H-infinity Discrete Time Fuzzy Controller Design Based on Bilinear Matrix Inequality

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Abstract

This paper presents an H_∞ controller synthesis method for discrete time fuzzy dynamic systems based on a piecewise smooth Lyapunov function. The basic idea of the proposed approach is to construct controllers for the fuzzy dynamic systems in such a way that a piecewise smooth Lyapunov function can be used to establish the global stability with H_∞ performance of the resulting closed loop fuzzy control systems. It is shown that the control laws can be obtained by solving a set of Bilinear Matrix Inequalities (BMIs). An example is given to illustrate the application of the proposed method.

Keywords : bilinear matrix inequalities, control, fuzzy systems, output feedback, T-S models, stability

1. Introduction

Fuzzy systems have been used to represent various nonlinear systems and fuzzy logical control (FLC) has proved to be a successful control approach for certain complex nonlinear systems, see [1]-[8] for example. Despite the increasing number of industrial applications of fuzzy control, the development of systematic methods for analysis and design of fuzzy control systems is still lagging behind.

Recently, there have appeared a number of stability analysis and controller design results in fuzzy control literature [9]-[17], where the Takagi-Sugeno's fuzzy models are used. The stability of the overall fuzzy system is determined by checking a Lyapunov equation or a Linear Matrix Inequality (LMI). It is required that a common positive definite matrix P can be found to satisfy the Lyapunov equation or the LMI for all the local models. However this is a difficult problem to solve since such a matrix might not exist in many cases, especially for highly nonlinear complex systems. The controller designs are also based on a common positive definite matrix P. Most recently, a stability result of fuzzy systems using a piecewise quadratic Lyapunov function has been reported [18]. It is also demonstrated in the paper that the piecewise Lyapunov function is a much richer class of Lyapunov function candidates than the common Lyapunov function candidates and thus it is able to deal with a larger class of fuzzy dynamic systems. In fact, the common Lyapunov function is a special case of the more general piecewise Lyapunov function.

During the last few years, we have proposed a number of new methods for the systematic analysis and design of fuzzy logic controllers based on a so-called fuzzy dynamic model which is similar to the Takagi-Sugeno's model [19]-[22]. The basic idea of these methods is to design a feedback controller for each local model and to construct a global controller from the local controllers in such a way that global stability of the closed loop fuzzy control system is guaranteed. However, for the methods based on the piecewise Lyapunov function, certain restrictive boundary conditions have to be imposed.

Motivated from the results of piecewise continuous Lyapunov functions in [18], we have developed some stability analysis methods for fuzzy dynamic systems based on piecewise Lyapunov functions in [23]-[26] recently. The work presented in this paper is an extension of the preliminary results [22]-[26]. In this paper we will propose a new constructive controller synthesis method for the fuzzy dynamic systems based on a new stability theorem. It should be noted that with this kind of piecewise Lyapunov function, the restrictive boundary condition existing in our previous controller design

Manuscript received May. 27, 2006; revised Jun. 12, 2006.
The work described in this paper was partially supported by a grant from City University of Hong Kong [Project No.: SRG 7001954] and a grant from National Natural Science Foundation of China [No. 60574080].

can be removed and global stability of the resulting closed loop system can be easily established. Under the proposed piecewise Lyapunov function based approach, the conservatism arising from common Lyapunov function based approach for stabilization of fuzzy system can be reduced. Moreover, the design procedure is to solve a set of BMIs.

The rest of the paper is organized as follows. Section 2 introduces the discrete time fuzzy dynamic model and the stability theorem. Section 3 presents an H_∞ controller synthesis method for fuzzy dynamic systems. A numerical example is shown in section 4. Finally, conclusions are given in section 5.

2. Fuzzy dynamic model and its piecewise quadratic stability

The following fuzzy dynamic model proposed in [14]-[26] can be used to represent a complex discrete-time system with both fuzzy inference rules and local analytic linear models as follows.

$$R^l:\ \text{IF } x_1 \text{ is } F_1^l \text{ AND } \dots\ x_n \text{ is } F_n^l$$
$$\text{THEN } x(t+1) = A_l x(t) + B_l u(t) + D_l v(t) + a_l \quad (2.1)$$
$$z(t) = H_l x(t) + G_l u(t) \quad l = 1,2,\dots,m,$$

where R^l denotes the l-th fuzzy inference rule, m the number of inference rules, F_j^l (j=1,2,...,n) the fuzzy sets, $x(t) \in \Re^n$ the state variables, $u(t) \in \Re^p$ the control outputs, $z(t) \in \Re^r$ the controlled outputs, $v(t) \in \Re^q$ the disturbances which belong to $l_2[0,\infty)$, $(A_l, B_l, D_l, H_l, G_l, a_l)$ the l-th local model of the fuzzy system (2.1), and a_l are the offset terms.

Let $\mu_l(x(t))$ be the normalized membership function of the inferred fuzzy set F^l where $F^l = \sum_{i=1}^n F_i^l$ and

$$\sum_{l=1}^m \mu_l = 1. \quad (2.2)$$

By using a centre-average defuzzifier, product inference and singleton fuzzifier, the dynamic fuzzy model (2.1) can be expressed by the following global model

$$x(t+1) = A(\mu)x(t) + B(\mu)u(t) + D(\mu)v(t) + a(\mu) \quad (2.3)$$
$$z(t) = H(\mu)x(t) + G(\mu)u(t)$$

where

$$A(\mu) = \sum_{l=1}^m \mu_l A_l,\ B(\mu) = \sum_{l=1}^m \mu_l B_l,\ D(\mu) = \sum_{l=1}^m \mu_l D_l,$$
$$a(\mu) = \sum_{l=1}^m \mu_l a_l,\ H(\mu) = \sum_{l=1}^m \mu_l H_l,\ G(\mu) = \sum_{l=1}^m \mu_l G_l.$$

The objective of this section is to design a suitable controller for the system (2.3) with a guaranteed performance in the H_∞ sense, that is, given a prescribed level of disturbance attenuation $\gamma > 0$, find a controller such that the induced l_2-norm of the operator from v(t) to the controlled output z(t) is less than γ under zero initial conditions,

$$\|z(t)\|_2 < \gamma \|v(t)\|_2$$

for all nonzero $v(t) \in l_2$. In this case, the closed loop control system is said to be globally stable with disturbance attenuation γ.

Remark 2.1: It is noted that the system models defined in (2.1) or (2.3) are in fact affine systems instead of linear systems. They include an additional offset term. These models have much improved function approximation capabilities [27].

Define m regions in the state space as follows,

$$\bar{S}_l = S_l \cup \partial S_l,\ l = 1,2,\dots,m \quad (2.4)$$

where

$$S_l = \{x \mid \mu_l(x) > \mu_i(x),\quad i = 1,2,\dots,m,\ i \ne l\}, \quad (2.5)$$

and its boundary

$$\partial S_l = \{x \mid \mu_l(x) = \mu_i(x),\ i = 1,2,\dots,m,\ i \ne l\}. \quad (2.6)$$

And also define L as the set of region indexes, $L_0 \subseteq L$ as the set of indexes for regions that contain the origin and $L_1 \subseteq L$ the set of indexes for the regions that do not contain the origin. Then the global model of the fuzzy dynamic system can also be expressed in each region by

$$\begin{aligned} x(t+1) &= (A_l + \Delta A_l(\mu))x(t) + (B_l + \Delta B_l(\mu))u(t) \\ &\quad + (D_l + \Delta D_l(\mu))v(t) + a_l + \Delta a_l(\mu) \end{aligned} \quad (2.7)$$
$$z(t) = (H_l + \Delta H_l(\mu))x(t) + (G_l + \Delta G_l(\mu))u(t)$$

for $x(t) \in \bar{S}_l$, where

$$\Delta A_l(\mu) = \sum_{i=1,i\ne l}^m \mu_i \Delta A_{li},\ \Delta B_l(\mu) = \sum_{i=1,i\ne l}^m \mu_i \Delta B_{li},$$
$$\Delta D_l(\mu) = \sum_{i=1,i\ne l}^m \mu_i \Delta D_{li},\ \Delta a_l(\mu) = \sum_{i=1,i\ne l}^m \mu_i \Delta a_{li},$$
$$\Delta H_l(\mu) = \sum_{i=1,i\ne l}^m \mu_i \Delta H_{li},\ \Delta G_l(\mu) = \sum_{i=1,i\ne l}^m \mu_i \Delta G_{li},$$
$$\Delta A_{li} = A_i - A_l,\ \Delta B_{li} = B_i - B_l,\ \Delta D_{li} = D_i - D_l,$$
$$\Delta a_{li} = a_i - a_l,\ \Delta H_{li} = H_i - H_l,\ \Delta G_{li} = G_i - G_l.$$

It should be noted that many membership functions could be equal to zero, that is, many fuzzy rules could be inactive when the l-th subsystem plays a dominant role, that is, $x(t) \in \bar{S}_l$.

