provide the tool of multi-resolution, so user can see the region of interest, efficiently.

If we select only the important data to reduce the amount of data, it is no more rectilinear data. The selected data is more scattered (random, unstructured) data rather than regular data. Volumetric scattered data is volumetric data that has not been sampled on a cuberille grid. The data may have been sampled in some random manner. Scattered data interpolation from $\Re^3 \rightarrow \Re$ consists of constructing a function $f = (x, y, z)$ such that $f(x_i, y_i, z_i) = F_i$, i =1, N where V= $\{v_i = (x_i, y_i, z_i) \in \Re^3$, $i = 1,...N\}$ is a set of distinct and non-coplanar data points and $F = (F_1, ..., F_N)$ is a real data vector as shown in (Figure 1). F_i is a function value at x_i, y_i, and z_i.

In order to visualize the collection of data that have little specified connectivity among data points, numerical techniques are utilized to model the data[4].

One possible way of interpolating volumetric scattered data is a piecewise linear interpolation. The quality of a piecewise linear interpolation over tetrahedral domain depends on the specific tetrahedrization of the data points. Therefore, one main task is to provide the best 3-D domain consisting of tetrahedral[5]. The quality of a piecewise linear interpolation in space can be improved by considering not only positional information, but also intensity gray values for the given volumetric scattered data points.



(Figure 1) Volumetric scattered data

3. Wavelet Transformation

It is common to visualize volumetric data by selecting the important data points among the all volumetric data points in the web based visualization. In this case, the uniform point sets become the non-uniform sets. Wavelets are mathematical tool for hierarchically decomposing functions[6, 7]. They allow a function to be described in terms of a coarse overall shape, plus details that range from broad to narrow. We select the important data by wavelet transform. In the process, we can obtain the weight factor for weighted α-shapes.

We treat a volume as the coefficients corresponding to a three-dimensional piecewise-constant basis and wavelet-transform them. Then, we omit small coefficients under an allowable error. The volume is represented by the remained coefficients. This means that we can express an original set of data using smaller set of data. This makes us find the important data points among all data points.

Our approach consists of two stages. The first stage makes use of wavelet based image compression that is extended in three dimensions. The second stage makes use of the difference of gray value of two-neighbor position in x, y, z direction. In common image compression, an original volume data $f(x)$ are represented as a weighted sum of basis functions $v_1(x),...v_m(x)$.

$$f(x) = \sum_{i=1}^{m} c_i v_i(x) \tag{3.1}$$

The coefficient $c_1,...c_m$ is an original volumetric data set. The original volume data $f(x)$ can be approximated with fewer coefficients using different basis as follows.

$$\hat{f}(x) = \sum_{i=1}^{\hat{m}} \hat{c}_i \hat{v}_i(x) \tag{3.2}$$

If $\hat{m}$ is smaller than m and $\|f(x) - \hat{f}(x)\|$ is smaller than error tolerance(ε), we can find a set of basis function that approximates the original volume data with few coefficients. A user specifies the error tolerance(ε). The square of the L^2 norm error is given by following equations.

$$\left\| f(x) - \hat{f}(x) \right\|^2 = \sum_{i=\hat{m}+1}^{m} (c_{\pi(i)})^2 \tag{3.3}$$

In equation (3.3), $\pi(i)$is a permutation of *i 1,2,.m*. The algorithm of weight assignment for volumetric scattered data can be described as follows.

1. Compute wavelet coefficient representing a volumetric in three-dimensional wavelet basis.
2. Sort the coefficients in order of decreasing magnitude.
3. Find the smallest wavelet coefficient $\hat{m}$ for which satisfies $\sum_{i=\hat{m}}^{m} (c_{\pi(i)})^2 \leq \varepsilon^2$ where ε^2 is an allowable L^2 error.
4. Replace original coefficient as zero for all of smaller wavelet coefficients than $\hat{m}$.
5. Reconstruct volumetric scattered data based on using new coefficients of step 4.
6. Calculate the difference of gray value from 6-degree (front, back, left, right, top and bottom) based on reconstructed data of step 5 in L^2 norm manner.
7. Assign weights for each data point according to the calculated values in step 6.

