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Robust Indirect Adaptive Fuzzy Controller for Balancing and Position Control of Inverted Pendulum System

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Abstract

In the paper a robust indirect adaptive fuzzy controller is proposed for balancing and position control of the inverted pendulum system. Because balancing control rules of the pendulum and position control rules of the cart can be opposite, it is difficult to design an adaptive fuzzy controller that satisfy both objectives. To stabilize the pendulum at a specified position, the proposed fuzzy controller consists of a robust indirect adaptive fuzzy controller for balancing and a supervisory fuzzy controller which emulates heuristic control strategy and arbitrate two control objectives. It is proved that the signals in the overall system are bounded. Simulation results are given to verify the proposed adaptive fuzzy control method.

Key words : Robust indirect adaptive fuzzy control, Robustness, Balancing and position control, Inverted pendulum.

1. Introduction

The inverted pendulum system is a typical example of an unstable nonlinear control system which is difficult to control. In fact, the inverted pendulum problem has been widely used as an example to test new control concepts as well as demonstrate the effectiveness of modern control theory. Control objectives of the inverted pendulum system can be swinging-up the pendulum, balancing of the pendulum at the upright position, and regulation of the cart at an arbitrary specified position. There have been a large amount of research efforts: stabilization of double inverted pendulum on an inclined rail[1], swing up and stabilization of the pendulum with scheduled control input[2], attitude control of a triple-inverted pendulum[3], control of the pendulum with a angular motion type cart[4], swing up control of the pendulum with tree search technique[5], and linear control of the pendulum with a random search[6].

Also, many researchers have used stabilizing problem of the inverted pendulum for demonstrating the effectiveness of their adaptive and learning control methods. After Barto et. al.[7] proposed the neural network-based balancing controller, a number of neural network-based learning controllers have been developed [8, 9, 10, 17]. Kim and Bien[14] proposed balancing controller based on the adaptive fuzzy control method. Also, Ha[11] proposed a fuzzy control scheme with multiple rule bases and rule supervisors for swing-up, balancing, and position control. However, while taking nonlinear nature of the pendulum system into account, it is difficult to design rule supervisors and arbitrating rule bases which are implemented to achieve above three objectives. Therefore, most of researches are focused on the balancing control of the pendulum system.

In this paper, a balancing and position control method of an inverted pendulum system is designed using the indirect adaptive fuzzy control. Moreover, the robustness of the proposed fuzzy control method is considered. To stabilize the pendulum at an arbitrary specified position, we propose an indirect adaptive fuzzy control system with a robust adaptation law and a supervisory fuzzy control law which emulates heuristic control strategy and arbitrate two control objectives. It is showed that all the signals in the overall pendulum control system are bounded. In Section 2, we describe multi-objective control problem of the inverted pendulum system. In Section 3, to solve the control problem of the pendulum system, an indirect adaptive fuzzy control method is proposed. In Section 4, simulation results are given to verify the effectiveness of the proposed method. Section 5 concludes this paper.

2. Problem Description

Let x_1 be the angle of the pendulum with respect to the vertical line, x_2 be the angular velocity, x_3 be the position of the cart and x_4 be the position velocity. Then the dynamic equations of the inverted pendulum system are [12][17]

$$\dot{x_1} = x_2,$$

$$\dot{x_2} = \frac{gsinx_1 - \frac{mlx_2^2 cosx_1 sinx_1}{M+m} + \frac{cosx_1}{M+m}u}{l(\frac{4}{3} - \frac{mcos^2 x_1}{M+m})}, \tag{1}$$

$$\dot{x_3} = x_4 ,$$

$$\dot{x_4} = \frac{mlx_2^2 \sin x_1 - \dot{x_2} cosx_1 - bx_4 + u}{M+m} \tag{2}$$

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where $g = 9.8m/s^2$ is the acceleration due to gravity, M is the mass of the cart, m is the mass of the pole, l is the length of the pole, u is the applied force.

In this paper, balancing of the pendulum and position control of the cart is considered. Swing up controllers of pendulum were proposed by some researchers [2, 6, 11]. It can be easily designed by using fuzzy rule tables which are constructed by examining mathematical model and understanding its dynamic behavior. Also, a fuzzy controller for balancing of the pendulum and a fuzzy controller for regulation of the cart can be easily designed by observing the dynamic behavior of the inverted pendulum system, independently.

