Fuzzy Preference Based Interactive Fuzzy Physical Programming and Its Application in Multi-objective Optimization

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Interactive Fuzzy Physical Programming (IFPP) developed in this paper is a new efficient multi-objective optimization method, which retains the advantages of physical programming while considering the fuzziness of the designer's preferences. The fuzzy preference function is introduced based on the model of linear physical programming, which is used to guide the search for improved solutions by interactive decision analysis. The example of multi-objective optimization design of the spindle of internal grinder demonstrates that the improved preference conforms to the subjective desires of the designer.

Key Words : Multi-objective Optimization, Physical Programming, Fuzzy Number, Fuzzy Preference, Interactive Fuzzy Physical Programming, Interactive Decision-making, Spindle of NC Internal Grinder

1. Introduction

Generally, product design has been taken as a multi-objective problem and the decision-making, and design process is actually an optimizing process, considering multi-restricted conditions. Multi-objective optimization has been researched and applied widely. Some new algorithms for multi-objective optimization appear, such as collaborative optimization (Tappeta et al., 1997 ; Huang et al., 2005a), interactive multi-objective optimization (Tappeta et al., 1997 ; Huang et al., 2005b), physical programming (Messac et al., 1996 ; Huang et al., 2005c) and VEGA (Vector Evaluated Genetic Algorithm) (Tappeta et al., 2001). These algorithms have specific characteristics, and have found applications in various engineering practical problems.

Physical programming is an efficient multi-objective optimization method first developed by Messac in 1995. It captures the designer's preferences using a preference function, and places the design process into a more flexible and natural framework. Physical programming provides the means to reliably employ optimization with minimal prior knowledge thereof. Once the designer's preferences are articulated, obtaining the corresponding optimal design is a non-iterative process.

Human participation is one of the important resources to the fuzziness of engineering system (Huang et al., 2004, 2005d). It is no doubt that

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the process of the designers' recognition and inference has fuzziness, which could result in the fuzziness of his decision-making. While the preference function is the map from the designer's decision to the optimization model, it contains fuzziness because of the shortage of information and experience.

Through the man-machine interactive interface, interactive decision-making enables the designer to control the optimization process to some extent, which can effectively improve the design efficiency and design result, thus avoiding time wasted in an errant direction during the design process.

2. Mathematical Model of Physical Programming Based on Fuzzy Preference Function

2.1 Physical programming (Tappeta et al., 2000; Messac et al., 2001)

Physical programming is a new effective multidisciplinary optimization method, which reduces the computational intensity of large problems and places the design process into a more flexible and natural framework (Messac, 1996). It has been successfully applied to control, structure design, interactive design, and robust design.

Within the physical programming procedure, the engineer expresses objectives with respect to each design metric using four different classes. Each class comprises two cases, hard and soft. The preference functions take the form of a spline segment that can be defined by its value and slope at its left and right boundaries.

Usually, Genetic algorithms are used to solve the model (Tian et al., 2002) ; it can obtain the global optimum even when local optima exist.

2.2 Model of fuzzy linear physical programming

In physical programming, the boundary of the preference region is a constant of practical physical significance, the value of which is evaluated by the designer. Because the precise value cannot be given in most cases, this paper considers that