4. Weighted Alpha Shapes

4.1 Alpha shapes

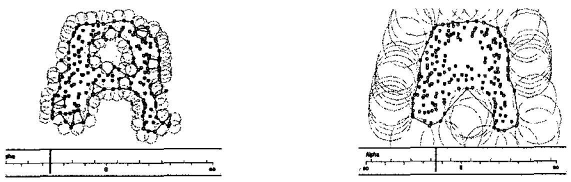
The α-shapes of finite point set are a polytope that is uniquely determined by the set and a real number α.

Definition 1 : Let α be a sufficiently small but otherwise arbitrary positive real. The α-hull of *S* is the intersection of all closed discs with radius 1/α that contain all the points.

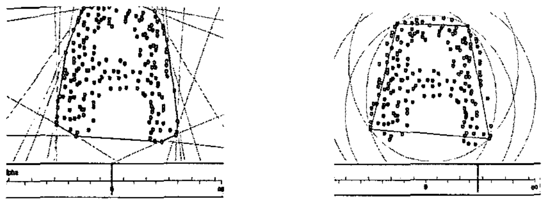
Definition 2 : For arbitrary negative reals α, the α-hull is defined as the intersection of all closed complements of discs (where these discs have radii - 1/α) that contain all the points of *S*.

If we define a generalized disc of radius 1/α as a disc of radius 1/α if α>0, the complement of a disc of radius - 1/α if α < 0, and a half plane if α=0, then definition 1 and 2 could be combined as follows: for an arbitrary real α and a set *S* of points in the plane, the α-hull of *S* is the intersection of all closed generalized discs of radius 1/α that contain all the points of *S*. In (Figure 2), the results of α-shapes, which have negative value of α. In (Figure 3), two pictures are the results of α-shapes, which have positive value of α. The value of α is increased from top to bottom [8, 9].

Alpha shapes provide a mathematical framework to make the geometric shape of a set of points in three-dimensions. However, α-shapes give good results for point sets of roughly uniform density, it does not give for non-uniform point sets. In order to be effective in non-uniform point sets, it needs to change the value of



(Figure 2) α - shapes of negative α for α increasing from top to bottom



(Figure 3) α - shapes of positive α for α increasing from top to bottom

alpha(radius of sphere) locally depending on the intensity of a point set.

It is a polytope in a fairly general sense : it can be concave and even disconnected; it can contain two-dimensional patches of triangles and one-dimensional strings of edges; and its components can be as small as single points. The parameter α controls the maximum curvature of any cavity of the polytope.

4.2 Weighted alpha shapes

The weighted α shapes method is defined for a finite set of weighted points. Let $S \subseteq \Re^d \times \Re$ be such a set. A weighted point is denoted as $p = (p', \omega)$, with $p' \in \Re^d$ its location and $\omega \in \Re$ its weight. For a weighted point p and a real α define $p_{+\alpha} = (p', \omega + \alpha)$. So p and $p_{+\alpha}$ share the same location and their weights differ by α. In other words, it is a polytope uniquely determined by the points, their weights, and a parameter $\alpha \in \Re$ that controls the desired level of detail.

Given a finite set of points, each with a real weight, the regular triangulation is a unique simplicial complex whose underlying space is the convex hull of the point set. If all weights are the same then it equals the Delaunay triangulation of the points.

In reconstruction an interpolant from volumetric scattered from point data it is rarely the case that the points are uniformly dense every in 3-D space. Indeed, the density often varies with the curvature. If α is chosen so that the α-shape produces a piecewise linear surface in sparse regions, it would not be represented in detail in denser regions. Conversely, if α is chosen so that dense regions are nicely modeled then the α-shape will get holes and disconnected in sparse regions. The assignment of large weight in sparse region and of small weights in dense regions can be used to take care of this undesirable effect [10].

The previous weighted α-shapes consider about positional information, only. In other words, the weight fore each data point is based on Euclidian distance. The quality of interplant in volumetric trivariate space depends not only on the distribution of the data points in $\Re^3$, but also on the data value (intensity). Wavelet coefficients can provide the description in terms of a coarse overall shape, plus details that range from broad to narrow with an approximation coefficients and detail coefficients, respectively. The intensity values of points in sparse region may be more distinctive than the intensity values of points in dense region. We can improve the quality of an approximation by using wavelet coefficient as weight for each point. Intuitively, a large weight favors and a small weight discourages connections to neighboring points.