However, it is difficult to design fuzzy controller that satisfies two control objectives, that is, balancing of the pendulum and regulation of the cart at arbitrary position. To solve the problem, Ha [11] uses another rule base to arbitrate both objectives. Because control rules for balancing of the pendulum and control rules for position control of the cart can be opposite, it is very difficult to obtain the arbitration rules. In addition, it is more difficult to design an adaptive fuzzy control algorithm that satisfy both objectives. Therefore many adaptive fuzzy control methods are only applied to the balancing control of the pendulum system.

In the paper, to solve this multi-objective adaptive control problem of pendulum system, we propose a robust indirect adaptive fuzzy control method with supervisory fuzzy position control law. Also, we show the robustness of the proposed control system.

3. Design of a Robust Indirect Adaptive Fuzzy Controller

3.1 Robust indirect adaptive fuzzy control with supervisory fuzzy control law

We consider two control objectives of the inverted pendulum system. The first objective is to control the cart at its target position from an arbitrary initial position. The second objective is to balance the pendulum at the upright position. To achieve this two objectives, an adaptive fuzzy control system with a supervisory fuzzy controller is developed considering its dynamic behavior as well as heuristic control strategy to stabilize the pendulum at a specified position. Fig. 1 shows heuristic position control strategy of the pendulum system. The heuristic control algorithm can be described as follows.

Step 1. Balance the pendulum at an arbitrary position.
Step 2. If cart is not at the specified position, move cart toward the opposite direction of the specified position until angle error of pendulum reaches to certain value that is determined according to the distance from the desired position.
Step 3. Stabilize the pendulum according to the appropriate performance indices that are selected based on the balancing experiences.
Step 4. If the cart is at the specified position, stop. Otherwise, continue with Step 2.

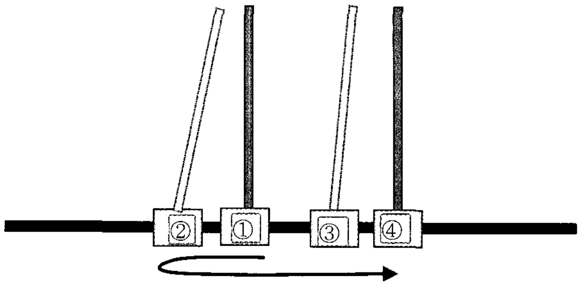

Fig 1. Graphical description of heuristic position control strategy of the inverted pendulum system.

The proposed controller is designed based on the above heuristic control strategy by using the fuzzy control theory. Overall structure of the proposed controller is shown in Fig.2.

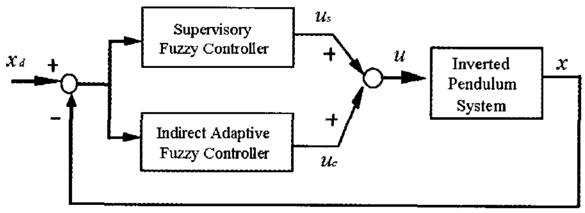

Fig 2. Overall structure of the proposed control system.

Fuzzy control can be used as an effective means to capture the human expertized knowledge and achieve multiple control objectives of nonlinear systems. Therefore, the fuzzy position controller is designed based on the above heuristic control strategy by using the fuzzy control theory. Table 1 shows the fuzzy rule table for position control of the inverted pendulum system. From the heuristic position control strategy, the fuzzy position controller should be activated after balancing the pendulum. Supervisory controller decides whether the system is in the balancing phase or position control phase by monitoring the plant states. Then supervisory controller activate the fuzzy position controller according to system state as follows.

$$u_s(x_3, x_4) = \begin{cases} u_p, & \text{if } |x_1| \le \alpha \text{ and } |x_2| \le \beta, \\ 0, & \text{if } |x_1| > \alpha \ or \ |x_2| > \beta, \end{cases} \quad (3)$$

where α and β are arbitrary positive constants.

Table 1. Fuzzy rule table for the position control of the inverted pendulum system.

