Bull. Korean Math. Soc. **43** (2006), No. 2, pp. 333–341

GENERALIZED TOEPLITZ ALGEBRA OF A CERTAIN NON-AMENABLE SEMIGROUP

Sun Young Jang

Abstract. We analyze a detailed picture of the algebraic structure of C^*-algebras generated by isometric representations of the non-amenable semigroup $P = \{0, 2, 3, \dots, n, \dots\}$.

1. Introduction

Let S denote a countable discrete semigroup with unit e and $\mathcal{B}$ be a unital C^*-algebra. A map $W : S \to \mathcal{B}, x \mapsto W_x$ is called an *isometric homomorphism* if $W_e = 1$, W_x is an isometry and $W_{xy} = W_x W_y$ for all $x, y \in S$. If $\mathcal{B}$ is the C^*-algebra $\mathcal{B}(H)$ of all bounded linear operators of a non-zero Hilbert space H, we call (H, W) an *isometric representation* of S.

If S is left-cancellative, then we can have a specific isometric representation of S, called the *left regular isometric representation* on the Hilbert space $l^2(S)$. The left regular isometric representation $\mathcal{L} : S \to \mathcal{B}(l^2(S)), x \mapsto \mathcal{L}_x$ is defined by the equation

$$(\mathcal{L}_x f)(z) = \begin{cases} f(y), & \text{if } z = xy \text{ for some } y \in M, \\ 0, & \text{if } z \notin xM. \end{cases}$$

In fact, when $\{\delta_x \mid x \in S\}$ is the canonical orthonormal basis of the Hilbert space $l^2(S)$ defined by

$$\delta_x(y) = \begin{cases} 1, & \text{if } x = y, \\ 0, & \text{otherwise,} \end{cases}$$

Received February 14, 2005.
2000 Mathematics Subject Classification: 46L05, 47C15, 47B35.
Key words and phrases: isometric homomorphism, left regular isometric representation, reduced semigroup C^*-algebra, semigroup C^*-algebra, Toeplitz algebra.
This paper is partially supported by 2004 Research Fund of University of Ulsan.

we have that $\mathcal{L}_x(\delta_y) = \delta_{xy}$ for all $x, y \in S$.

In order to make things explicit, let us consider the semigroup $\mathbb{N}$ of all natural numbers. The isometry $\mathcal{L}_1$ of the left regular isometric representation $\mathcal{L} : \mathbb{N} \to \mathcal{B}(l^2(\mathbb{N})), x \mapsto \mathcal{L}_x$ is the unilateral shift of $l^2(\mathbb{N})$ on the canonical orthonormal basis $\{\delta_n \mid n \in \mathbb{N}\}$.

Among C^*-algebras generated by isometries, the C^*-algebra generated by the left regular isometric representation of a left cancellative semigroup can be considered as the appropriate analogue for the Toeplitz algebra. The C^*-algebra generated by the left regular isometric representation of a left cancellative semigroup S has several names. We shall call it the *reduced semigroup C^*-algebra*, and denote it $C^*_{red}(S)$ as in the paper [5].

Besides the reduced semigroup C^*-algebra, we will consider the semigroup C^*-algebra introduced by G. J. Murphy [8]. The *semigroup C^*-algebra* is obtained by enveloping all isometric representations of S, and is denoted by $C^*(S)$. From the construction of $C^*(S)$ we have naturally a canonical isometric homomorphism V from S to $C^*(S)$. It follows from the definition of the semigroup C^*-algebra it has the following universal property: If V is the canonical isometric homomorphism from S into the semigroup C^*-algebra $C^*(S)$, then for any isometric homomorphism W from S into a unital C^*-algebra B there exists a unique homomorphism from the semigroup C^*-algebra $C^*(S)$ into the unital C^*-algebra B sending a canonical isometry V_x to an isometry W_x for each $x \in S$.