For convenience, we introduce the following notation,

$$\bar{A}_l = \begin{bmatrix} A_l & a_l \\ 0 & 1 \end{bmatrix},\ \bar{B}_l = \begin{bmatrix} B_l \\ 0 \end{bmatrix},\ \bar{D}_l = \begin{bmatrix} D_l \\ 0 \end{bmatrix},\ \bar{x} = \begin{bmatrix} x \\ 1 \end{bmatrix},$$
$$\bar{H}_l = \begin{bmatrix} H_l & 0 \end{bmatrix},\ \Delta\bar{A}_l = \begin{bmatrix} \Delta A_l & \Delta a_l \\ 0 & 0 \end{bmatrix},\ \Delta\bar{B}_l = \begin{bmatrix} \Delta B_l \\ 0 \end{bmatrix}, \quad (2.8)$$
$$\Delta\bar{D}_l = \begin{bmatrix} \Delta D_l \\ 0 \end{bmatrix},\ \Delta\bar{H}_l = \begin{bmatrix} \Delta H_l & 0 \end{bmatrix}.$$

where it is assumed that $a_l = 0$ and $\Delta a_l = 0$ for all $l \in L_0$. Then using this notation, the system model (2.7) can be expressed as

$$\bar{x}(t+1)=(\bar{A}_l+\Delta\bar{A}_l(\mu))\bar{x}(t)+(\bar{B}_l+\Delta\bar{B}_l(\mu))u(t)+(\bar{D}_l+\Delta\bar{D}_l(\mu))v(t),\quad x(t)\in\bar{S}_l$$
$$z(t)=(\bar{H}_l+\Delta\bar{H}_l(\mu))\bar{x}(t)+(G_l+\Delta G_l(\mu))u(t) \tag{2.9}$$

For purpose of stability analysis and subsequent use, we introduce the following upper bounds for the uncertainty term of the fuzzy system (2.7) or (2.9),

$$\begin{aligned}&[\Delta A_l(\mu)\quad \Delta a(\mu)]^T[\Delta A_l(\mu)\quad \Delta a(\mu)]\le[E_{lA}\quad E_{la}]^T[E_{lA}\quad E_{la}],\\ &[\Delta B_l(\mu)]^T[\Delta B_l(\mu)]\le E_{lB}{}^T E_{lB},\\ &[\Delta D_l(\mu)][\Delta D_l(\mu)]^T\le E_{lD}E_{lD}{}^T,\\ &[\Delta H_l(\mu)]^T[\Delta H_l(\mu)]\le E_{lH}{}^T E_{lH},\ [\Delta G_l(\mu)]^T[\Delta G_l(\mu)]\le E_{lG}{}^T E_{lG}.\end{aligned} \tag{2.10a}$$

Then

$$\begin{aligned}&[\Delta\bar{A}_l(\mu)]^T[\Delta\bar{A}_l(\mu)]\le E_{l\bar{A}}^T E_{l\bar{A}}=[E_{lA}\quad E_{la}]^T[E_{lA}\quad E_{la}],\\ &[\Delta\bar{B}_l(\mu)]^T[\Delta\bar{B}_l(\mu)]\le E_{l\bar{B}}^T E_{l\bar{B}}=E_{lB}^T E_{lB},\\ &[\Delta\bar{D}_l(\mu)][\Delta\bar{D}_l(\mu)]^T\le E_{l\bar{D}}E_{l\bar{D}}^T=\begin{bmatrix}E_{lD}E_{lD}^T & 0\\ 0 & 0\end{bmatrix},\\ &[\Delta\bar{H}_l(\mu)]^T[\Delta\bar{H}_l(\mu)]\le E_{l\bar{H}}^T E_{l\bar{H}}=\begin{bmatrix}E_{lH}^T E_{lH} & 0\\ 0 & 0\end{bmatrix}.\end{aligned} \tag{2.10b}$$

It is noted that there are many ways to obtain these bounds, the interested readers can refer to [19-26] for details.

With such a state space partition, we proposed a number of controller design methods based on a piecewise Lyapunov function. The key idea is to design a local controller for each region based on the subsystem (2.7), and then to use the piecewise Lyapunov function to establish the global stability of the resulting closed loop fuzzy control system. Due to the discontinuity of the function across the boundaries of the region, certain boundary conditions are developed to ensure the stability of the system [19-22]. However, most of these boundary conditions are very restrictive in the sense that they are not checkable a priori or very hard to check. Recently, the authors in [18] independently introduced a different kind of piecewise Lyapunov functions and developed a stability result based on this piecewise Lyapunov function for continuous time systems. The key idea is to make the piecewise Lyapunov function continuous across the region boundaries and thus avoid the boundary conditions we encountered in our design.

As in [18], to reduce the conservatism of the stability result the S-procedure can be used. Construct matrices, $\bar{E}_l=[E_l\quad e_l], l\in L$ with $e_l=0$ for $l\in L_0$ such that

$$\bar{E}_l\begin{bmatrix}x\\1\end{bmatrix}\ge 0,\quad x\in\bar{S}_l,\quad l\in L. \tag{2.11}$$

It should be noted that the above vector inequality means that each entry of the vector is nonnegative.

Remark 2.2: A systematic procedure for constructing these matrices $\bar{E}_l, l\in L$ for a given fuzzy dynamic system can be found in [18]. The procedure is directly based on the information in the fuzzy rule base. The interested readers please refer to [18] for details.

Then we are ready present the following stability result [24].

Theorem 2.1: Consider the fuzzy dynamic system (2.1) with $u\equiv 0$ and $v\equiv 0$. If there exist a set of positive constants $\varepsilon_l, l=1,2,\cdots,m$, a set of symmetric matrices $P_l, l\in L_0, \bar{P}_l, l\in L_1$, symmetric matrices U_l, W_l and $Q_{lj}, l,j\in\Omega$, such that U_l, W_l and Q_{lj} have nonnegative entries, and the following LMIs are satisfied,

$$0<P_l-E_l^T U_l E_l,\quad l\in L_0 \tag{2.12}$$

$$0>\begin{bmatrix}A_l^T P_l A_l-P_l+\frac{1}{\varepsilon_l}E_{lA}^T E_{lA}+E_l^T W_l E_l & A_l^T P_l\\ P_l A_l & -(\frac{1}{\varepsilon_l}I-P_l)\end{bmatrix},\quad l\in L_0 \tag{2.13}$$

$$0<\bar{P}_l-\bar{E}_l^T U_l\bar{E}_l,\quad l\in L_0 \tag{2.14}$$

$$0>\begin{bmatrix}\bar{A}_l^T\bar{P}_l\bar{A}_l-\bar{P}_l+\frac{1}{\varepsilon_l}E_{l\bar{A}}^T E_{l\bar{A}}+\bar{E}_l^T W_l\bar{E}_l & \bar{A}_l^T\bar{P}_l\\ \bar{P}_l\bar{A}_l & -(\frac{1}{\varepsilon_l}I-\bar{P}_l)\end{bmatrix},\quad l\in L_1 \tag{2.15}$$

$$0>\begin{bmatrix}A_l^T P_j A_l-P_l+\frac{1}{\varepsilon_l}E_{lA}^T E_{lA}+E_l^T Q_{lj}E_l & A_l^T P_j\\ P_j A_l & -(\frac{1}{\varepsilon_l}I-P_j)\end{bmatrix},\quad l,j\in\Omega\cap L_0 \tag{2.16}$$

$$0>\begin{bmatrix}\bar{A}_l^T\bar{P}_j\bar{A}_l-\bar{P}_l+\frac{1}{\varepsilon_l}E_{l\bar{A}}^T E_{l\bar{A}}+\bar{E}_l^T Q_{lj}\bar{E}_l & \bar{A}_l^T\bar{P}_j\\ \bar{P}_j\bar{A}_l & -(\frac{1}{\varepsilon_l}I-\bar{P}_j)\end{bmatrix},\quad l,j\in\Omega\cap L_1 \tag{2.17}$$

$$0>\begin{bmatrix}\bar{A}_l^T\bar{P}_j\bar{A}_l-\bar{P}_l+\frac{1}{\varepsilon_l}E_{l\bar{A}}^T E_{l\bar{A}}+\bar{E}_l^T Q_{lj}\bar{E}_l & \bar{A}_l^T\bar{P}_j\\ \bar{P}_j\bar{A}_l & -(\frac{1}{\varepsilon_l}I-\bar{P}_j)\end{bmatrix},$$
$$l\in L_1,\ j\in L_0,\ l,j\in\Omega \tag{2.18}$$

$$0>\begin{bmatrix}\bar{A}_l^T\bar{P}_j\bar{A}_l-\bar{P}_l+\frac{1}{\varepsilon_l}E_{l\bar{A}}^T E_{l\bar{A}}+\bar{E}_l^T Q_{lj}\bar{E}_l & \bar{A}_l^T\bar{P}_j\\ \bar{P}_j\bar{A}_l & -(\frac{1}{\varepsilon_l}I-\bar{P}_j)\end{bmatrix},$$
$$l\in L_0,\ j\in L_1,\ l,j\in\Omega \tag{2.19}$$

where we define $\bar{P}_j=[I_{n\times n}\quad 0_{n\times 1}]^T P_j[I_{n\times n}\quad 0_{n\times 1}]$ for $j\in L_0$ in (2.18), and $\bar{P}_l=[I_{n\times n}\quad 0_{n\times 1}]^T P_l[I_{n\times n}\quad 0_{n\times 1}]$ for $l\in L_0$ in (2.19), and the set Ω represents all possible transitions from one region to another, that is, $\Omega:=\{l,j\,|\,x(t)\in S_l,\ x(t+1)\in S_j,\ j\ne l\}$, then the fuzzy dynamic system is globally exponentially stable, that is, $x(t)$ tends to zero exponentially for every continuous piecewise trajectory in the state space.