$u_p = FPC(x_3, x_4)$		x_3				
		NB	NS	ZO	PS	PB
x_4	PB	ZO	ZO	ZO	PB	PB
	PS	ZO	ZO	ZO	PS	PB
	ZO	NS	NS	ZO	PS	PS
	NS	NB	NS	ZO	ZO	ZO
	NB	NB	NB	ZO	ZO	ZO

Balancing control rules can be learned using the indirect adaptive fuzzy algorithm. However, the control action in the position control phase generates similar situation produced by external disturbance and the balancing rules will be modified according to the performance decision as shown in Fig. 3 [13]. It should be noted that the balancing rules will be continuously modified according to the performance evaluation as long as the states are disturbed by the control action in the position control phase, even though the states may move satisfactorily toward the band. This implies that when there exist frequent control actions by the forced disturbance, the control performance can be eventually deteriorated by the repeated modification of balancing rules [13][14].

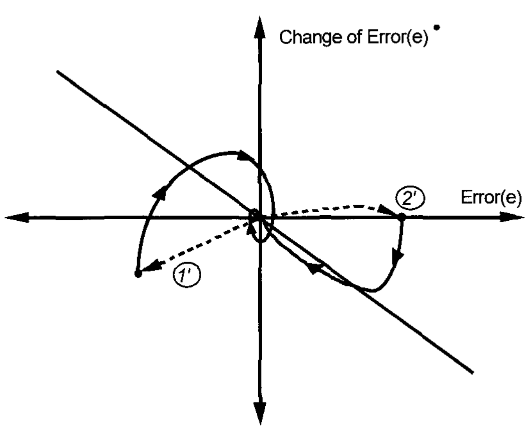

Fig 3. State trajectories in the position control phase.

To overcome this problem, the robust indirect adaptive fuzzy control method[14] is used to learn the balancing rules of the pendulum. The system (1) can be described by a class of single-input nonlinear systems as

$$\ddot{x}_1 = f(x_1,\dot{x}_1) + g(x_1,\dot{x}_1)\,u, \quad y = x_1, \tag{4}$$

where f and g are bounded continuous functions, and $u \in R$ and $y \in R$ are input force and pendulum angle of the inverted pendulum system, respectively. Let $\underline{x} = (x,\dot{x})^T = (x_1,x_2)^T$. Let us denote the output tracking error $e = y_m - y$ where y_m is a given reference signal. The error state vector $\underline{e} = (e,\dot{e})^T = (e_1,e_2)^T$. Also, let us define the tracking error metric as $s = \underline{c}^T\underline{e}$ where $\underline{c} = (c_1,c_2)^T$ such that all roots of $h(p) = c_2 p + c_1 = 0$ are in the left-half plane. We replace $f(\underline{x})$ and $g(\underline{x})$ by the fuzzy logic systems $u_f(\underline{x}/\underline{\theta}_f)$ and $u_g(\underline{x}/\underline{\theta}_g)$,

$$u_f(\underline{x}/\underline{\theta}_f) = \hat{f}(\underline{x}/\underline{\theta}_f) = \underline{\xi}_f^T(\underline{x})\underline{\theta}_f, \quad u_g(\underline{x}/\underline{\theta}_g) = \hat{g}(\underline{x}/\underline{\theta}_g) = \underline{\xi}_g^T(\underline{x})\underline{\theta}_g, \tag{5}$$

where $\underline{\xi}_f(\underline{x})$ and $\underline{\xi}_g(\underline{x})$ are fuzzy basis vectors, $\underline{\theta}_f$ and $\underline{\theta}_g$ are corresponding parameter vectors of each fuzzy system. Then, the following indirect adaptive fuzzy control law can be applied [14].

$$u_c = \frac{1}{u_g(\underline{x}/\underline{\theta}_g)}\left[-u_f(\underline{x}/\underline{\theta}_f) + \ddot{y}_m + k_d s + \sum_{i=1}^{2} c_i e_{i+1}\right], \tag{6}$$

where k_d is positive constants such that all roots of $h(p) = p^2 + (k_d c_2 + c_1)p + k_d c_1 = 0$ are in the left- half plane. Consequently, the final control law is

$$u = u_c + u_s(x_3, x_4). \tag{7}$$

where $u_s(x_3, x_4)$ denotes the output of supervisory fuzzy position controller (3).