Ever since L. A. Coburn proved his well-known theorem, which asserts that the C^*-algebra generated by a non-unitary isometry on a separable infinite dimensional Hilbert space does not depend on the particular choice of the isometry [1], many authors have taken an interest in the generalization of Coburn's theorem. It is sometimes called the uniqueness property of the C^*-algebras generated by isometries. If the C^*-algebras generated by isometries have the uniqueness property, the structures of those C^*-algebras are to some extent independent of the choice of isometries on a Hilbert space. The uniqueness property of C^*-algebras generated by isometries describes when the reduced semigroup C^*-algebra $C^*_{red}(S)$ and the semigroup C^*-algebra $C^*(S)$ are isomorphic or when the reduced semigroup C^*-algebra $C^*_{red}(S)$ has a universal property for certain kinds of isometric representations of S [2, 3, 4, 6, 7]. As good examples of the uniqueness property, we note that all the C^*-algebras generated by the isometric representations of the semigroup $\mathbb{N}$ of all natural numbers are isomorphic to the Toeplitz algebra. The C^*-algebras generated by one parameter semigroup of isometries and the Cuntz algebras are also remarkable examples of the C^*-algebras of isometries which have the uniqueness property.

A. Nica introduced the guasi-lattice group (G, S), the covariant isometry representations of semigroups and the amenability problem of quasi lattice ordered groups in order to find the condition that the reduced semigroup C^*-algebra $C^*_{red}(S)$ has a universal property for certain kinds of isometric representations of S [9]. The partially ordered group (G, S) is quasi-lattice ordered group if every finite subset of G with an upper bound in S has a least upper bound in S. The amenability problem, which asks when the left regular isometric representations have the universal property of the covariant isometric representations, was also investigated in [6]. The quasi-lattice ordered group is an appropriate concept for the universal property of the reduced semigroup C^*-algebras.

In this paper we show that the uniqueness property is much dependent on the order structure of the semigroup S, by analyzing the structure of the reduced semigroup C^*-algebra $C^*_{red}(P)$ of P and the semigroup C^*-algebra $C^*(P)$ of P, where $P = \{0, 2, 3, \dots\}$.

The semigroup $P = \{0, 2, 3, \dots\}$ is a generating subsemigroup of the integer group $\mathbb{Z}$. By Coburn's result it is known that the reduced semigroup C^*-algebra $C^*(\mathbb{N})$ of $\mathbb{N}$ is isomorphic to the semigroup $C^*(\mathbb{N})$ which is isomorphic to the Toeplitz algebra. We show that the reduced semigroup C^*-algebra $C^*_{red}(P)$ is isomorphic to the Toeplitz algebra $C^*(\mathbb{N})$, but we also show that $C^*_{red}(P)$ is not isomorphic to $C^*(P)$ by using the order structure of P in Proposition 2.7. Our semigroup P is abelian and really simple one, but not quasi lattice ordered.

2. Main result

Let G be a countable discrete group and S a subsemigroup of G with the unit e. We define an order on G as follows: Two elements x and y in S are comparable when $x \in yS$ or $y \in xS$. If x is contained in yS , then x is larger than y and we denote it by $y \leq x$. This relation makes (G, S) a pre-ordered group. If the unit e of S is the only invertible element of S, $(G, S, \leq)$ is a partially ordered group.

We can identify a maximal and a minimal element in S in the following sense: an element $x_0 \in S$ is maximal if and only if $x_0 \leq x$ implies that $x = x_0$ and an element x_1 is minimal if and only if $x \leq x_1$ implies that $x_1 = x$ for $x \in S$.

The reduced semigroup C^*-algebra $C^*(S)$ is generated by $\{\mathcal{L}_x \mid x \in$

$S\}$, where $\mathcal{L}$ is the left regular isometric representation of S. In fact, $C^*(S)$ is the closed linear span of $\{\mathcal{L}_{x_1}\mathcal{L}^*_{x_2}\cdots\mathcal{L}^*_{x_{2k}}\mathcal{L}_{x_{2k+1}} \mid x_i \in S\}$. If the semigroup S is the semigroup $\mathbb{N}$ of natural numbers, then $C^*(\mathbb{N})$ is the Toeplitz algebra. So sometimes the reduced semigroup C^*-algebra $C^*(S)$ is called a generalized Toeplitz algebra.

PROPOSITION 2.1. *If the unit of S is the only invertible element of S, then $\{\mathcal{L}_x\mathcal{L}^*_y \mid x, y \in S\}$ is linearly independent.*

Proof. First, we can see that $\mathcal{L}_x\mathcal{L}^*_y(\delta_y) = \delta_x$ for all $x, y \in S$, so $\mathcal{L}_x\mathcal{L}^*_y$ never can be zero for any $x,\ y \in S$.