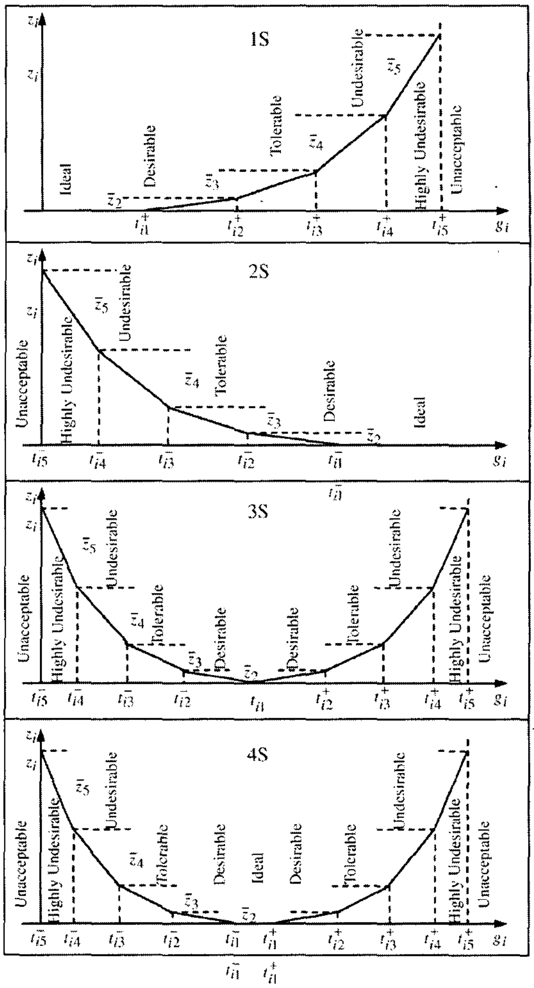

Fig. 1 Class function regions for the generic i-th objective

it is relatively closer to the designers' intents for the preference function, in which the boundary of preference region is defined as a fuzzy number.

Linear physical programming is a simplified form of physical programming. The soft class function is shown in Figure 1. The model of linear physical programming (Messac et al., 1996) takes the form :

$$\min_{d_{is}^-, d_{is}^+, x} J = \sum_{i=1}^{n_{sc}} \sum_{s=2}^{5} (\bar{w}_{is}^- d_{is}^- + \bar{w}_{is}^+ d_{is}^+) \qquad (1)$$

subject to

$$g_i - d_{is}^+ \leq t_{i(s-1)}^+ \ ; \ d_{is}^+ \geq 0 \ ; \ g_i \leq t_{i5}^+$$

(for all i in class 1S, 3S 4S,

$i=1, 2, \cdots, n_{sc}, s=2, \cdots, 5$)

$$g_i - d_{is}^- \le \tilde{t}_{i(s-1)}^- \; ; \; d_{is}^- \ge 0 \; ; \; g_i \le \tilde{t}_{i5}^-$$
(for all i in class 2S, 3S 4S,
$i = 1, 2, \cdots, n_{sc}, s = 2, \cdots, 5$)

In the case of 1S, the boundary of the preference region is represented by a fuzzy number set, $(\tilde{t}_{i1}^+, \tilde{t}_{i2}^+, \tilde{t}_{i3}^+, \tilde{t}_{i4}^+, \tilde{t}_{i5}^+)$. The fuzzy preference ranges are defined as follow:

Ideal range $(g_i \le \tilde{t}_{i1}^+)$
Desirable range $(\tilde{t}_{i1}^+ \le g_i \le \tilde{t}_{i2}^+)$
Tolerable range $(\tilde{t}_{i2}^+ \le g_i \le \tilde{t}_{i3}^+)$
Undesirable range $(\tilde{t}_{i3}^+ \le g_i \le \tilde{t}_{i4}^+)$
Highly Undesirable range $(\tilde{t}_{i4}^+ \le g_i \le \tilde{t}_{i5}^+)$
Unacceptable rang $(g_i \ge \tilde{t}_{i5}^+)$

From the characteristics of aggregate preference functions, the length of the s-th range of the i-th criterion is defined as

$$\tilde{\tilde{t}}_{is}^+ = \tilde{t}_{is}^+ - \tilde{t}_{i(s-1)}^+ \; ; \; \tilde{\tilde{t}}_{is}^- = \tilde{t}_{is}^- - \tilde{t}_{i(s-1)}^- \; ; \; (2 \le s \le 5) \quad (2)$$

The magnitude of the slopes of the class function of the generic i-th criterion takes the form

$$\tilde{w}_{is}^+ = \tilde{z}_s / \tilde{\tilde{t}}_{is}^+ \; ; \; \tilde{w}_{is}^- = \tilde{z}_s / \tilde{\tilde{t}}_{is}^- \; ; \; (2 \le s \le 5) \quad (3)$$