Proof: See [24] for details. ∇∇

The above conditions are linear matrix inequalities in the variables Q_{lj}, U_l, and W_l. A solution to those inequalities ensures $V(x)$ defined in (2.20) to be a Lyapunov function for the fuzzy dynamic system. The LMI in (2.12) or (2.14) for each region guarantees that the function is positive and the LMI in (2.13) or (2.15) for each region guarantees that the function decreases along all system trajectories. The LMIs in (2.16)-(2.19) guarantee that the function decreases when the state of the system transits from one region to another. In addition, $E_l^T U_l E_l$, $E_l^T W_l E_l$, $\bar{E}_l^T U_l \bar{E}_l$, $\bar{E}_l^T W_l \bar{E}_l$, $E_l^T Q_{lj} E_l$, and $\bar{E}_l^T Q_{lj} \bar{E}_l$ in those LMIs are terms of the S-procedure used to reduce the conservatism of the Lyapunov function.

Remark 2.3: The stability checking of the fuzzy dynamic system in eqn. (2.12)-(2.19) can be easily facilitated by a commercially available software package Matlab LMI toolbox [28].

Remark 2.4: The set Ω can be determined by the reachability analysis [29]. If it is possible for the transitions happen between all regions, then $\Omega = L \times L$, which is defined as a set of $\{l, j \mid l, j \in L, j \neq l\}$.

3. H_∞ Controller synthesis

In this section, we will address the controller synthesis problem for the discrete time fuzzy dynamic systems introduced in the section 2. The proposed controller synthesis approach is based on the local subsystem defined in each region. However, the interactions from other subsystems must be accounted for in order to guarantee the stability of the global system.

Consider the fuzzy system in each region

$$\begin{aligned} x(t+1) &= (A_l + \Delta A_l(\mu))x(t) + (B_l + \Delta B_l(\mu))u(t) \\ &\quad + (D_l + \Delta D_l(\mu))v(t) + a_l + \Delta a_l(\mu) \end{aligned} \tag{3.1}$$

$$z(t) = (H_l + \Delta H_l(\mu))x(t) + (G_l + \Delta G_l(\mu))u(t)$$

For $x(t) \in \bar{S}_l$, or in the more compact form,

$$\bar{x}(t+1) = (\bar{A}_l + \Delta\bar{A}_l(\mu))\bar{x}(t) + (\bar{B}_l + \Delta\bar{B}_l(\mu))u(t) + (\bar{D}_l + \Delta\bar{D}_l(\mu))v(t),$$
$$x(t) \in \bar{S}_l \tag{3.2}$$
$$z(t) = (\bar{H}_l + \Delta\bar{H}_l(\mu))\bar{x}(t) + (G_l + \Delta G_l(\mu))u(t)$$

With the following piecewise controller,

$$u(t) = K(x)x = \begin{cases} K_l x(t) & x(t) \in \bar{S}_l, \quad l \in L_0 \\ \bar{K}_l \bar{x}(t) & x(t) \in \bar{S}_l, \quad l \in L_1 \end{cases}. \tag{3.3}$$

the global closed loop system can be described by the following equation,

$$\bar{x}(t+1) = \bar{A}_c(\mu)\bar{x}(t) + \bar{D}_c(\mu)v(t) \tag{3.4}$$
$$z(t) = \bar{H}_c(\mu)\bar{x}(t)$$

where $\bar{A}_c(\mu) = \bar{A}(\mu) + \bar{B}(\mu)K(x)$, $\bar{D}_c(\mu) = \bar{D}(\mu)$, $\bar{H}_c(\mu) = \bar{H}(\mu) + G(\mu)K(x)$.

The equ.(3.4) can also be expressed in each local region as follows,

$$\bar{x}(t+1) = \bar{A}_{cl}(\mu)\bar{x}(t) + \bar{D}_{cl}(\mu)v(t) \quad x(t) \in \bar{S}_l \tag{3.5}$$
$$z(t) = \bar{H}_{cl}(\mu)\bar{x}(t)$$

where $\bar{A}_{cl}(\mu) = \bar{A}_l + \Delta\bar{A}_l(\mu) + (\bar{B}_l + \Delta\bar{B}_l(\mu))\bar{K}_l$, $\bar{D}_{cl} = \bar{D}_l + \Delta\bar{D}_l(\mu)$, $\bar{H}_{cl}(\mu) = \bar{H}_l + \Delta\bar{H}_l(\mu) + (G_l + \Delta G_l(\mu))\bar{K}_l$.

For $l \in L_0$, (3.5) becomes

$$x(t+1) = A_{cl}(\mu)x(t) + D_{cl}(\mu)v(t) \quad x(t) \in \bar{S}_l \tag{3.6}$$
$$z(t) = H_{cl}(\mu)x(t)$$

where $A_{cl}(\mu) = A_l + \Delta A_l(\mu) + (B_l + \Delta B_l(\mu))K_l$, $D_{cl} = D_l + \Delta D_l(\mu)$, $H_{cl}(\mu) = H_l + \Delta H_l(\mu) + (G_l + \Delta G_l(\mu))K_l$.

Then we are ready to present the following lemma.

Lemma 3.1: Given a constant $\gamma > 0$, the fuzzy system (3.1) or (3.5) are globally stable with disturbance attenuation γ, if there exist a set of symmetric matrices $P_l, l \in L_0, \bar{P}_l, l \in L_1$, symmetric matrices U_l, W_l and Q_{lj}, $l, j \in \Omega \cap L_0$, such that U_l, W_l and Q_{lj} have nonnegative entries, and the following inequalities are satisfied,

$$0 < P_l - E_l^T U_l E_l \tag{3.7}$$

$$\begin{aligned} 0 > A_{cl}^T P_l A_{cl} - P_l + E_l^T W_l E_l + A_{cl}^T P_l D_{cl}(\gamma^2 I - D_{cl}^T P_l D_{cl})^{-1} \\ D_{cl}^T P_l A_{cl} + H_{cl}^T H_{cl} \end{aligned}, \tag{3.8}$$

with $\gamma^2 I - D_{cl}^T P_l D_{cl} > 0$, for $l \in L_0$,

$$0 < \bar{P}_l - \bar{E}_l^T U_l \bar{E}_l \tag{3.9}$$

$$\begin{aligned} 0 > \bar{A}_{cl}^T \bar{P}_l \bar{A}_{cl} - \bar{P}_l + \bar{E}_l^T W_l \bar{E}_l + \bar{A}_{cl}^T \bar{P}_l \bar{D}_{cl}(\gamma^2 I - \bar{D}_{cl}^T \bar{P}_l \bar{D}_{cl})^{-1} \\ \bar{D}_{cl}^T \bar{P}_l \bar{A}_{cl} + \bar{H}_{cl}^T \bar{H}_{cl} \end{aligned} \tag{3.10}$$

with $\gamma^2 I - \bar{D}_{cl}^T \bar{P}_l \bar{D}_{cl} > 0$, for $l \in L_1$,

$$\begin{aligned} 0 > A_{cl}^T P_j A_{cl} - P_l + E_l^T Q_{lj} E_l + A_{cl}^T P_j D_{cl}(\gamma^2 I - D_{cl}^T P_j D_{cl})^{-1} \\ D_{cl}^T P_j A_{cl} + H_{cl}^T H_{cl} \end{aligned}, \tag{3.11}$$

with $\gamma^2 I - D_{cl}^T P_j D_{cl} > 0$, for $l, j \in \Omega \cap L_0$,

$$\begin{aligned} 0 > \bar{A}_{cl}^T \bar{P}_j \bar{A}_{cl} - \bar{P}_l + \bar{E}_l^T Q_{lj} \bar{E}_l + \bar{A}_{cl}^T \bar{P}_j \bar{D}_{cl}(\gamma^2 I - \bar{D}_{cl}^T \bar{P}_j \bar{D}_{cl})^{-1} \\ \bar{D}_{cl}^T \bar{P}_j \bar{A}_{cl} + \bar{H}_{cl}^T \bar{H}_{cl} \end{aligned}, \tag{3.12}$$

with $\gamma^2 I - \bar{D}_{cl}^T \bar{P}_j \bar{D}_{cl} > 0$, for $l, j \in \Omega \cap L_1$,

$$\begin{aligned} 0 > \bar{A}_{cl}^T \bar{P}_j \bar{A}_{cl} - \bar{P}_l + \bar{E}_l^T Q_{lj} \bar{E}_l + \bar{A}_{cl}^T \bar{P}_j \bar{D}_{cl}(\gamma^2 I - \bar{D}_{cl}^T \bar{P}_j \bar{D}_{cl})^{-1} \\ \bar{D}_{cl}^T \bar{P}_j \bar{A}_{cl} + \bar{H}_{cl}^T \bar{H}_{cl} \end{aligned}, \tag{3.13}$$

with $\gamma^2 I - \bar{D}_{cl}{}^T \bar{P}_j \bar{D}_{cl} > 0$, for $l, j \in \Omega$, $l \in L_1$, $j \in L_0$, and

$$0 > \bar{A}_{cl}^T \bar{P}_j \bar{A}_{cl} - \bar{P}_l + \bar{E}_l^T Q_{lj} \bar{E}_l + \bar{A}_{cl}^T \bar{P}_j \bar{D}_{cl} (\gamma^2 I - \bar{D}_{cl}{}^T \bar{P}_j \bar{D}_{cl})^{-1} \bar{D}_{cl}^T \bar{P}_j \bar{A}_{cl} + \bar{H}_{cl}^T \bar{H}_{cl} \quad (3.14)$$

with $\gamma^2 I - \bar{D}_{cl}{}^T \bar{P}_j \bar{D}_{cl} > 0$, for $l, j \in \Omega$, $l \in L_0$, $j \in L_1$, where we define $\bar{P}_j = [I_{n\times n} \quad 0_{n\times 1}]^T P_j [I_{n\times n} \quad 0_{n\times 1}]$ for $j \in L_0$ in (3.13), and $\bar{P}_l = [I_{n\times n} \quad 0_{n\times 1}]^T P_l [I_{n\times n} \quad 0_{n\times 1}]$ for $l \in L_0$ in (3.14).