Applying (7) to (4), after some manipulations, we obtain the tracking error dynamic equation

$$\ddot{e} = -\underline{k}^T\underline{e} + [u_f(\underline{x}/\underline{\theta}_f) - f(\underline{x})] + [u_g(\underline{x}/\underline{\theta}_g) - g(\underline{x})]u_c - g(\underline{x})u_s \tag{8}$$

where $\underline{k} = (k_1,k_2)^T = (k_d c_1, k_d c_2 + c_1)^T$ such that all roots of $h(p) = p^2 + k_2 p + k_1 = 0$ are in the left-half plane.

Let us define the optimal parameter vectors $\underline{\theta}_f^*$ and $\underline{\theta}_g^*$ as follows:

$$\underline{\theta}_f^* \equiv argmin_{\underline{\theta}_f \in \Omega_{\theta_f}}[sup_{\underline{x} \in \Omega_x} \mid f(\underline{x}) - u_f(\underline{x}/\underline{\theta}_f) \mid], \tag{9}$$

$$\underline{\theta}_g^* \equiv argmin_{\underline{\theta}_g \in \Omega_{\theta_g}}[sup_{\underline{x} \in \Omega_x} \mid g(\underline{x}) - u_g(\underline{x}/\underline{\theta}_g) \mid], \tag{10}$$

where Ω_{θ_f}, Ω_{θ_g} and Ω_x are the sets of suitable bounds on $\underline{\theta}_f$, $\underline{\theta}_g$ and $\underline{x}$, respectively. Let us be the approximation error

$$w = (f(\underline{x}) - u_f(\underline{x}/\underline{\theta}_f^*)) - (g(\underline{x}) - u_g(\underline{x}/\underline{\theta}_g^*))u_c. \tag{11}$$

Then the tracking error dynamic equation(7) can be rewritten as

$$\ddot{e} = -\underline{k}^T\underline{e} - \underline{\xi}_f^T(\underline{x})\underline{\phi}_f - \underline{\xi}_g^T(\underline{x})\underline{\phi}_g u_c - g(\underline{x})u_s - w, \tag{12}$$

where $\underline{\phi}_f = \underline{\theta}_f^* - \underline{\theta}_f$ and $\underline{\phi}_g = \underline{\theta}_g^* - \underline{\theta}_g$. From (12), it is noted that, since ξ_f and ξ_g are bounded, when $\underline{\phi}_f, \underline{\phi}_g, u_c, w$, and $g(\underline{x})u_s$ are bounded, the tracking error $\underline{e}$ will be bounded.

Also, to overcome continual improper modification of the parameters of fuzzy system, we consider an adaptive law [14]

$$\dot{\underline{\theta}}_f = -\eta_1 \cdot (\dot{s} + \lambda s) \cdot \xi_f(\underline{x}), \quad \dot{\underline{\theta}}_g = -\eta_2 \cdot (\dot{s} + \lambda s) \cdot \xi_g(\underline{x}) \cdot u_c, \tag{13}$$

where η_1 and η_2 are adaptation rates and λ is a positive constant. Using the control law (7), time derivative of the tracking error metric can then be written as

$$\dot{s} = -k_d s - \underline{\xi}_f^T(\underline{x})\underline{\phi}_f - \underline{\xi}_g^T(\underline{x})\underline{\phi}_g u_c - g(\underline{x})u_s - w. \tag{14}$$

3.2 Robustness analysis

Let us assume that $g(\underline{x})u_s$ is bounded by d, $d + |w| < \epsilon$, and $\lambda = k_d + 1/k_d$. For clarification, the notations are simplified as follows: $u_f \triangle u_f(\underline{x}/\underline{\theta}_f)$, $u_f^* \triangle u_f(\underline{x}/\underline{\theta}_f^*)$, $u_g \triangle u_g(\underline{x}/\underline{\theta}_g)$, and $u_g^* \triangle u_g(\underline{x}/\underline{\theta}_g^*)$.