Then we define

$$\tilde{\tilde{w}}_{is}^+ = \tilde{w}_{is}^+ - \tilde{w}_{i(s-1)}^+ \; ; \; \tilde{\tilde{w}} = \tilde{w}_{is}^+ - \tilde{w}_{i(s-1)}^+ \; ; \; w_{i1}^+ = w_{i1}^- = 0 \quad (4)$$

Based on the description above, the final fuzzy linear physical programming problem model takes the following form

$$\min_{d_{is}^-, d_{is}^+, x} \tilde{J} = \sum_{i=1}^{n_{sc}} \sum_{s=2}^{5} (\tilde{\tilde{w}}_{is}^- d_{is}^- + \tilde{\tilde{w}}_{is}^+ d_{is}^+) \quad (5)$$

subject to

$$g_i - d_{is}^+ \le \tilde{t}_{i(s-1)}^+ \; ; \; d_{is}^+ \ge 0 \; ; \; g_i \le \tilde{t}_{i5}^+$$
(for all i in class 1S, 3S 4S,
$i = 1, 2, \cdots, n_{sc}, s = 2, \cdots, 5$)

$$g_i - d_{is}^- \le \tilde{t}_{i(s-1)}^- \; ; \; d_{is}^- \ge 0 \; ; \; g_i \le \tilde{t}_{i5}^-$$
(for all i in class 2S, 3S 4S,
$i = 1, 2, \cdots, n_{sc}, s = 2, \cdots, 5$)

3. Interactive Fuzzy Physical Programming

Interactive fuzzy physical programming takes into account the designer's physical understanding of the desired design outcomes by forming fuzzy preferences during the optimization process, which enables the designer to control the optimization process to some extent. The flow chart of interactive decision-making is shown in Figure 2.

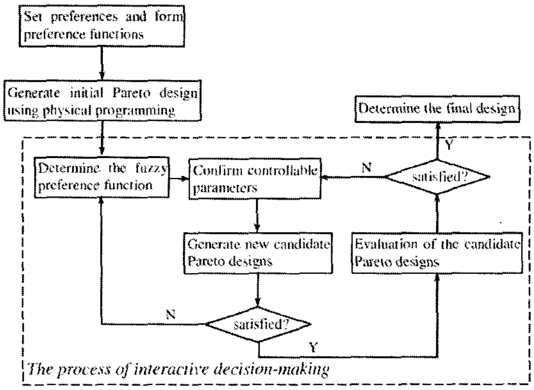

Fig. 2 Flow chat of interactive decision-making

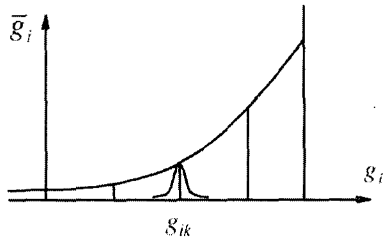

Fig. 3 Determining the boundary of fuzzy preference function

3.1 Fuzzy preference

As shown in Figure 3, suppose that the preference function of the i-th objective belongs to Class 1S, the boundary, g_{ik}, is defined as a normal fuzzy number, $\tilde{g}_{ik}$. Its membership function takes the form

$$\mu_{\tilde{g}_{ik}}(g_i) = e^{-\left[\frac{g_i - g_{ik}}{\delta_{ik}}\right]^2}, \; \delta_{ik} > 0 \quad (6)$$

where δ_{ik} is the fuzzy parameter of the k-th boundary of preference function, which can be defined referring to the robust design method of confirming variation of design objective (Huang et al., 2005f).

3.2 Generate candidate solutions

A set of initial candidate solutions will be generated by initializing a threshold λ and a step size. The threshold λ is obtained by the method of fuzzy comprehensive evaluation (Huang et al., 2005f). Theoretically, the step size is smaller with the result that the final solution more reaches the ideal solution, while the calculation process is more complicated. The preference function will change based on the changeable boundaries of the preference region, but the objective functions and constraints are unchangeable. Therefore, all the new generated solutions belong to the set of Pareto solutions.

