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Comparison of Different Mathematical Models for Describing the Complete Lactation of Akkaraman Ewes in Turkey

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ABSTRACT : This study was carried out to investigate the use of three different mathematical models (Wood, Quadratic and Cubic) for describing the lactation curve of Akkaraman ewes. Data were collected from 42 ewes that were three years of age and from the same flock raised in The State Farm of Gözlü in Konya Province. All ewes lambed in March. They were hand milked twice daily and the first milk test was performed with in the first month after lambing (mean = 27.8 day, SD = 4.26) in an attempt to describe the peak yield. The differences between estimated total milk yields by the models were not significant. The models were adequate for describing total milk yield. The differences between peak yields were not significant. The Wood model estimated the time of peak yield earlier than the other models and observed values (p<0.01). Especially the Cubic model's peak time was very close to really peak time (34.30 vs. 35.33 days). R^2 values of the models ranged from 85.85% to 96.20%. The Cubic model gave the best R^2 value (p<0.01). Correlation coefficients between descriptive values of the models changed from 0.32 to 1.00. Total milk yields of the models were highly correlated (above 0.99) with the total milk yield calculated by the Fleischmann method (p<0.01). As a result the Cubic model showed the best fit to the data collected from Akkaraman ewes and allowed a suitable description of the shape of the lactation curve. (**Key Words :** Akkaraman, Ewes, Lactation Curve, Milk Yield, Mathematical Model)

INTRODUCTION

Sheep populations of Turkey consist of 25 million heads (Anonymous, 2003). Akkaraman is one of the fat tailed sheep breeds of Turkey. It has the largest population (above 40% of the total sheep population) among the sheep breeds of the country (Ertuğrul, 1997). The Akkaraman breed is distributed throughout central Anatolia. It is very hardy and strives well under poor feeding and extreme climatic conditions. The husbandry is typically extensive, with animals kept in simple sheep-sheds during winter, when they are fed on straw. In some flocks, animals receive some hay and limited amount of concentrates for a short period before and after lambing. Mating takes place in September and October, with lambing in February and March. Lambs are weaned at about 2-3 months of age, after which ewes are milked. The milking period is 2-3 months. Marketable milk yield (during the milking period) is about 40-50 kg. Ewes body weight and greasy fleece weight averages are 40-50 kg and 1.5-2.0 kg respectively. Tail weight generally changes from 4 to 6 kg.

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Sheep milk is an important product for Turkey. Total milk production of Turkey is about 8.4 million tons. Approximately 7.82% of total milk production is produced by sheep (Anonymous, 2003). Akkaraman breed has an important share in this production. Studies on lactation curves of native sheep breeds of Turkey are newly carried out. The lack of studies on the complete lactation of native sheep breed is particularly due to the fact that in most sheep production systems are extensive. In these systems, lambs are allowed to suck for at least 30 days post lambing. Milk recording is difficult and requires extra labour. Generally it is started after weaning. However in some studies especially carried out in the state farms, milk recording was started at beginning of the lactation in order to estimate whole lactation milk yield (Vanlı et al., 1984; Dayıoğlu and Akyurt, 1988; Akmaz, 1994).

Lactation curve characteristics of native sheep breeds should be determined. It is important for improving the milk yields of these breeds and obtaining desired lactation shape. Models of lactation curves allow the evaluation of genetic and environmental factors on components of milk production such as persistency (Gengler, 1996), maximum production and days to maximum production (Masselin et al., 1987; Gipson and Grossman, 1990). The curves could help in the management of herds, particularly in culling and in assessing nutritional and health status of animals (Sauvant and Fehr, 1975; Dudouet, 1982). Curves are also useful for developing unbiased comparison methods among animals with incomplete lactation records for genetic evaluation purposes (Keown and Van Vleck, 1973), and are required for the test-day method of genetic evaluation (Guo and Swalve, 1995). Animals with a high milk yield potential can identified by using this information before the whole lactation completed. Also, lactation curves can be used to evaluate a suitable time to end milking (Chang et al., 2001).