Then, we show the robustness of proposed adaptive fuzzy control method. Consider the Lyapunov function candidate as

$$V(s,\underline{\phi}_f,\underline{\phi}_g) = \frac{1}{2}\left(\frac{1}{k_d}s^2 + \frac{1}{\eta}\underline{\phi}_f^T\underline{\phi}_f + \underline{\phi}_g^T\underline{\phi}_g\right). \tag{15}$$

Differentiating the Lyapunov function V with respect to time,

$$\dot{V}=\frac{1}{k_d}s\dot{s}+\frac{1}{\eta_1}\dot{\underline{\phi}}_f^T\underline{\phi}_f+\frac{1}{\eta_2}\dot{\underline{\phi}}_g^T\underline{\phi}_g$$
$$=\frac{1}{k_d}s\dot{s}+(\dot{s}+\lambda s)\,\underline{\xi}_f^T(\underline{\theta}_f^*-\underline{\theta}_f)$$
$$+(\dot{s}+\lambda s)\,\underline{\xi}_g^T(\underline{\theta}_g^*-\underline{\theta}_g)$$
$$<-[s^2-\frac{1}{k_d}\epsilon|s|+((u_f^*-u_f)+(u_g^*-u_g)u_c)^2$$
$$-\epsilon|(u_f^*-u_f)+(u_g^*-u_g)u_c|\,] \quad (16)$$
$$<0,\ \forall\ |s|>(\frac{1}{2}+\frac{1}{k_d})\epsilon,$$

$$or\ |(u_f^*-u_f)+(u_g^*-u_g)u_c|>(1+\frac{1}{2k_d})\epsilon. \quad (17)$$

From (17), $\dot{V}<0$ outside a compact region Ω, where the set Ω is defined as

$$\Omega\triangleq\{(s,\underline{\phi}_f,\underline{\phi}_g)\,|\,|s|\le(\frac{1}{2}+\frac{1}{k_d})\epsilon,$$
$$|\underline{\xi}_f^T\underline{\phi}_f+\underline{\xi}_g^T\underline{\phi}_g u_c|\le(1+\frac{1}{2k_d})\epsilon\}. \quad (18)$$

Since $V(s,\underline{\phi}_f,\underline{\phi}_g)$ is a scalar function with continuous partial derivatives in Ω^c and satisfy $V(s,\underline{\phi}_f,\underline{\phi}_g)>0$, $\dot{V}(s,\underline{\phi}_f,\underline{\phi}_g)\le 0$ and $V(s,\underline{\phi}_f,\underline{\phi}_g)$ belongs to class K_∞ [15], s is shown to be uniformly bounded. From the input-to-state stability theory [15] and the tracking error metric $c_2\dot{e}+c_1e=s$, if the input s is bounded, then tracking error e and $\dot{e}$ are bounded. Also, boundedness of tracking errors means boundedness of states $\underline{x}$.

From (13) and (14), the adaptation law can be rewritten as

$$\begin{bmatrix}\dot{\underline{\phi}}_f\\ \dot{\underline{\phi}}_g\end{bmatrix}=-\begin{bmatrix}\eta_1\,\underline{\xi}_f\,\underline{\xi}_f^T & \eta_1\,\underline{\xi}_f\,\underline{\xi}_g^T u_c\\ \eta_2\,\underline{\xi}_g\,\underline{\xi}_f^T u_c & \eta_2\,\underline{\xi}_g\,\underline{\xi}_g^T u_c^2\end{bmatrix}\cdot\begin{bmatrix}\underline{\phi}_f\\ \underline{\phi}_g\end{bmatrix}$$
$$+(\frac{s}{k_d}-w-g\,u_s)\begin{bmatrix}\eta_1\,\underline{\xi}_f\\ \eta_2\,\underline{\xi}_g\,u_c\end{bmatrix}$$
$$=-\begin{bmatrix}\eta_1 I & 0\\ 0 & \eta_2 I\end{bmatrix}\begin{bmatrix}\underline{\xi}_f\\ \underline{\xi}_g u_c\end{bmatrix}\cdot[\underline{\xi}_f^T\ \underline{\xi}_g^T u_c]\begin{bmatrix}\underline{\phi}_f\\ \underline{\phi}_g\end{bmatrix}$$
$$+(\frac{s}{k_d}-w-g\,u_s)\begin{bmatrix}\eta_1 I & 0\\ 0 & \eta_2\end{bmatrix}\begin{bmatrix}\underline{\xi}_f\\ \underline{\xi}_g u_c\end{bmatrix} \quad (19)$$