Let explain algorithm with a simple example. In resolution 4, original data has 4 pixels. And those intensities are 9,7,3, and 5 as shown in <Table 1>.

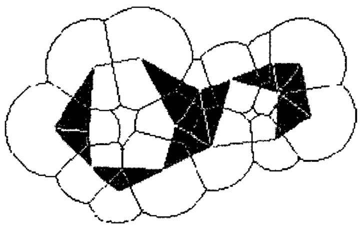
〈Table 1〉 Wavelet Coefficients Volumetric Scattered Data

Resolution	Average	Detail Coefficient
4	[9,7,3,5]	
2	[8,4]	[1,-1]
1	[6]	[2]

We can obtain coefficients of Haar basis function. If we remove [-1] only, then original data is changed as [9,7,4,4]. If we remove [1,-1]. Then, original data is changed as [8,8,4,4]. Now, many pixels may have the same value such as [8,8] or [4,4]. In this case, we select the data points, which have more different gray value from the adjacent data points. In this case, the second and third data points have more deferent gray values. Next, we calculate the difference of gray value from right and left for the first 8. The first 8 has 0, and the second 8 has 4. In this case, we assign weight to the first 8 as 0.0, and to the second 8 as 2.0 (4.0 divided by 2). In the same manner, the first 4 has 4, and the second 4 has 0. So, we assign weight to the first 4 as 2.0 (4.0 divided by 2) and to the second 4 as 0.0. In three-dimensional case, we calculate the difference of gray value from 6 directions (i.e. front, back, left, right, top, and bottom) for the current pixel in an averaged L^2 norm manner (i.e. $\sum_{bottom}^{top} \sum_{back}^{front} \sum_{left}^{right} L_2 \Big/ 6$).

If all weights are zero, then it will coincide with the unweighted α- shapes. The polytope is not necessarily connected and it is not necessarily the same as the closure of its interior. To explain the idea of weighted α-shapes, we illustrate the 2-D version of weighted α-shapes in (Figure 4). The shape of a finite set of weighted points is defined in terms of a decomposition of the union of corresponding balls into convex sets. This decomposition is defined by the (weighted) Voronoi cells of the balls. The shaded triangles are the dual complex of the collection of weighted points, which is the result of α-shapes with a certain parameter α on the weighted Voronoi diagram.

The result of weighted α-shapes does not necessary to connect all the given points. If we think in terms of spheres for 3-D extension, we may think of radii, which are α+ω, where α is the parameter of α-shapes and ω is the weight of data point. Therefore, a sphere possibly may have unequal radii according to weight of each point ω_i [11-13]



(Figure 4) Weight α-shapes

Two ways of assigning weight are considered as follows:

1. min-max : calculated the minimum and maximum value of the difference of gray value among all the given data points in L_2 norm manner and linear interpolate for current data point in the range from 0 to α.

$$\omega_i = \alpha * \frac{L_{2(i)} - L_{2\min}}{L_{2\max} - L_{2\min}} \tag{4.1}$$

2. barycentric weighting : calculate the sum of all the difference of gray value for each data point in L_2 norm manner and take a portion of α according to the current data point with respect to the sum of all the difference of gray value.

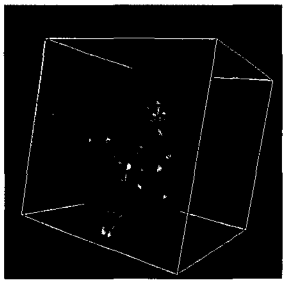
$$\omega_i = \alpha * \frac{L_{2(i)}}{\sum_{i=1}^{N} L_{2(i)}} \tag{4.2}$$

5. Visualization

The results of volumetric scattered data interpolation are visualized by using OPENGL in Visual C++ developing environment. Alpha shapes are generated by using CGAL (Computational Geometry Algorithms Library). α is a parameter $0 \leq \alpha \leq \infty$. For $\alpha = \infty$, the α-shape is the convex hull of a point set. As α decreases, the α-shape shrinks and develops cavities, as soon as a sphere of radius can be put inside.