Proof: It is easily seen that equ.(3.8) and (3.10)-(3.14) imply the following inequalities respectively,

$$A_{cl}^T P_l A_{cl} - P_l + E_l^T W_l E_l < 0, \quad l \in L_0$$

$$\bar{A}_{cl}^T \bar{P}_l \bar{A}_{cl} - \bar{P}_l + \bar{E}_l^T W_l \bar{E}_l < 0, \quad l \in L_1$$

$$A_{cl}^T P_j A_{cl} - P_l + E_l^T Q_{lj} E_l < 0, \quad l, j \in \Omega \cap L_0,$$

$$\bar{A}_{cl}^T \bar{P}_j \bar{A}_{cl} - \bar{P}_l + \bar{E}_l^T Q_{lj} \bar{E}_l < 0, \quad l, j \in \Omega \cap L_1,$$

$$\bar{A}_{cl}^T \bar{P}_j \bar{A}_{cl} - \bar{P}_l + \bar{E}_l^T Q_{lj} \bar{E}_l < 0, \quad l, j \in \Omega, \quad l \in L_1, \quad j \in L_0$$

$$\bar{A}_{cl}^T \bar{P}_j \bar{A}_{cl} - \bar{P}_l + \bar{E}_l^T Q_{lj} \bar{E}_l < 0, \quad l, j \in \Omega, \quad l \in L_0, \quad j \in L_1$$

and thus it follows from Theorem 2.1 and its proof that the closed loop system is globally stable.

Now we show the disturbance attenuation performance, that is $v(t) \neq 0$. Consider the Lyapunov function,

$$V(x) = \begin{cases} x^T P_l x, & x \in \bar{S}_l, l \in L_0 \\ \begin{bmatrix} x \\ 1 \end{bmatrix}^T \bar{P}_l \begin{bmatrix} x \\ 1 \end{bmatrix}, & x \in \bar{S}_l, l \in L_1 \end{cases} \quad (3.15)$$

or in a more compact form,

$$V(x) = \bar{x}^T \bar{P}_l \bar{x}, \; x \in \bar{S}_l, l \in L. \quad (3.16)$$

Then its difference along the solution of the system (3.1) or (3.5) can be described as follows,

$$\begin{aligned}
\Delta V(t) &:= V(t+1) - V(t) \\
&= \bar{x}(t+1)^T \bar{P}_j \bar{x}(t+1) - \bar{x}(t)^T \bar{P}_l \bar{x}(t) \\
&= \bar{x}(t)^T (\bar{A}_{cl}^T \bar{P}_j \bar{A}_{cl} - \bar{P}_l) \bar{x}(t) + v(t)^T \bar{D}_{cl}^T \bar{P}_j \bar{A}_{cl} \bar{x}(t) + \bar{x}(t)^T \bar{A}_{cl}^T \bar{P}_j \bar{D}_{cl} v(t) \\
&\quad + v(t)^T \bar{D}_{cl}^T \bar{P}_j \bar{D}_{cl} v(t) \\
&\le \bar{x}(t)^T [-\bar{E}_l^T Q_{lj} \bar{E}_l - \bar{A}_{cl}^T \bar{P}_j \bar{D}_{cl} (\gamma^2 I - \bar{D}_{cl}{}^T \bar{P}_j \bar{D}_{cl})^{-1} \bar{D}_{cl}^T \bar{P}_j \bar{A}_{cl} - \bar{H}_{cl}^T \bar{H}_{cl}] \bar{x}(t) \\
&\quad + v(t)^T \bar{D}_{cl}^T \bar{P}_j \bar{A}_{cl} \bar{x}(t) + \bar{x}(t)^T \bar{A}_{cl}^T \bar{P}_j \bar{D}_{cl} v(t) + v(t)^T \bar{D}_{cl}^T \bar{P}_j \bar{D}_{cl} v(t) \\
&\le \bar{x}(t)^T [-\bar{A}_{cl}^T \bar{P}_j \bar{D}_{cl} (\gamma^2 I - \bar{D}_{cl}{}^T \bar{P}_j \bar{D}_{cl})^{-1} \bar{D}_{cl}^T \bar{P}_j \bar{A}_{cl} - \bar{H}_{cl}^T \bar{H}_{cl}] \bar{x}(t) \\
&\quad + v(t)^T \bar{D}_{cl}^T \bar{P}_j \bar{A}_{cl} \bar{x}(t) + \bar{x}(t)^T \bar{A}_{cl}^T \bar{P}_j \bar{D}_{cl} v(t) + v(t)^T \bar{D}_{cl}^T \bar{P}_j \bar{D}_{cl} v(t) \\
&= -z(t)^T z(t) + \gamma^2 v(t)^T v(t) - w(t)^T M(t) w(t) \qquad (3.17)
\end{aligned}$$

where $M(t) = \gamma^2 I - \bar{D}_{cl}^T \bar{P}_j \bar{D}_{cl}$, $w(t) = v(t) - M(t)^{-1} \bar{D}_{cl}^T \bar{P}_j \bar{A}_{cl} \bar{x}$, and $j = l$ when the state stays in the region S_l and $j \neq l$ when the state transits from the region S_l to S_j. Then it follows from (3.17) that

$$\Delta V(t) \le -z(t)^T z(t) + \gamma^2 v(t)^T v(t) \quad (3.18)$$

which implies that

$$V(x(\infty)) - V(x(0)) \le -\sum_{t=0}^{\infty} z(t)^T z(t) + \sum_{t=0}^{\infty} \gamma^2 v(t)^T v(t), \quad (3.19)$$

that is, with $x(0) = 0$,

$$\| z \|_2 \le \gamma \| v \|_2,$$

and thus the proof is completed. ∇∇

Then based on the Lemma 3.1, we have the following result.

Theorem 3.1: Given a constant $\gamma > 0$, the system (3.1) or (3.5) is globally stable with disturbance attenuation γ, if there exist a set of positive constants $\varepsilon_l, l = 1, 2, \cdots, m$, a set of positive definite symmetric matrices $P_l, l \in L_0, \bar{P}_l, l \in L_1$, symmetric matrices W_l and $Q_{lj}, l, j \in \Omega$, such that W_l and Q_{lj} have nonnegative entries, and the following BMIs are satisfied,

$$0 < \begin{bmatrix} P_l & P_l \\ P_l & [\varepsilon_l I + 2\gamma^{-2}(D_l D_l^T + E_{lD} E_{lD}^T)]^{-1} \end{bmatrix}, \quad l \in L_0 \quad (3.20)$$

$$0 > \Psi_l := \begin{bmatrix}
\Omega_l & (A_l + B_l K_l)^T P_l & 0 & 0 & 0 & K_l^T E_{lB}^T & K_l^T G_l^T & K_l^T E_{lG}^T \\
P_l(A_l + B_l K_l) & -P_l & P_l D_l & P_l E_{lD} & P_l & 0 & 0 & 0 \\
0 & D_l^T P_l & -\tfrac{1}{2}\gamma^2 I & 0 & 0 & 0 & 0 & 0 \\
0 & E_{lD}^T P_l & 0 & -\tfrac{1}{2}\gamma^2 I & 0 & 0 & 0 & 0 \\
0 & P_l & 0 & 0 & -\tfrac{1}{\varepsilon_l} I & 0 & 0 & 0 \\
E_{lB} K_l & 0 & 0 & 0 & 0 & -\tfrac{\varepsilon_l}{2} I & 0 & 0 \\
G_l K_l & 0 & 0 & 0 & 0 & 0 & -\tfrac{1}{4} I & 0 \\
E_{lG} K_l & 0 & 0 & 0 & 0 & 0 & 0 & -\tfrac{1}{4} I
\end{bmatrix}$$