Suppose that there exist $\{\lambda_i \mid \lambda_i \in \mathbb{C},\ 1 \le i \le n\}$ and $\{(x_i, y_i) \mid x_i, y_i \in S, (x_i, y_i) \ne (x_j, y_j) \text{ for } i \ne j,\ 1 \le i, j \le n\}$ such that

$$\sum_{i=1}^{n} \lambda_i \mathcal{L}_{x_i}\mathcal{L}^*_{y_i} = 0.$$

We can divide $\{y_i \mid 1 \le i \le n\}$ into two kinds of subsets; the one consists of elements which are comparable with any other element of $\{y_i \mid 1 \le i \le n\}$ and the other is the rest.

Let y_{i_0} be the element of $\{y_i \mid 1 \le i \le n\}$ which is not comparable with any other element of $\{y_i \mid 1 \le i \le n\}$. Since $\mathcal{L}_{x_i}\mathcal{L}^*_{y_i}(\delta_{y_{i_0}}) = \delta_{x_i}$ for $y_i = y_{i_0}$, we can have

$$\sum_{i=1}^{n} \lambda_i \mathcal{L}_{x_i}\mathcal{L}^*_{y_i}(\delta_{y_{i_0}}) = \lambda_{l_1}\delta_{x_{l_1}} + \cdots + \lambda_{l_k}\delta_{x_{l_k}} = 0,$$

if $y_{l_j} = y_{i_0}$ for $j = 1, \ldots, k$ and $\{l_1, \ldots, l_k\} \subset \{1, 2, \ldots, n\}$. Since $x_{l_r} \ne x_{l_s}$ if $l_r \ne l_s$ for $1 \le r, s \le k$, we have $\lambda_{l_j} = 0$ for $j = i, \ldots, k$.

Next, since $\{y_1, \ldots, y_n\}$ is finite, we can consider a minimal element y_{i_1} of some chain of $\{y_1, \ldots, y_n\}$. If we look at prudently the term $\sum \lambda_i \mathcal{L}_{x_i}\mathcal{L}^*_{y_i}(\delta_{y_{i_1}})$, we can see that only the terms with y_{i_1} may not be zero. So we have

$$\sum_{i=1}^{n} \lambda_i \mathcal{L}_{x_i}\mathcal{L}^*_{y_i}(\delta_{y_{i_1}}) = \lambda_{m_1}\delta_{x_{m_1}} + \cdots + \lambda_{m_p}\delta_{x_{m_p}} = 0,$$

if $y_{m_j} = y_{i_1}$ for $j = 1, \ldots, p$ and $\{m_1, m_2, \ldots, m_p\} \subset \{1, 2, \ldots, n\}$.

By the similar computation as the above, we can see that $\lambda_{m_j} = 0$ for $j = 1, \ldots, p$. So we can exclude those terms in the two cases and have a reduced form of $\sum_{j=1}^{<n} \lambda_j \mathcal{L}_{x_j} \mathcal{L}_{y_j}^*$. Continuing this process in the reduced form, we obtain that all λ_i's are zero because $\{y_1, \dots, y_n\}$ is finite. □

PROPOSITION 2.2. *If the unit e of S is the only invertible element, then $C_{red}^*(S)$ acts irreducibly on $l^2(S)$.*

Proof. If the operator T in $B(l^2(S))$ commutes with $C_{red}^*(S)$ and $[T_{x,y}]_{x,y \in S}$ denotes the matrix representation of T with respect to the canonical orthonormal basis $\{\delta_x \mid x \in S\}$ of $l^2(S)$, then we have

$$T_{x,y} = \langle T(\delta_y), \delta_x \rangle = \langle T\mathcal{L}_x^*(\delta_y), \delta_e \rangle = \langle T(\delta_e), \mathcal{L}_y^*(\delta_x) \rangle.$$

So $T_{x,y} \neq 0$ only when both $x \in yS$ and $y \in xS$. Since the unit of S is the only invertible element, it happens that both $x \in yS$ and $y \in xS$ only when $x = y$. So T is a diagonal operator. Furthermore we have $T_{x,x} = T_{e,e}$ for all $x \in S$ since $\mathcal{L}_x$ is an isometry. It follows that $C_{red}^*(S)$ acts irreducibly on $l^2(S)$. □

If $P = \{0, 2, 3, \dots\}$, then the ordered group $(\mathbb{Z}, P)$ is a partially ordered group, but the order structure of $(\mathbb{Z}, P)$ is different from that of $(\mathbb{Z}, \mathbb{N})$.