Several models of lactation curves have been developed (Wood, 1967; Neal and Thornley, 1983; Grossman and Koops, 1988; Morant and Gnanasakthy, 1989; Rook et al., 1993; Cappio-Borlino et al., 1997; Dijkstra et al., 1997; Pollot, 1999; Pollot and Gootwine, 2000). The Wood model has been used in most lactation curve model studies, because it includes the basic features of lactation curves with only three parameters that allows the calculation of some key measurements such as average production, maximum production and day to maximum production (Wood, 1967). This has made the Wood model the most widely used function for the description of lactation curves. Most of the alternative models are also based on Wood model (Cobby Le Du, 1978; Wilmink, 1987; Papajsik and Bodero, 1988). They usually incorporate additional parameters to improve the fitness and have been developed from experimental data (Morant and Gnanasakthy, 1989). These alternative models have generally been applied to lactation data for dairy cattle. Most studies on the lactation curve of sheep have been based on the Wood model (Torres-Hernandez and Hohenboken, 1980; Sakul and Boylan, 1992; Portolano et al., 1996). However, little systematic comparison has been made of several models to determine the best suited for unimproved sheep lactation data. Dag et al. (2005), evaluated four different mathematical functions (Wood, Inverse Polynomial, Quadratic and Cubic models) for describing the lactation curve of unimproved Awassi ewes. R^2 values of the models ranged from 0.724 to 0.977. The Cubic model gave the best R^2 value. Especially the Inverse Polynomial model was insufficient for describing the data. This study results showed that the Cubic model gave the best fit to the data collected from unimproved Awassi ewes and allowed a suitable description of the shape of the lactation curve. The Inverse Polynomial model was not evaluated in the study because of giving the lowest fit as mentioned above.

The aim of the present paper was to compare the Wood, Quadratic and Cubic models in their application to the lactation of Akkaraman ewes.

MATERIALS AND METHODS

The experiment was carried out on the lactations of 42 Akkaraman ewes that were three years of age and belonged to the same flock raised in The State Farm of Gözlü in Konya Province (38° 27'N, 32° 22'E and 930 m above sea level). The other age groups were not taken into consideration because of having less number of ewes. Age at first lambing was approximately 24 months. All ewes lambed in March. They were hand milked twice daily and the first milk test was performed with in the first month after lambing (mean = 27.8 day, SD = 4.26) in an attempt to describe the peak yield. All lambs sucked their dams freely until first milk recordings. They were then kept on a residual suckling regimen until 75 days of age, when they were weaned completely from milk. During the residual suckling period, lambs joined their dams after morning and evening milking for residue suckling for a period of 30 minutes at a time. The lactating ewes were grazed from April to December and were kept and fed indoors throughout the winter.

Milk yield was recorded fortnightly and total milk yield (TMY) through the lactation was calculated by using Fleischmann method:

$$\text{TMY} = y_1 t_1 + \sum \left((y_i + y_{i+1}) / 2 (t_{i+1} - t_i) \right)$$

Where TMY is total milk yield; y_1 is yield at first milk record, t_1 is interval between lambing and first recording; y_i is yield of the record i and t_i is interval between the record i and the record (i+1), (i = 1, … k) (Ruiz et al., 2000).

Three different mathematical models were used for describing the lactation curve. These were Wood, Quadratic and Cubic model. The Wood model:

$$Y_{(t)} = at^b e^{(-ct)}$$

peak yield for this model is

$$Y_{max} = a(b/c)^b e^{-b}$$

occurs at time b/c; the Quadratic model:

$$Y_{(t)} = a + bt + ct^2$$

peak yield for this model is

$$Y_{max} = a + b(-b/2c) + c(-b/2c)^2,$$

time at peak yield t = -b/2c, the Cubic model:

$$Y_{(t)} = a + bt + ct^2 + dt^3$$

Table 1. Estimates of the model parameters

Parameters	Models		
	Wood	Quadratic	Cubic
a	0.508±0.029	0.618±0.031	0.549±0.028
b	0.2535±0.0274	0.0031±0.0006	0.0107±0.0008
c	0.014049±0.000540	-0.000051±0.000004	-0.000195±0.000014
d	-	-	0.000000713±0.000000265

Table 2. Comparison of the models for estimating total milk yield (TMY), peak yield (PY), time to peak yield (PT), Persistency (P) and goodness-of-fit statistics (R^2 and MSE values)