$l \in L_0$ (3.21)

$$0 < \begin{bmatrix} \bar{P}_l & \bar{P}_l \\ \bar{P}_l & [\varepsilon_l I + 2\gamma^{-2}(\bar{D}_l \bar{D}_l^T + E_{l\bar{D}} E_{l\bar{D}}^T)]^{-1} \end{bmatrix}, \quad l \in L_1 \quad (3.22)$$

$$0 > \bar{\Psi}_l := \begin{bmatrix}
\bar{\Omega}_l & (\bar{A}_l + \bar{B}_l \bar{K}_l)^T \bar{P}_l & 0 & 0 & 0 & \bar{K}_l^T E_{l\bar{B}}^T & \bar{K}_l^T G_l^T & \bar{K}_l^T E_{lG}^T \\
\bar{P}_l(\bar{A}_l + \bar{B}_l \bar{K}_l) & -\bar{P}_l & \bar{P}_l \bar{D}_l & \bar{P}_l E_{l\bar{D}} & \bar{P}_l & 0 & 0 & 0 \\
0 & \bar{D}_l^T \bar{P}_l & -\tfrac{1}{2}\gamma^2 I & 0 & 0 & 0 & 0 & 0 \\
0 & E_{l\bar{D}}^T \bar{P}_l & 0 & -\tfrac{1}{2}\gamma^2 I & 0 & 0 & 0 & 0 \\
0 & P_l & 0 & 0 & -\tfrac{1}{\varepsilon_l} I & 0 & 0 & 0 \\
E_{l\bar{B}} \bar{K}_l & 0 & 0 & 0 & 0 & -\tfrac{\varepsilon_l}{2} I & 0 & 0 \\
G_l \bar{K}_l & 0 & 0 & 0 & 0 & 0 & -\tfrac{1}{4} I & 0 \\
E_{lG} \bar{K}_l & 0 & 0 & 0 & 0 & 0 & 0 & -\tfrac{1}{4} I
\end{bmatrix}$$

$l \in L_1$ (3.23)

$$0 < \begin{bmatrix} P_j & P_j \\ P_j & [\varepsilon_l I + 2\gamma^{-2}(D_l D_l^T + E_{lD} E_{lD}^T)]^{-1} \end{bmatrix}, \quad l, j \in \Omega \cap L_0 \quad (3.24)$$

$$0 > \Psi_{lj} := \begin{bmatrix}
\Omega_{lj} & (A_l + B_l K_l)^T P_j & 0 & 0 & 0 & K_l^T E_{lB}^T & K_l^T G_l^T & K_l^T E_{lG}^T \\
P_j(A_l + B_l K_l) & -P_j & P_j D_l & P_j E_{lD} & P_j & 0 & 0 & 0 \\
0 & D_l^T P_j & -\tfrac{1}{2}\gamma^2 I & 0 & 0 & 0 & 0 & 0 \\
0 & E_{lD}^T P_j & 0 & -\tfrac{1}{2}\gamma^2 I & 0 & 0 & 0 & 0 \\
0 & P_j & 0 & 0 & -\tfrac{1}{\varepsilon_l} I & 0 & 0 & 0 \\
E_{lB} K_l & 0 & 0 & 0 & 0 & -\tfrac{\varepsilon_l}{2} I & 0 & 0 \\
G_l K_l & 0 & 0 & 0 & 0 & 0 & -\tfrac{1}{4} I & 0 \\
E_{lG} K_l & 0 & 0 & 0 & 0 & 0 & 0 & -\tfrac{1}{4} I
\end{bmatrix}$$

$l, j \in \Omega \cap L_0$ (3.25)

$$0 < \begin{bmatrix} \bar{P}_j & \bar{P}_j \\ \bar{P}_j & [\varepsilon_l I + 2\gamma^{-2}(\bar{D}_l \bar{D}_l^T + E_{l\bar{D}} E_{l\bar{D}}^T)]^{-1} \end{bmatrix}, \quad l, j \in \Omega \cap L_1 \quad (3.26)$$

$$0>\bar{\Psi}_{lj}:=\begin{bmatrix} \bar{\Omega}_{lj} & (\bar{A}_l+\bar{B}_l\bar{K}_l)^T\bar{P}_j & 0 & 0 & 0 & \bar{K}_l^T E_{l\bar{B}}^T & \bar{K}_l^T G_l^T & \bar{K}_l^T E_{lG}^T \\ \bar{P}_j(\bar{A}_l+\bar{B}_l\bar{K}_l) & -\bar{P}_j & \bar{P}_j\bar{D}_l & \bar{P}_j E_{l\bar{D}} & \bar{P}_j & 0 & 0 & 0 \\ 0 & \bar{D}_l^T\bar{P}_j & -\frac{1}{2}\gamma^2 I & 0 & 0 & 0 & 0 & 0 \\ 0 & E_{l\bar{D}}^T\bar{P}_j & 0 & -\frac{1}{2}\gamma^2 I & 0 & 0 & 0 & 0 \\ 0 & \bar{P}_j & 0 & 0 & -\frac{1}{\varepsilon_l} I & 0 & 0 & 0 \\ E_{l\bar{B}}\bar{K}_l & 0 & 0 & 0 & 0 & -\frac{\varepsilon_l}{2} I & 0 & 0 \\ G_l\bar{K}_l & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} I & 0 \\ E_{lG}\bar{K}_l & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} I \end{bmatrix}$$

$$l,j\in\Omega\cap L_1 \qquad (3.27)$$

$$0<\begin{bmatrix} \bar{P}_j & \bar{P}_j \\ \bar{P}_j & [\varepsilon_l I+2\gamma^{-2}(\bar{D}_l\bar{D}_l^T+E_{l\bar{D}}E_{l\bar{D}}^T)]^{-1} \end{bmatrix},$$

$$l,j\in\Omega,\quad l\in L_1,\quad j\in L_0 \qquad (3.28)$$

$$0>\bar{\Psi}_{lj}:=\begin{bmatrix} \bar{\Omega}_{lj} & (\bar{A}_l+\bar{B}_l\bar{K}_l)^T\bar{P}_j & 0 & 0 & 0 & \bar{K}_l^T E_{l\bar{B}}^T & \bar{K}_l^T G_l^T & \bar{K}_l^T E_{lG}^T \\ \bar{P}_j(\bar{A}_l+\bar{B}_l\bar{K}_l) & -\bar{P}_j & \bar{P}_j\bar{D}_l & \bar{P}_j E_{l\bar{D}} & \bar{P}_j & 0 & 0 & 0 \\ 0 & \bar{D}_l^T\bar{P}_j & -\frac{1}{2}\gamma^2 I & 0 & 0 & 0 & 0 & 0 \\ 0 & E_{l\bar{D}}^T\bar{P}_j & 0 & -\frac{1}{2}\gamma^2 I & 0 & 0 & 0 & 0 \\ 0 & P_j & 0 & 0 & -\frac{1}{\varepsilon_l} I & 0 & 0 & 0 \\ E_{l\bar{B}}\bar{K}_l & 0 & 0 & 0 & 0 & -\frac{\varepsilon_l}{2} I & 0 & 0 \\ G_l\bar{K}_l & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} I & 0 \\ E_{lG}\bar{K}_l & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} I \end{bmatrix}$$

$$l,j\in\Omega,\quad l\in L_1,\quad j\in L_0 \qquad (3.29)$$

$$0<\begin{bmatrix} \bar{P}_j & \bar{P}_j \\ \bar{P}_j & [\varepsilon_l I+2\gamma^{-2}(\bar{D}_l\bar{D}_l^T+E_{l\bar{D}}E_{l\bar{D}}^T)]^{-1} \end{bmatrix},$$

$$l,j\in\Omega,\quad j\in L_1,\quad l\in L_0 \qquad (3.30)$$

$$0>\bar{\Psi}_{lj}:=\begin{bmatrix} \bar{\Omega}_{lj} & (\bar{A}_l+\bar{B}_l\bar{K}_l)^T\bar{P}_j & 0 & 0 & 0 & \bar{K}_l^T E_{l\bar{B}}^T & \bar{K}_l^T G_l^T & \bar{K}_l^T E_{lG}^T \\ \bar{P}_j(\bar{A}_l+\bar{B}_l\bar{K}_l) & -\bar{P}_j & \bar{P}_j\bar{D}_l & \bar{P}_j E_{l\bar{D}} & \bar{P}_j & 0 & 0 & 0 \\ 0 & \bar{D}_l^T\bar{P}_j & -\frac{1}{2}\gamma^2 I & 0 & 0 & 0 & 0 & 0 \\ 0 & E_{l\bar{D}}^T\bar{P}_j & 0 & -\frac{1}{2}\gamma^2 I & 0 & 0 & 0 & 0 \\ 0 & P_j & 0 & 0 & -\frac{1}{\varepsilon_l} I & 0 & 0 & 0 \\ E_{l\bar{B}}\bar{K}_l & 0 & 0 & 0 & 0 & -\frac{\varepsilon_l}{2} I & 0 & 0 \\ G_l\bar{K}_l & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} I & 0 \\ E_{lG}\bar{K}_l & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} I \end{bmatrix}$$

$$l,j\in\Omega,\quad j\in L_1,\quad l\in L_0 \qquad (3.31)$$

where

$$\Omega_l=-P_l+\frac{2}{\varepsilon_l}E_{lA}^T E_{lA}+4(H_l^T H_l+E_{lH}^T E_{lH})+E_l^T W_l E_l$$

$$\bar{\Omega}_l=-\bar{P}_l+\frac{2}{\varepsilon_l}E_{l\bar{A}}^T E_{l\bar{A}}+4(\bar{H}_l^T\bar{H}_l+E_{l\bar{H}}^T E_{l\bar{H}})+\bar{E}_l^T W_l\bar{E}_l,$$

$$\Omega_{lj}=-P_l+\frac{2}{\varepsilon_l}E_{lA}^T E_{lA}+4(H_l^T H_l+E_{lH}^T E_{lH})+E_l^T Q_{lj}E_l,$$

$$\bar{\Omega}_{lj}=-\bar{P}_l+\frac{2}{\varepsilon_l}E_{l\bar{A}}^T E_{l\bar{A}}+4(\bar{H}_l^T\bar{H}_l+E_{l\bar{H}}^T E_{l\bar{H}})+\bar{E}_l^T Q_{lj}\bar{E}_l,$$

and we define $\bar{P}_j=[I_{n\times n}\quad 0_{n\times 1}]^T P_j[I_{n\times n}\quad 0_{n\times 1}]$ for $j\in L_0$ in (3.28) and (3.29), and $\bar{P}_l=[I_{n\times n}\quad 0_{n\times 1}]^T P_l[I_{n\times n}\quad 0_{n\times 1}]$ for $l\in L_0$ in (3.30) and (3.31).