If we put $p_n = \mathcal{L}_n \mathcal{L}_n^*$ and $q_n = I - p_n$ for each $n \in P$, then the projection p_n is the orthogonal projection onto the closed linear span of $\{\delta_n, \delta_{n+2}, \dots\}$ and q_n is the orthogonal projection onto the closed linear span of $\{\delta_0, \delta_2, \delta_3, \dots, \delta_{n-1}\}$.

PROPOSITION 2.3. *Let $\mathcal{B}$ be the C^*-subalgebra of $C_{red}^*(P)$ generated by $\{p_n \mid n \in P\}$. Then the strong closure of $\mathcal{B}$ is a maximal abelian von-Neumann algebra of $\mathcal{B}(l^2(P))$.*

Proof. Since p_n and p_m commute for each $n, m \in P$, $\mathcal{B}$ is abelian. Furthermore, if we put $\delta = \delta_0 + \sum_{k=2}^{\infty} \delta_k / 2^k$, then δ is a cyclic vector for $\mathcal{B}$. If we put $\mathcal{B}' = \{x \in \mathcal{B}(l^2(P)) \mid xy = yx \text{ for all } y \in \mathcal{B}\}$, then $\mathcal{B}$ is abelian. And we can see by an easy computation that δ is a separating vector for $\mathcal{B}$. This implies that $\mathcal{B}'$ is maximal abelian in $\mathcal{B}(l^2(P))$. Thus from the maximal abelianess of $\mathcal{B}'$ we have that $\mathcal{B}'$ is equal to the double commutant $\mathcal{B}''$ of $\mathcal{B}$. Since $\mathcal{B}$ has the identity operator of $\mathcal{B}(l^2(P))$, $\mathcal{B}''$ is the strong closure of $\mathcal{B}$. □

The group C^*-algebra of an abelian group is, of course, itself abelian and so not very interesting from the point of view of C^*-theory. But the reduced semigroup C^*-algebra and the semigroup C^*-algebra may not be abelian, and are moreover primitive for a large class of abelian semigroups.

PROPOSITION 2.4. *The commutator ideal $\mathcal{Z}(C^*_{red}(P))$ of $C^*_{red}(P)$ is the algebra $\mathcal{K}(l^2(P))$ of compact operators on the Hilbert space $l^2(P)$.*

Proof. Since $C^*_{red}(P)$ is generated by $\mathcal{L}_2$ and $\mathcal{L}_3$, it is enough to see how these operators act on $l^2(P)$. The operator $I - \mathcal{L}_2\mathcal{L}_2^*$ is of finite rank, and so is contained in $\mathcal{K}(l^2(P))$. Therefore, $\mathcal{K}(l^2(P))$ and the commutator ideal $\mathcal{Z}(C^*_{red}(P))$ have non-empty intersection. Since $C^*_{red}(P)$ acts irreducibly on $l^2(P)$, the commutator ideal $\mathcal{Z}(C^*_{red}(P))$ contains the algebra $\mathcal{K}(l^2(P))$ of compact operators [10, 6.1.4].

Furthermore $C^*_{red}(P)/\mathcal{K}(l^2(P))$ is abelian because $I - \mathcal{L}_2\mathcal{L}_2^*$ and $I - \mathcal{L}_3\mathcal{L}_3^*$ are contained in $\mathcal{K}(l^2(P))$. Hence $\mathcal{Z}(C^*_{red}(P))$ is equal to $\mathcal{K}(l^2(P))$. □

Though there are many interesting simple group C^*-algebras, the reduced semigroup C^*-algebras are rarely simple for a large and natural class of semigroups. In fact, there are many prime reduced semigroup C^*-algebras and it is still open when the reduced semigroup C^*-algebra is prime. We can see that $C^*_{red}(P)$ is prime from the Proposition 2.4.

The semigroup P has two generators, so apparently $C^*_{red}(P)$ is generated by two non-unitary isometries. However, the following theorem shows us that $C^*_{red}(P)$ is generated by a non-unitary isometry.