	Models			
	Fleischmann/really	Wood	Quadratic	Cubic
TMY (*l*)	$89.58±3.81^{ns}$	$91.23±3.97^{ns}$	$91.61±3.89^{ns}$	$91.05±3.90^{ns}$
PY (*l*)	$0.783±0.035^{ns}$	$0.784±0.033^{ns}$	$0.691±0.028^{ns}$	$0.730±0.029^{ns}$
PT (day)	$35.33±1.52^{A}$	$17.91±1.16^{B}$	$29.44±2.89^{A}$	$34.30±1.76^{A}$
P (%)	$79.05±0.75^{b}$	$86.83±0.52^{a}$	$79.69±0.83^{b}$	$81.67±0.86^{b}$
R^2 (%)	-	$86.85±1.36^{C}$	$90.91±0.97^{B}$	$96.20±0.50^{A}$
MSE	-	$0.00051±0.00010^{ns}$	$0.00253±0.00220^{ns}$	$0.00013±0.00002^{ns}$

$^{a, b, c}$ $p<0.05$; $^{A, B, C}$ $p<0.01$ (in the same row), ^{ns} not significant.

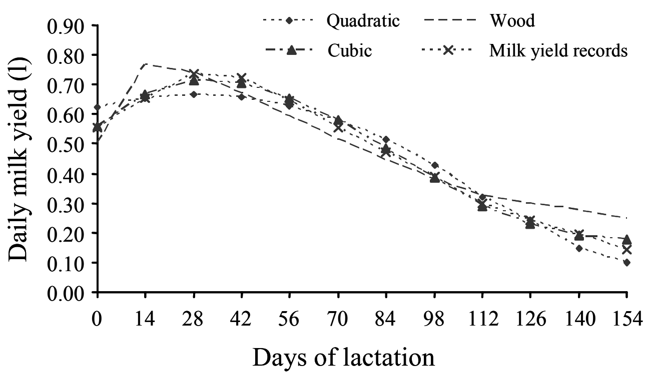

Figure 1. Shape of lactation curve according to the models.

peak yield for this model is

$$Y_{max} = a + b\left(-c - \sqrt{c^2 - 3bd} / 3d\right) + c\left(-c - \sqrt{c^2 - 3bd} / 3d\right)^2 + d\left(-c - \sqrt{c^2 - 3bd} / 3d\right)^3$$

time at peak yield:

$$t = -c - \sqrt{c^2 - 3bd / 3d}$$

Where $Y_{(t)}$ is the milk yield at day x from lambing, e is the base of natural logarithm, a, b, c and d are the parameters which characterize the shape of the curve and which were estimated from a nonlinear regression analysis using the Minitab program (Minitab, 1995).

Persistency (P) was calculated as:

$$P(\%) = \frac{\sum_{i=1}^{k} (pi+1)/pi}{k} \times 100$$

Where pi is the yield of the record i that starts at peak time and *k* is the record number from peak time to the end of lactation.

The parameters obtained were used to calculate the predicted yields in the original equations above. Residuals, defined as the absolute values of the difference between the predicted yield and real data of daily milk yield, were calculated and then the mean square error (MSE) for each lactation curve fitted was calculated and averaged for each model.

The models were compared in respect of their MSE, R^2, TMY, peak yield (PY), peak time (PT) and persistency (P). Also the correlation coefficients among these characteristics were determined for evaluating the fitness of the models.

RESULTS

The parameters of the models were given in Table 1. The parameter a, which represents the estimated production at the moment of lambing was lower for the Wood model in comparison with the Quadratic model. The b parameter, responsible for the rising phase of the curve, showed the highest value for the Wood model. The parameter c, which represents the pattern of the decline in milk production at time t, showed the highest value for the Wood model. The lactation curves for the models were given in Figure 1. It is clearly seen that from Figure 1, the Wood model overestimated milk yields for the last four records in comparison with observed milk yields and estimated milk yields by Quadratic and Cubic models.

Total milk yields (TMY), peak yield and day of peak yield calculated by Fleischmann and the models were given in Table 2. Also R^2 and mean square error (MSE) values of the models were given in the same table. Total milk yields

Table 3. Correlation coefficients between descriptive values of the models

	FTMY	WTMY	QTMY		RPT	WPT	QPT
WTMY	0.991**			WPT	0.417*		
QTMY	0.995**	0.996**		QPT	0.433*	0.809**	
CTMY	0.994**	0.997**	1.000**	CPT	0.623**	0.853**	0.856**
	RPY	WPY	QPY		RP	WP	QP
WPY	0.974**			WP	0.436*		
QPY	0.975**	0.986**		QP	0.767**	0.390*	
CPY	0.980**	0.990**	0.992**	CP	0.907**	0.322ns	0.820**