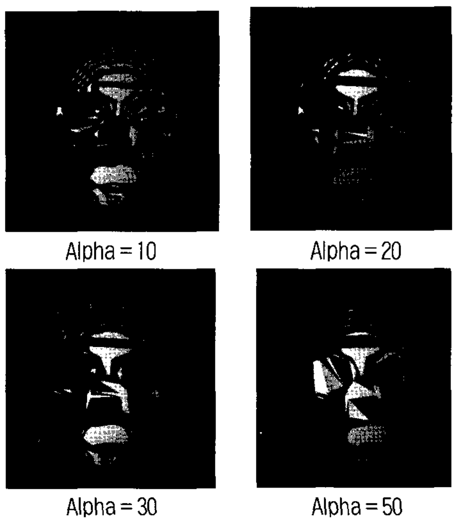
5.1 Various common factors

This original volumetric skull data size is 262144 (64*64*64 = 262144). Data reduction is achieved by using wavelet transformation. Data reduction rate is 15956/262144 = 6.07 %. Threshold value is 50 in the range from 0 to 255 as shown in (Figure 5). Marching Cube algorithm needs all of data points. But, alpha shapes algorithm needs 6.07 % only.

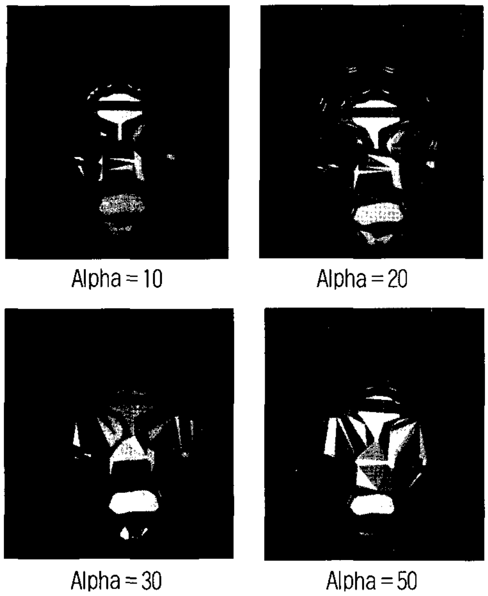


(Figure 5) Original Image

Common factor method means we assign the same weight for each data points. As α decreases, the α-shapes method shrinks and develops cavities. As α increases, α-shapes method expands and develops convex hull. Common factor 0, and 10 are applied as shown in (Figure 6), and (Figure 7), respectively.



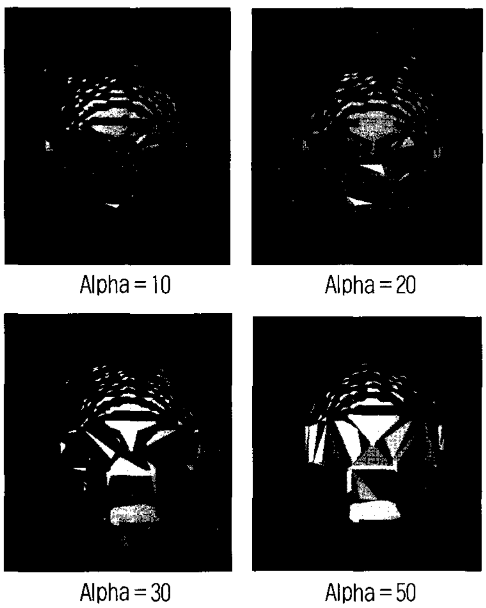
(Figure 6) Common factor 0



(Figure 7) Common factor 10

5.2 Weighted α-shapes based on Min-Max

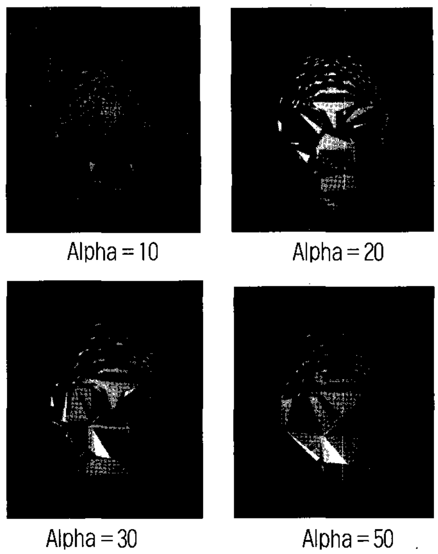
The weight for data point is assigned based on the minimum and maximum value of the difference of gray value among all the given data points in L2 norm manner and linearly interpolate for current data point in the range from 0 to α. The results are shown in (Figure 8).