Proof: According to Lemma 3.1, we know that the system (3.1) or (3.5) is globally stable with disturbance attenuation γ, if the conditions (3.7)-(3.14) are satisfied. Because P_l *and* $\bar{P}_l$ are positive definite symmetric matrices, the conditions (3.7) and (3.9) are satisfied naturally. We will show that (3.20) and (3.21) imply (3.8).

We will first show that the inequality (3.20) implies $\gamma^2 I-D_{cl}^T P_l D_{cl}>0$, $l\in L_0$. It follows from (3.20) using Schur Complement Lemma A.2 that

$$P_l-P_l[\varepsilon_l I+2\gamma^{-2}(D_l D_l^T+E_{lD}E_{lD}^T)]P_l>0,$$

Using Lemma A.4, the left hand side of the above inequality implies that

$$\begin{aligned} & P_l-P_l[\varepsilon_l I+2\gamma^{-2}(D_l D_l^T+E_{lD}E_{lD}^T)]P_l \\ &=P_l-P_l[\varepsilon_l I+\gamma^{-2}(2D_l D_l^T+2E_{lD}E_{lD}^T)]P_l \\ &\le P_l-P_l[\varepsilon_l I+\gamma^{-2}(D_l D_l^T+E_{lD}E_{lD}^T+D_l E_{lD}^T+E_{lD}D_l^T)]P_l \\ &\le P_l-P_l[\varepsilon_l I+\gamma^{-2}(D_l D_l^T+\Delta D_l\Delta D_l^T+D_l\Delta D_l^T+\Delta D_l D_l^T)]P_l \qquad (3.32) \\ &=P_l-P_l[\varepsilon_l I+\gamma^{-2}(D_l+\Delta D_l)(D_l+\Delta D_l)^T)]P_l \\ &=P_l-P_l[\varepsilon_l I+\gamma^{-2}D_{cl}D_{cl}^T)]P_l \\ &=P_l-\gamma^{-2}P_l D_{cl}D_{cl}^T P_l-\varepsilon_l P_l P_l \end{aligned}$$

which implies that

$$P_l-\gamma^{-2}P_l D_{cl}D_{cl}^T P_l-\varepsilon_l P_l P_l>0.$$

Multiplying D_{cl}^T and D_{cl} from the left hand side and the right hand side of the above inequality respectively leads to,

$$D_{cl}^T P_l D_{cl}-\gamma^{-2}D_{cl}^T P_l D_{cl}D_{cl}^T P_l D_{cl}-\varepsilon_l D_{cl}^T P_l P_l D_{cl}\ge 0.$$

Since $P_l>0$, there exists a small enough constant $\delta>0$ such that

$$D_{cl}^T P_l D_{cl}-\gamma^{-2}D_{cl}^T P_l D_{cl}D_{cl}^T P_l D_{cl}-\delta\varepsilon_l D_{cl}^T P_l D_{cl}\ge 0,$$

that is,

$$(I-\gamma^{-2}D_{cl}^T P_l D_{cl}-\delta\varepsilon_l I)D_{cl}^T P_l D_{cl}\ge 0,$$

which implies that

$$(I-\gamma^{-2}D_{cl}^T P_l D_{cl}-\delta\varepsilon_l I)\ge 0.$$

Thus the desired result follows directly from the above inequality.

We then show that the inequality (3.21) implies the inequality (3.8). It is noted that via the Matrix Inversion Lemma A.3 the right hand side of the inequality (3.8) can be expressed as,

$$\begin{aligned} RH:=\ & A_{cl}^T P_l A_{cl}-P_l+E_l^T W_l E_l+A_{cl}^T P_l D_{cl}(\gamma^2 I-D_{cl}^T P_l D_{cl})^{-1} \\ & D_{cl}^T P_l A_{cl}+H_{cl}^T H_{cl} \\ =\ & A_{cl}^T(P_l^{-1}-\gamma^{-2}D_{cl}D_{cl}^T)^{-1}A_{cl}-P_l+E_l^T W_l E_l+H_{cl}^T H_{cl} \\ =\ & [A_l+\Delta A_l+(B_l+\Delta B_l)K_l]^T[P_l^{-1}-\gamma^{-2}(D_l+\Delta D_l)(D_l+\Delta D_l)^T]^{-1} \\ & [A_l+\Delta A_l+(B_l+\Delta B_l)K_l]-P_l+E_l^T W_l E_l+[H_l+\Delta H_l \\ & +(G_l+\Delta G_l)K_l]^T[H_l+\Delta H_l+(G_l+\Delta G_l)K_l] \end{aligned}$$

Let $\Theta=[P_l^{-1}-2\gamma^{-2}(D_l D_l^T+E_{lD}E_{lD}^T)]^{-1}$, which is positive definite via (3.20). Using Lemma A.1, we have

$$
\begin{aligned}
RH \le\ & (A_l + B_lK_l)^T\Theta(A_l + B_lK_l) + (A_l + B_lK_l)^T\Theta(\Delta A_l + \Delta B_lK_l) \\
& +(\Delta A_l + \Delta B_lK_l)^T\Theta(A_l + B_lK_l) + (\Delta A_l + \Delta B_lK_l)^T\Theta(\Delta A_l + \Delta B_lK_l) \\
& -P_l + E_l^TW_lE_l + 2(H_l + \Delta H_l)^T(H_l + \Delta H_l) + 2[(G_l + \Delta G_l)K_l]^T \\
& [(G_l + \Delta G_l)K_l] \\
\le\ & A_l + B_lK_l)^T\Theta(A_l + B_lK_l) + (A_l + B_lK_l)^T\Theta(\frac{1}{\varepsilon_l}I - \Theta)^{-1}\Theta \\
& (A_l + B_lK_l) + \frac{1}{\varepsilon_1}(\Delta A_l + \Delta B_lK_l)^T(\Delta A_l + \Delta B_lK_l) - P_l + E_l^TW_lE_l \\
& +4(H_l^TH_l + \Delta H_l^T\Delta H_l) + 4K_l^T(G_l^TG_l + \Delta G_l^T\Delta G_l)K_l \\
\le\ & (A_l + B_lK_l)^T\Theta(A_l + B_lK_l) + (A_l + B_lK_l)^T\Theta(\frac{1}{\varepsilon_l}I - \Theta)^{-1}\Theta(A_l + B_lK_l) \\
& + \frac{1}{\varepsilon_1}(\Delta A_l + \Delta B_lK_l)^T(\Delta A_l + \Delta B_lK_l) - P_l + E_l^TW_lE_l \\
& + 4(H_l^TH_l + \Delta H_l^T\Delta H_l) + 4K_l^T(G_l^TG_l + \Delta G_l^T\Delta G_l)K_l \\
\le\ & (A_l + B_lK_l)^T[\Theta^{-1} - \varepsilon_lI]^{-1}(A_l + B_lK_l) + \frac{2}{\varepsilon_l}(\Delta A_l^T\Delta A_l + \\
& K_l^T\Delta B_l^T\Delta B_lK_l) - P_l + E_l^TW_lE_l + 4(H_l^TH_l + \Delta H_l^T\Delta H_l) \\
& +4K_l^T(G_l^TG_l + \Delta G_l^T\Delta G_l)K_l \\
\le\ & (A_l + B_lK_l)^T[\Theta^{-1} - \varepsilon_lI]^{-1}(A_l + B_lK_l) + \frac{2}{\varepsilon_l}(E_{lA}^TE_{lA} + K_l^TE_{lB}^TE_{lB}K_l) - P_l \\
& +E_l^TW_lE_l + 4(H_l^TH_l + E_{lH}^TE_{lH}) + 4K_l^T(G_l^TG_l + E_{lG}^TE_{lG})K_l \\
\le\ & (A_l + B_lK_l)^T[\Theta^{-1} - \varepsilon_lI]^{-1}(A_l + B_lK_l) + \frac{2}{\varepsilon_l}E_{lA}^TE_{lA} - P_l + E_l^TW_lE_l \\
& + 4(H_l^TH_l + E_{lH}^TE_{lH}) + K_l^T(\frac{2}{\varepsilon_l}E_{lB}^TE_{lB} + 4G_l^TG_l + 4E_{lG}^TE_{lG})K_l
\end{aligned} \tag{3.33}
$$

On the other hand, the following inequality

$$
\begin{aligned}
0 >\ & (A_l + B_lK_l)^T[\Theta^{-1} - \varepsilon_lI]^{-1}(A_l + B_lK_l) + \frac{2}{\varepsilon_l}E_{lA}^TE_{lA} - P_l + E_l^TW_lE_l \\
& + 4(H_l^TH_l + E_{lH}^TE_{lH}) + K_l^T(\frac{2}{\varepsilon_l}E_{lB}^TE_{lB} + 4G_l^TG_l + 4E_{lG}^TE_{lG})K_l
\end{aligned} \tag{3.34}
$$

implies (3.8). Using Schur complement formulas, it is easily shown that the inequality (3.34) is in turn equivalent to the bilinear matrix inequality (3.21). Thus, we have shown that the inequality (3.21) implies (3.8). Following the similar procedure, we can also show that the inequality (3.22) and (3.23) implies $\gamma^2I - \bar{D}_{cl}{}^T\bar{P}_l\bar{D}_{cl} > 0$ and the inequality (3.10). Similarly, we can also show that other inequalities (3.24)-(3.31) imply the other conditions in Lemma 3.1. Therefore, it can be concluded from the Lemma 3.1 that the closed loop control system is globally stable with disturbance attenuation γ and thus the proof is completed. ∇∇

It is noted that the matrix inequalities in (3.20)-(3.31) are the BMIs. They can be solved by V-K iteration method [30]. The details of the solution procedure can be summarized in the following algorithm.