** $p<0.01$; * $p<0.05$; ns: not significant.

FTMY: Fleischmann total milk yield; WTMY: Wood total milk yield; QTMY: Quadratic total milk yield; CTMY: Cubic total milk yield; RPT: Really peak time; WPT: Wood peak time; QPT: Quadratic peak time; CPT: Cubic peak time; RPY: Really peak yield; WPY: Wood peak yield; QPY: Quadratic peak yield; CPY: Cubic peak yield; RP: Really persistency; WP: Wood persistency; QP: Quadratic persistency; CP: Cubic persistency.

changed from 89.58 L to 91.61 L for the models. The differences among total milk yields of the models were not significant. Also the differences among the peak yields of the models were not significant. Peak time of the Wood model was found to be lower than the peak times of the other models and test day records ($p<0.01$).

The persistency value of the Wood model was higher than the other persistency values ($p<0.05$). It is also clearly seen from the Figure 1. Towards the end of the lactation, the Wood model overestimated the milk yields. Therefore the persistency of this model was found to be higher. The Cubic model persistency was very close to really persistency. Anyway, the curves of Cubic model and observed values fitted to each other (Figure 1).

R^2 values of the models ranged from 85.85% to 96.20%. The differences between R^2 values of the models were significant ($p<0.01$). The Cubic model gave the best R^2 value (96.20%), when the Wood model gave the lowest R^2 value (85.85%).

Although, the lowest MSE value was found for the Cubic model, the differences between the MSE values were not significant. Therefore the goodness-of-fit of the models should be evaluated by using R^2 and other descriptive values.

Correlation coefficients between descriptive values of the models were given in Table 3. Correlation coefficients among total milk yields were significant ($p<0.01$). This situation is also caused by the approximation of the total milk yields estimated by the models to the Fleischmann total milk yield. The correlation coefficients among peak time values were changed from 0.417 to 0.856. The Cubic model's peak time was highly correlated with really peak time ($p<0.01$). Also there were highly significant relationships between the peak yields of the models and really peak yield ($p<0.01$). The highest correlation coefficient for the persistency values was found between really and the Cubic model persistency values.

DISCUSSION

In this paper the use of three mathematical functions for describing the complete lactation curve of Akkaraman ewes were investigated. Although the lactation of sheep has been extensively reported, especially the studies on the complete lactation of native sheep breeds are rare due to extensive management practices. Suckling period lasts at least two months. The suckling period coincides with the rising phase and peak time of lactation, making them impossible to estimate. In this study, milk test was performed with in the first month after lambing for describing peak yield.

TMY, PY, PT, P, R^2 and MSE values were evaluated for the fitness of the models. Really total milk yields and estimated total milk yields by the models were close to each other. A similar situation was observed for the peak yields. However the Wood model estimated the time of peak yield earlier than the other models and observed values. Similarly in some studies the Wood model forecasted peak yield earlier than observed values and the other models (Portolano et al., 1996; Franci et al., 1999). Especially the Cubic models peak time was very close to peak time determined from test day records (34.30 vs. 35.33 days). It therefore seems that, the Wood model can be used for describing the lactation curves which beginning at low milk yields and reaching peak earlier than average.

The day of peak yield was reported as 30.28 days for the lactation curves of first lambing Comisana ewes with winter lambing (Portolano et al., 1996); 39.2 days for Sarda ewes (Cappio-Borlino et al., 1997) and 27 days for typical improved Awassi lactation curve (Pollot and Gootwine, 2000). Peak time in this research is between these ranges.

Actual persistency value and estimated persistency values of the models were lower in comparison to the persistency values of the dairy ewes (Portolano et al., 1996; Franci et al., 1999; Pollot and Gootwine, 2000).

Ruiz et al. (2000) determined R^2 values of six mathematical models ranging from 93% to 97% and they suggested a nonlinear variable decay model for describing the lactation curve of Latxa sheep. R^2 values of the Cubic and Quadratic models were similar to these findings. R^2 value of the Wood model in this study was higher than the findings reported by Portolano et al. (1996) and Franci et al. (1999) for the R^2 values of the Wood model.

CONCLUSION

The best estimations of total milk yield, peak time and persistency made by the Cubic model. This model also showed the highest R^2 value (96.20%). As a result, the Cubic model showed the best fit to the data collected from Akkaraman ewes and allowed a suitable description of the shape of the lactation curve.

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