(Figure 8) Weighted α-shapes based on Min-Max

5.3 Based on Barycentric-weighting

The weight for data point is assigned based on the portion of α according to the current data point with respect to the sum of all the difference of intensity value. The results are shown in (Figure 9).

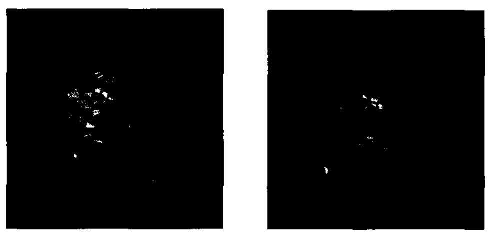


(Figure 9) Weighted α-shapes based on Barycentric-weighting

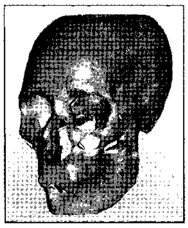
5.4 Comparison of results

We have a chance to compare the results between weighted alpha shapes based on Min-Max and based on Barycentric Weighting. As shown in (Figure 10), and (Figure 11). The left image is generated by un-weighted alpha shapes. The right image is generated by weighted alpha shapes based on barycentric weighting method. In the left image, we can find hole (cavity). In the right image, the hole is filled. Therefore, It is possible that the problem of un-weighted (common weighted) alpha shapes can be improved by using weighted alpha shapes techniques.

If alpha chosen so that the alpha shapes produce a piecewise linear surface in less distinctive regions, it will be clumsy and hide details in more distinctive regions. Conversely, if alpha is chosen so that more distinctive regions are nicely modeled then the alpha-shape will get holes and break apart in less distinctive regions. The assignment of large weights in more distinctive regions and of small weights in less distinctive regions can be used to take care of this undesirable effect as shown in (Figure 10) and (Figure 11).



(Figure 10) α-shape common weight = 0, α=10 on the left image and weighted α-shape based on barycentric weighting, α=10 on the right image.



(Figure 11) α-shape common weight = 0, α=10 on the left image and weighted α-shape based on barycentric weighting, α=10 on the right image by using GEMVIEW visualization package.

6. Conclusions

This paper describes a method to achieve different level of detail for the given volumetric data by assigning weight to points. The relation between wavelet transformation and alpha shapes method was used to define the different level of resolution. Wavelets are mathematical tool for hierarchically decomposing functions. They allow a function to be described in terms of a coarse overall shape, plus details that range from broad to narrow. In other words, wavelets provide a method for representing the level of detail. We apply this feature for describing the ranking of importance for each data points. We treat a volumetric scattered data as the coefficients corresponding to a three-dimensional piecewise constant basis functions of wavelet transformation.

The quality of interpolant in volumetric trivariate space depends not only on the distribution of the data points in $\Re^3$, but also on the data value (intensity). The α - shape considers the positional information, only. Wavelet transformation deals with intensity value of data points. The connection between wavelet and alpha shape was studied for visualizing volumetric scattered data points.

We assign weight value for each data point by using wavelet coefficient. The given volumetric scattered data points, each with a real weight, is triangulated by using the concept of weighted alpha shapes In constructing an interpolant from scattered data points, it is the case that the points may be irregularly dense everywhere on the interpolant. We assign relatively large weights for important points and small weights for less important points based on wavelet coefficient, respectively. The intensity values of points in sparse region may be more distinctive than the intensity values of points in dense region. Points which have distinctive intensity value around a neighborhood may counted as more important points than less distinctive points based on wavelet coefficients. This scheme can be useful to achieve a high quality volumetric scattered data interpolation.

For further research, relationship between positional information and intensity value need to be investigated. Also, formalization of relationship between parameter α for α-shapes and weight value ω based on wavelet coefficient is pretty much challenging task for feature research. We assume that one level of α can be maximally weighted by α. So the maximum radius of a level α can be increase by twice (i.e. α+α=2α).

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