Let us simplify the expressions as

$$\underline{\phi}_{fg}=\begin{bmatrix}\underline{\phi}_f\\ \underline{\phi}_g\end{bmatrix},\ \underline{\xi}_{fg}=\begin{bmatrix}\underline{\xi}_f\\ \underline{\xi}_g u_c\end{bmatrix},\quad \text{and}\quad \Gamma=\begin{bmatrix}\eta_1 I & 0\\ 0 & \eta_2\end{bmatrix}. \quad (20)$$

Then, adaptation law (19) can be rewritten as

$$\dot{\underline{\phi}}_{fg}=-\Gamma\,\underline{\xi}_{fg}\,\underline{\xi}_{fg}^T\,\underline{\phi}_{fg}+(\frac{s}{k_d}-w-g\,u_s)\Gamma\,\underline{\xi}_{fg}\,. \quad (21)$$

Since $\underline{\xi}_{fg}$ is a piecewise continuous and bounded, when $\underline{\xi}_{fg}$ is persistently exciting, it can be shown [14] that the nominal system of (21), $\dot{\underline{\phi}}_{fg}=-\Gamma\,\underline{\xi}_{fg}\,\underline{\xi}_{fg}^T\,\underline{\phi}_{fg}$, has an exponentially stable equilibrium state $\underline{\phi}_{fg}=0$. Since $\underline{\xi}_{fg}$, s, w and gu_s are bounded, the remain term $\rho_{fg}=(\frac{s}{k_d}-w-gu_s)\Gamma\underline{\xi}_{fg}$ is bounded. Then, it can be shown that if the system (21) has the exponentially stable nominal dynamics and the $\underline{\rho}_{fg}$ is bounded, then the parameter error vector $\underline{\phi}_{fg}$ is bounded [15].

From the equation (16),

$$\dot{V}<-\frac{1}{2}s^2-\frac{1}{2}[s^2-2\frac{\epsilon}{k_d}s+(\frac{\epsilon}{k_d})^2]+\frac{1}{2}(\frac{\epsilon}{k_d})^2$$
$$-\frac{1}{2}((u_f^*-u_f)+(u_g^*-u_g)u_c)^2$$
$$-\frac{1}{2}[((u_f^*-u_f)+(u_g^*-u_g)u_c)^2$$
$$-2\epsilon((u_f^*-u_f)+(u_g^*-u_g)u_c)+\epsilon^2]+\frac{1}{2}\epsilon^2,$$
$$<-\frac{1}{2}s^2-\frac{1}{2}((u_f^*-u_f)+(u_g^*-u_g)u_c)^2$$
$$+\frac{1}{2}(1+\frac{1}{k_d^2})\epsilon^2. \quad (22)$$

Integrating both side of (22), we obtain

$$\int_0^\infty[s^2+((u_f^*-u_f)+(u_g^*-u_g)u_c)^2]\,dt$$
$$<2V_0-2V_\infty+\int_0^\infty(1+\frac{1}{k_d^2})\,\epsilon^2\,dt. \quad (23)$$

If w and u_s tend to zero, $\epsilon\in\mathcal{L}_2$. Because V_0, V_∞ are bounded, if $\epsilon\in\mathcal{L}_2$, $s\in\mathcal{L}_2$. Also, from (14), we can find that $\dot{s}\in\mathcal{L}_\infty$. Therefore, from Barbalat's lemma [16], s converge to zero as $t\to\infty$. Since s converges to zero, the tracking error also converges to zero.

If the control law (7) and the adaptive law(13) are applied to control the inverted pendulum system(4), we can prove that the tracking error of pendulum is uniformly bounded and the error bound depends on the bound on approximation error w and additional supervisory fuzzy controller u_s. Also, whenever w and u_s tend to zero, tracking error converges to zero. Therefore, the proposed adaptive fuzzy controller can balance the pendulum within some error bounds. From the dynamics (2), we can see that the position state error is also bounded.