Algorithm 1:

V-Step. Given a fixed controller gain $K_l, l \in L_0, \bar{K}_l, l \in L_1$, solve the following optimization problem

$$
\min_{P_l, \bar{P}_l, W_l, Q_{lj}} \lambda_l, \lambda_{lj}
$$

s.t. (3.20), (3.22), (3.24), (3.26), (3.28), (3.30)

$\Psi_l - \lambda_lI < 0$, $\bar{\Psi}_l - \lambda_lI < 0$, $\Psi_{lj} - \lambda_{lj}I < 0$, and $\bar{\Psi}_{lj} - \lambda_{lj}I < 0$.

with P_l and $\bar{P}_l$ defined in (3.15) for a set of positive definite matrices $P_l, l \in L_0, \bar{P}_l, l \in L_1$.

K-Step. Using the matrices P_l and $\bar{P}_l$ obtained in Step V, solve the following optimization problem

$$
\min_{K_l, \bar{K}_l, W_l, Q_{lj}} \lambda_l, \lambda_{lj}
$$

s.t. $\Psi_l - \lambda_lI < 0$, $\bar{\Psi}_l - \lambda_lI < 0$, $\Psi_{lj} - \lambda_{lj}I < 0$, and $\bar{\Psi}_{lj} - \lambda_{lj}I < 0$,

for a set of matrices $K_l, l \in L_0, \bar{K}_l, l \in L_1$.

The above iteration stops when $\lambda_l < 0, l \in L, \lambda_{lj} < 0, l, j \in \Omega$.

In the case of $a_l \equiv 0$ for all $l \in L$ and $\Omega = L \times L$, we have the following corollary.

Corollary 3.1: Given a constant $\gamma > 0$, the system (3.1) is globally stable with disturbance attenuation γ, if there exist a set of positive constants $\varepsilon_l, l = 1, 2, \cdots, m$, a set of positive definite symmetric matrices $P_l, l \in L$, symmetric matrices W_l and Q_{lj} such that W_l and Q_{lj} have nonnegative entries, and the following BMIs are satisfied,

$$
0 < \begin{bmatrix} P_l & P_l \\ P_l & [\varepsilon_lI + 2\gamma^{-2}(D_lD_l^T + E_{lD}E_{lD}^T)]^{-1} \end{bmatrix}, \quad l \in L \tag{3.35}
$$

$$
0 > \Psi_l := \begin{bmatrix}
\Omega_l & (A_l + B_lK_l)^TP_l & 0 & 0 & 0 & K_l^TE_{lB}^T & K_l^TG_l^T & K_l^TE_{lG}^T \\
P_l(A_l + B_lK_l) & -P_l & P_lD_l & P_lE_{lD} & P_l & 0 & 0 & 0 \\
0 & D_l^TP_l & -\frac{1}{2}\gamma^2I & 0 & 0 & 0 & 0 & 0 \\
0 & E_{lD}^TP_l & 0 & -\frac{1}{2}\gamma^2I & 0 & 0 & 0 & 0 \\
0 & P_l & 0 & 0 & -\frac{1}{\varepsilon_l}I & 0 & 0 & 0 \\
E_{lB}K_l & 0 & 0 & 0 & 0 & -\frac{\varepsilon_l}{2}I & 0 & 0 \\
G_lK_l & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4}I & 0 \\
E_{lG}K_l & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4}I
\end{bmatrix}
$$

$l \in L$ (3.36)

$$
0 < \begin{bmatrix} P_j & P_j \\ P_j & [\varepsilon_lI + 2\gamma^{-2}(D_lD_l^T + E_{lD}E_{lD}^T)]^{-1} \end{bmatrix}, \quad l, j \in \Omega \cap L \tag{3.37}
$$

$$
0 > \Psi_{lj} := \begin{bmatrix}
\Omega_{lj} & (A_l + B_lK_l)^TP_j & 0 & 0 & 0 & K_l^TE_{lB}^T & K_l^TG_l^T & K_l^TE_{lG}^T \\
P_j(A_l + B_lK_l) & -P_j & P_jD_l & P_jE_{lD} & P_j & 0 & 0 & 0 \\
0 & D_l^TP_j & -\frac{1}{2}\gamma^2I & 0 & 0 & 0 & 0 & 0 \\
0 & E_{lD}^TP_j & 0 & -\frac{1}{2}\gamma^2I & 0 & 0 & 0 & 0 \\
0 & P_j & 0 & 0 & -\frac{1}{\varepsilon_l}I & 0 & 0 & 0 \\
E_{lB}K_l & 0 & 0 & 0 & 0 & -\frac{\varepsilon_l}{2}I & 0 & 0 \\
G_lK_l & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4}I & 0 \\
E_{lG}K_l & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4}I
\end{bmatrix}
$$

$l, j \in \Omega \cap L$ (3.38)

where

$$
\Omega_l = -P_l + \frac{2}{\varepsilon_l}E_{lA}^TE_{lA} + 4(H_l^TH_l + E_{lH}^TE_{lH}) + E_l^TW_lE_l,
$$

$$
\Omega_{lj} = -P_l + \frac{2}{\varepsilon_l}E_{lA}^TE_{lA} + 4(H_l^TH_l + E_{lH}^TE_{lH}) + E_l^TQ_{lj}E_l.
$$

Then the following simplified algorithm can be implemented.

Algorithm 2:

V-Step. Given a fixed controller gain $K_l, l \in L$, solve the following optimization problem

$$\min_{P_l, W_l, Q_{lj}} \lambda_l, \lambda_{lj}$$

$$s.t.\ (3.35), (3.37)\ \Psi_l - \lambda_l I < 0, \text{ and } \Psi_{lj} - \lambda_{lj} I < 0.$$

with P_l defined in (3.15) for a set of positive definite matrices $P_l, l \in L$.

K-Step. Using the matrices P_l obtained in V-Step, solve the following optimization problem

$$\min_{K_l, W_l, Q_{lj}} \lambda_l, \lambda_{lj}$$

$$s.t.\ \Psi_l - \lambda_l I < 0, \text{ and } \Psi_{lj} - \lambda_{lj} I < 0,$$

for a set of matrices $K_l, l \in L$.

The above iteration stops when $\lambda_l < 0, l \in L, \lambda_{lj} < 0, l, j \in \Omega$.

4. An example

Consider the modified Henon mapping model with external disturbance

$$\begin{cases} x_1(t+1) = -x_1^2(t) + 0.3x_2(t) + 1.4 + u(t) + 0.01\sin(0.02\pi t) \\ x_2(t+1) = x_1(t) \end{cases} \quad (4.1)$$

If we choose $u(t) \equiv 0$, the system dynamic appears in chaotic manner as shown in Fig. 1 with the initial condition $x(0) = [0.1 \ \ 0]^T$.

One of the most frequent objectives is to stabilize the chaotic system at one of its fixed points embedded in the attractor region. Obviously, we can get the two fixed points $x_f = [-1.5839 \ \ -1.5839]^T$ and $x_f = [0.8839 \ \ 0.8839]^T$ of the autonomous system of (4.1) by

$$\begin{cases} x_{f1}(t+1) = x_{f1} = -x_{f1}^2 + 0.3x_{f2} + 1.4 \\ x_{f2}(t+1) = x_{f2} = x_{f1} \end{cases}$$

We choose the point $x_f = [x_{f1} \ \ x_{f2}]^T = [0.8839 \ \ 0.8839]^T$ as the control goal. Thus the problem can be transformed into the H_∞ control problem at zero of the following error system:

$$\begin{cases} e_1(t+1) = -e_1^2(t) - 2x_{f1}e_1 + 0.3e_2(t) + u(t) + 0.01\sin(0.02\pi t) \\ e_2(t+1) = e_1(t) \\ z(t) = 0.1e_1(t) + 0.1u(t) \end{cases} \quad (4.2)$$

where $e_1(t) = x_1(t) - x_{f1}$ and $e_2(t) = x_2(t) - x_{f2}$ are the errors to the fixed point.