THEOREM 2.5. *The C^*-algebra $C^*_{red}(P)$ is generated by a single non-unitary isometry.*

Proof. The operator $\mathcal{L}_2^*\mathcal{L}_3$ acts on $l^2(P)$ as follows:

$$\mathcal{L}_2^*\mathcal{L}_3(\delta_n) = \begin{cases} 0, & \text{if } n = 0, \\ \delta_{n+1}, & \text{if } n \neq 0. \end{cases}$$

Hence $\mathcal{L}_2^*\mathcal{L}_3$ translates every element of the orthonormal basis $\{\delta_0, \delta_2, \delta_3, \dots\}$ of $l^2(P)$ except δ_0. Let K be the compact operator defined by

$$K(\delta_n) = \begin{cases} \delta_2, & \text{if } n = 0, \\ 0, & \text{if } n \neq 0. \end{cases}$$

Put $U = \mathcal{L}_2^*\mathcal{L}_3 + K$. By Proposition 2.3, U is contained in $C^*_{red}(P)$. U^*U is the identity operator on $l^2(P)$ because $\mathcal{L}_3^*\mathcal{L}_2\mathcal{L}_2^*\mathcal{L}_3$ is the projection onto the closed subspace spanned by $\{\delta_n \mid n \in P,\ n \neq 0\}$, K^*K is the projection onto the closed subspace $\mathbb{C}\delta_0$ and all other operators in the terms of U^*U are zero. Similarly, we see that UU^* is the projection onto the closed subspace spanned by $\{\delta_2, \delta_3, \dots\}$. In fact the operator

U sends δ_0 to δ_2 and δ_n to δ_{n+1} for $n \neq 0$. So the operator U is the unilateral shift on $l^2(P)$ with respect to the canonical orthonormal basis $\{\delta_n \mid n \in P\}$. Let $\mathcal{T}$ be the C^*-subalgebra of $C^*_{red}(P)$ generated by U. If we consider the compact operators T_1 and T_2 defined by

$$T_1(\delta_n) = \begin{cases} -\delta_3, & \text{if } n = 0, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad T_2(\delta_n) = \begin{cases} \delta_2, & \text{if } n = 0, \\ 0, & \text{otherwise,} \end{cases}$$

then we can show that $\mathcal{L}_2 = U^2 + T_1 + T_2$ because $U^2 + T_1 + T_2(\delta_n) = \delta_{n+2}$ for each $n \in P$. Similarly we can show that $\mathcal{L}_3 = U^3 + T_3 + T_4$ where

$$T_3(\delta_n) = \begin{cases} -\delta_4, & \text{if } n = 0, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad T_4(\delta_n) = \begin{cases} \delta_3, & \text{if } n = 0, \\ 0, & \text{otherwise.} \end{cases}$$

Since the algebra $\mathcal{T}$ contains the compact operator algebra $\mathcal{K}(l^2(P))$, the elements $U^2 + T_1 + T_2$ and $U^3 + T_3 + T_4$ are contained in $\mathcal{T}$. Hence we conclude that $C^*_{red}(P)$ is same as the algebra $\mathcal{T}$ because $C^*_{red}(P)$ is generated by $\mathcal{L}_2$ and $\mathcal{L}_3$. □

COROLLARY 2.6. *$C^*_{red}(P)$ is isomorphic to the Toeplitz algebra.*

From the point of view of C^*-algebras, amenability means that the canonical coincidence of two kinds C^*-algebras: One is the universal object obtained by enveloping a certain class of representation and the other is associated to a specific representations of the class.

A. Nica introduced the quasi-lattice ordered group (G, S), the covariant isometric representations of semigroups and the amenability problem of quasi-lattice ordered groups for the universal property of the reduced semigroup C^*-algebra $C^*_{red}(S)$.

The partially ordered group (G, S) is said to be *quasi-lattice ordered* if the following is satisfied: If $x_1, x_2, \ldots, x_n$ in G which have common upper bounds in S for any $n \geq 1$, they also have a least common upper bound in S.

The above condition can be expressed in another form consisting of two conditions:

1. Any x in SS^{-1} has a least upper bound in S.
2. Any s, t in S with the common upper bound has a least common upper bound.

If (G, S) is a quasi-lattice ordered group and $x_1, x_2, \ldots, x_n$ in G have a common upper bound in S, then their least common upper bound in S will be denoted by $\sigma(x_1, x_2, \ldots, x_n)$.