4. Simulation Example

We apply the proposed adaptive fuzzy controller to the balancing and position control of the inverted pendulum system (1). We choose $M=1\,Kg$, $m=0.1\,Kg$, $b=0.1$ and $l=0.5\,m$. Let the initial state $\underline{x}(0)=(8°,0,-20,0)^T$ and the parameter vectors $\underline{\theta}_f(0)=\underline{0}$, $\underline{\theta}_g(0)=\underline{1.2}$. We choose $\eta_1=1.0$, $\eta_2=0.01$ and $\lambda=1$. We directly integrate the differential equations of the closed-loop system and the adaptive law with a sampling time of 0.01 sec.

The simulation result for pendulum balancing and cart position control is shown in Fig. 4. This result shows that the proposed indirect adaptive fuzzy controller can be successfully applied to the balancing and position control of the inverted pendulum system. We can see that both angle error of the pendulum and position error of the cart are bounded. Fig. 5 shows the position state trajectory of the cart in the phase state plane. Fig. 6 shows the angle state trajectory of the pendulum in the state phase plane. The results show that the proposed adaptive controller can balance the pendulum near the specified position.

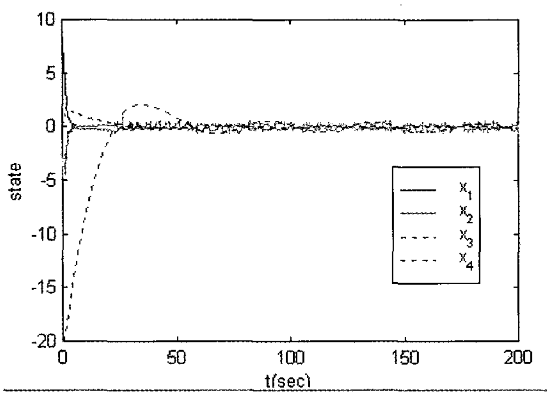

Fig 4. Simulation results for pendulum balancing and cart regulation.

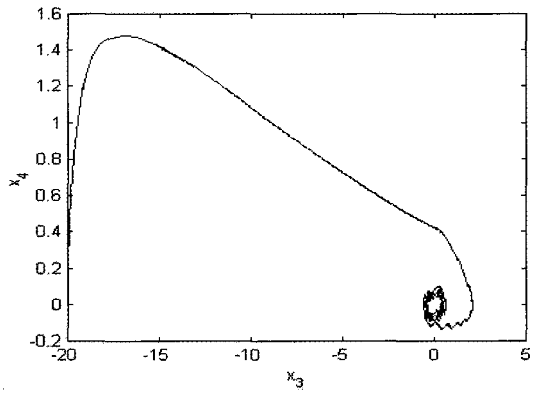

Fig 5. Position state trajectory of the cart in the state phase plane.

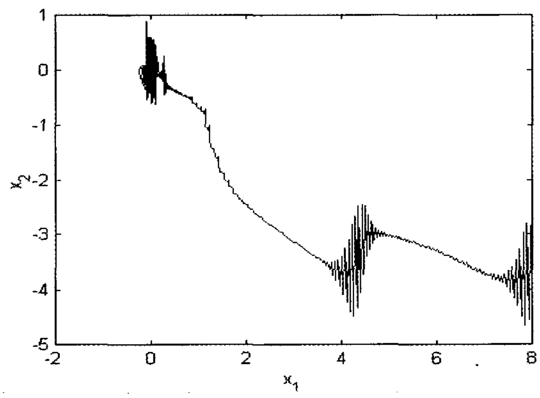

Fig 6. Angle state trajectory of the pendulum in the state phase plane.

5. Conclusion

In this paper, a robust indirect adaptive fuzzy controller is proposed for balancing and position control of the inverted pendulum system. To satisfy two control objectives, the proposed controller consists of an indirect adaptive fuzzy controller for balancing of pendulum, a supervisory fuzzy position controller which emulates heuristic position control strategy. It is proved that all the signals in the inverted pendulum system are bounded. Simulation example shows the effectiveness of the proposed control method.

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