The error system can be represented exactly by the following T-S fuzzy model when $e_1(t) \in [-d - 2x_{f1}, d - 2x_{f1}]$ where $d > 0$ is a constant:

R^1 : IF $e_1(t)$ is F_1

THEN $e(t+1) = A_1 e(t) + B_1 u(t) + D_1 v(t)$

$z(t) = H_1 e_1(t) + G_1 u(t)$

R^2 : IF $e_1(t)$ is F_2

THEN $e(t+1) = A_2 e(t) + B_2 u(t) + D_2 v(t)$

$z(t) = H_2 e_1(t) + G_2 u(t)$

where the fuzzy sets are chosen as

$$F_1(e_1(t)) = \frac{1}{2}\left(1 + \frac{e_1(t) + 2x_{f1}}{d}\right), \quad F_2(e_1(t)) = \frac{1}{2}\left(1 - \frac{e_2(t) + 2x_{f1}}{d}\right),$$

$d = 2$

and the other parameters are as follows:

$$A_1 = \begin{bmatrix} -d & 0.3 \\ 1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} d & 0.3 \\ 1 & 0 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$D_1 = D_2 = \begin{bmatrix} 0.01 \\ 0 \end{bmatrix}, \quad v(t) = \sin(0.02\pi t).$$

It is noted that the regions are

$$\bar{S}_1 = \{e_1(t) | -d - 2x_{f1} \le e_1 \le -2x_{f1}\}, \quad \bar{S}_2 = \{e_1(t) | -2x_{f1} \le e_1 \le d - 2x_{f1}\}.$$

Then, based on the technique developed in [18], the characterizing matrices E's can be obtained as follows,

$$E_1 = \begin{bmatrix} 0 & 0 \\ -0.5 & -0.8839 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 & 0 \\ 0.5 & 0.8839 \end{bmatrix}.$$

Now we set the parameters of controlled output as

$$H_1 = H_2 = [0.1 \ \ 0], \quad G_1 = G_2 = 0.1,$$

and we consider the following uncertainty bounds:

$$E_{1A} = E_{2A} = \begin{bmatrix} -0.4 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_{1B} = E_{2B} = E_{1D} = E_{2D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$E_{1H} = E_{2H} = [0 \ \ 0] \qquad E_{1G} = E_{2G} = 0.$$

We choose the initial controller gains by assigning closed loop poles of each subsystem at (0.1, 0.2), that is,

$$K_1 = [2.3 \ \ -0.32] \text{ for } \bar{S}_1 \quad \text{and} \quad K_2 = [-1.7 \ \ -0.32] \text{ for } \bar{S}_2.$$

With the disturbance attenuation $\gamma = 0.8$ and $\varepsilon_1 = \varepsilon_2 = 1$, the following solutions have been obtained via the Algorithm 2 after two iterations.

$$P_1 = \begin{bmatrix} 0.8628 & -0.0882 \\ -0.0882 & 0.1538 \end{bmatrix}, P_2 = \begin{bmatrix} 0.8526 & -0.0741 \\ -0.0741 & 0.1508 \end{bmatrix},$$

$$K_1 = [2.1821 \ \ -0.2988], \quad K_2 = [-1.8009 \ \ -0.2985], \quad \lambda_{\min} = -0.0084$$

Simulation results of stabilization to the desired fix point with initial conditions $x(0) = [0.1, 0]^T$ are shown in Fig.2 where the control input is added after $t > 100$ seconds.

5. Conclusions

In this paper, a new method is developed to design robust H_∞ controller for discrete time fuzzy dynamic systems based on a piecewise Lyapunov function. A constructive controller design algorithm is also given based on BMI techniques.

Appendix:

Lemma A.1: Let A and E be matrices of appropriate dimensions, and P be a symmetric matrix satisfying

$$\frac{1}{\varepsilon}I - P > 0, \quad \varepsilon > 0,$$

then

$$A^T PE + E^T PA + E^T PE \le A^T P(\frac{1}{\varepsilon}I - P)^{-1} PA + \frac{1}{\varepsilon}E^T E.$$

Lemma A.2 (Schur Complements): Given constant matrices $\Omega_1, \Omega_2, \Omega_3$, where $0 < \Omega_1 = \Omega_1^T$ and $0 < \Omega_2 = \Omega_2^T$, then $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$ if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^T & \Omega_1 \end{bmatrix} < 0.$$

Lemma A.3 (Matrix Inversion Lemma): For any real nonsingular matrices Σ_1, Σ_3 and real matrices Σ_2, Σ_4 with appropriate dimensions, it follows that,

$$(\Sigma_1 + \Sigma_2 \Sigma_3 \Sigma_4)^{-1} = \Sigma_1^{-1} - \Sigma_1^{-1} \Sigma_2 \left[\Sigma_3^{-1} + \Sigma_4 \Sigma_1^{-1} \Sigma_2 \right]^{-1} \Sigma_4 \Sigma_1^{-1}$$

Lemma A.4: Let X, Y be real constant matrices of compatible dimensions. Then

$$X^T Y + Y^T X \le \varepsilon X^T X + \varepsilon^{-1} Y^T Y$$

holds for any $\varepsilon > 0$.

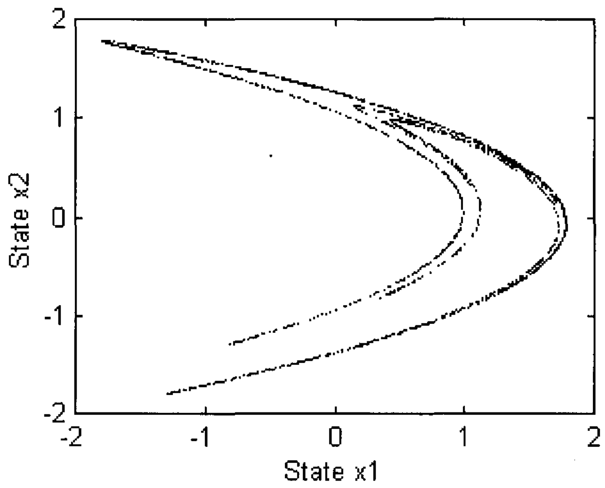
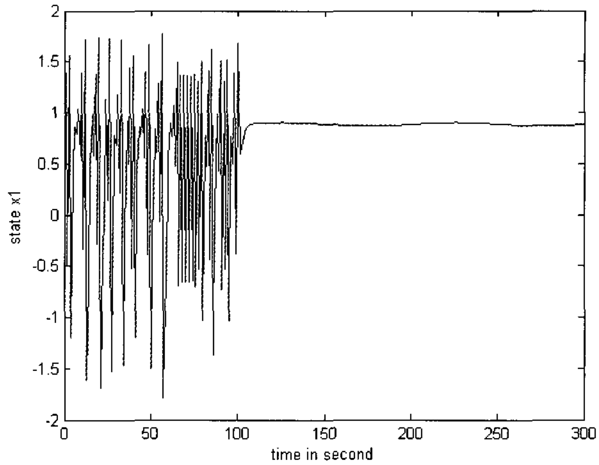
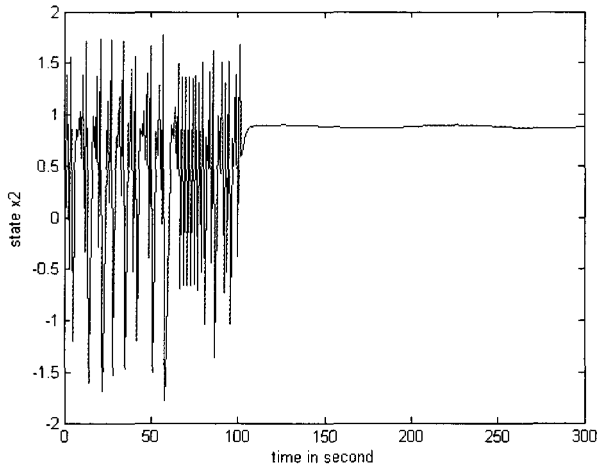


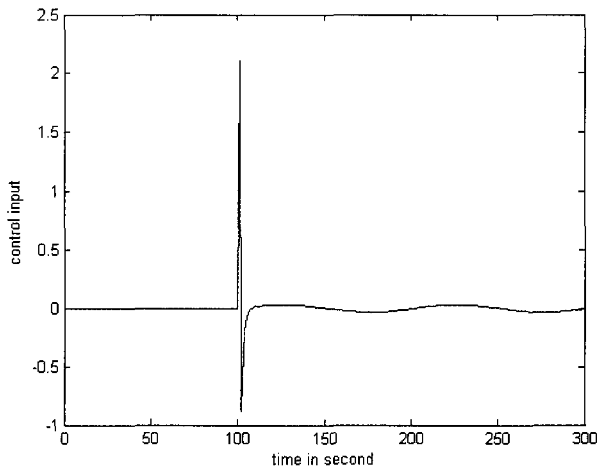
Fig. 1 The chaotic behavior of the unforced Henon Map



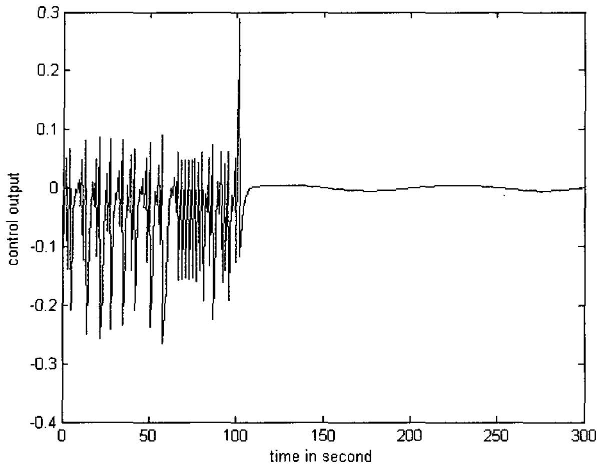
(a) Time responses of x1



(b) Time responses of x2



(c) The control input u(t)



(d) The controlled output z(t)

Fig. 2. The control results of the Henon system.

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