An isometric representation V of S on the Hilbert space H is said to be *covariant* if

$$V(s)V(t) = \begin{cases} V(\sigma(s,t)), & \text{if } s \text{ and } t \text{ have common upper bound} \\ 0, & \text{otherwise.} \end{cases}$$

It is clear that for the semigroup S of the quasi-lattice ordered group (G, S) the left regular isometric representation $\mathcal{L}$ of the semigroup S is a covariant representation. The reduced semigroup C^*-algebra $C^*_{red}(S)$ naturally plays the role of reduced C^*-algebra in the class of C^*-algebras of covariance isometric representations. Nica also constructed the full C^*-algebra $C^*_{cov}(G, S)$ of the C^*-algebras generated by enveloping conariant representations of (G, S) [9].

The quasi-lattice ordered group (G, S) is amenable if the reduced semigroup C^*-algebra $C^*_{red}(S)$ is isomorphic to $C^*_{cov}(G, S)$.

It is known that every abelian quasi-lattice ordered group is amenable. Our semigroup P is very simple and abelian. But it is not quasi-lattice ordered, because 2 and 3 in the semigroup $P = \{0, 2, 3, 4, \dots\}$ have common upper bounds 5 and 6. Since 5 and 6 are not comparable, 2 and 3 does not have a least common upper bound.

PROPOSITION 2.7. *The reduced semigroup C^*-algebra $C^*_{red}(P)$ is not isomorphic to the semigroup C^*-algebra $C^*(P)$.*

Proof. The left regular isometric representation $\mathcal{L}$ satisfies the relation:

$$\mathcal{L}_2^*\mathcal{L}_3(I - \mathcal{L}_2\mathcal{L}_2^*)(I - \mathcal{L}_3\mathcal{L}_3^*) = 0. \tag{1}$$

Let W be the isometric representation of P defined by $W_n = \mathcal{S}^n$ for $n = 0, 2, 3, \dots$, where $\mathcal{S}$ is the unilateral shift on $l^2(\mathbb{N})$. This representation does not satisfy the above relation, i.e.,

$$\mathcal{S}^{*2}\mathcal{S}^3(I - \mathcal{S}^2\mathcal{S}^{*2})(I - \mathcal{S}^3\mathcal{S}^{*3}) \neq 0. \tag{2}$$

Let $\mathcal{W}$ be the C^*-algebra generated by the isometric representation W of P. Since $C^*(P)$ has the universal property, there is a homomorphism from $C^*(P)$ to $\mathcal{W}$ sending V_n to W_n for each $n \in P$. But there does not exist a homomorphism from $C^*_{red}(P)$ to $\mathcal{W}$ sending $\mathcal{L}_n$ to W_n for each $n \in P$ because of equations (1) and (2). So the reduced semigroup C^*-algebra $C^*_{red}(P)$ is not isomorphic to the semigroup C^*-algebra $C^*(P)$. □

References

[1] L. A. Coburn, *The C^*-algebra generated by an isometry, I*, Bull. Amer. Math. Soc. **73** (1967), 722–726; *II*, Trans. Amer. Math. Soc. **137** (1969), 211–217.

[2] J. Cuntz, *Simple C^*-algebras generated by isometries*, Comm. Math. Phys. **57** (1977), no. 2, 173–185.

[3] K. R. Davidson and D. R. Pitts, *The algebraic structure of non-commutative analytic Toeplitz algebras*, Math. Ann. **311** (1998), no. 2, 275–303.

[4] R. G. Douglas, *On the C^*-algebra of a one-parameter semigroup of isometries*, Acta Math. **128** (1972), no. 3-4, 143–151.

[5] S. Y. Jang, *Reduced crossed products by semigroups of automorphisms*, J. Korean Math. Soc. **36** (1999), no. 1, 97–107.

[6] M. Laca and I. Raeburn, *Semigroup crossed products and the Toeplitz algebras of nonabelian groups*, J. Funct. Anal. **139** (1996), no. 2, 415–440.

[7] P. S. Muhly, *A structure theory for isometric representations of a class of semigroups*, J. Reine Angew. Math. **255** (1972), 135–154.

[8] G. J. Murphy, *Crossed products of C^*-algebras by semigroups of automorphisms*, Proc. London Math. Soc. (3) **68** (1994), no. 2, 423–448.

[9] A. Nica, *C^*-algebras generated by isometries and Wiener-Hopf operators*, J. Operator Theory **27** (1992), no. 1, 17–52.

[10] G. K. Pedersen, *C^*-algebras and their automorphism groups*, London Mathematical Society Monograph 14, Academic Press, London, 1979.

Department of Mathematics, University of Ulsan, Ulsan 680-748, Korea
E-mail: jsym@ulsan.ac.kr