Based on designers' basic satisfaction to the initial candidate solution obtained by the normative physical programming, the designer may want to improve upon some objective functions at the expense of certain other objective functions. This is called the designer's improving preferences. A controllable region is given to confine the value of the secondary objective. Through adjusting the threshold and the controllable region to match the improving preferences and controllable range, the candidates are obtained.

3.3 Evaluation of the candidate solutions

The candidate designs are evaluated with a qualitative-quantitative analysis method (Huang et al., 2002). The qualitative analysis is performed by evaluating the candidate solutions with the Analytic Hierarchy Process (AHP) (Huang et al., 2005e). The quantitative analysis is performed by evaluating the candidate designs with a quantitative criteria based on the preference functions of all the objectives. Supposing that the evaluation value determined with AHP is r_{AHP}, the evaluation values determined with quantitative analysis are r_1, r_2, r_3 and r_4. These evaluation values are combined into an evaluation vector

$$r=(r_1,\ r_2,\ r_3,\ r_4,\ r_{AHP}) \tag{7}$$

Based on the specified weights vector corresponding to all the evaluation criteria

$$W=(w_1,\ w_2,\ w_3,\ w_4,\ w_{AHP}) \tag{8}$$

The final evaluation value of the candidate design is

$$\text{grade}=W\cdot r^{T} \tag{9}$$

4. Example

4.1 Problem definition

Model representation of the grinding wheel spindle systems is the key problem in grinder design, and represents the entire system performance. In the static optimization design, the weight of the spindle represents its structure characteristics and working performance while bending deflection represents its stiffness. In this paper, both the minimum static bending deflection of the grinding wheel spindle extension, B, and the minimum weight of the entire spindle are selected as the design objectives. As shown in Figure 4, the static model of the grinding wheel spindle of the M2120A NC internal grinder is considered (Hu et al., 2004). The values of dimensions are shown in Table 1. k represents the stiffness coefficient of the bearing, k_1=1.14E7 N/m and k_2=7.75E7 N/m. The external force on B is F=577 N. The values of initial design objectives and variables are listed in Table 3.

The design vector is

$$x=(x_1,\ x_2,\ x_3)=(l_4,\ l_6,\ D_4) \tag{10}$$

The weight of the spindle takes the form

$$G=\frac{7800}{4}\pi\sum_{i=1}^{7}D_i^2 l_i \tag{11}$$

The static bending deflection of the grinding wheel

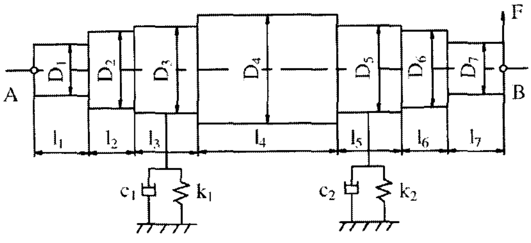

Fig. 4 The static model of the grinding wheel spindle

Table 1 The values of dimensions of the spindle system (m)

D_1	D_2	D_3	D_5	D_6	D_7	l_1	l_2	l_3	l_5	l_7
0.032	0.048	0.050	0.050	0.048	0.031	0.046	0.028	0.065	0.048	0.042

spindle extension, B, takes the form

$$Y_B = F\left[\frac{a^3}{3EI_a} + \frac{l_a}{3EI_l} + \frac{1}{k_1}\left(1+\frac{a}{l}\right)^2 + \frac{1}{k_2}\left(\frac{a}{l}\right)\right] \quad (12)$$

where $a = l_5/2 + l_6 + l_7$ and $l = l_3/2 + l_4 + l_5/2$. E represents the elastic ratio. I_a and I_l are the moment of inertia of a and l, respectively.

The mathematical model of the multi-objective optimization is formulated as follows:

$$\min(f_1(x), f_2(x))$$

$$f_1(x) = G = \frac{7800}{4}\pi\sum_{i=1}^{7} D_i^2 l_i$$

$$f_2(x) = Y_B = F\left[\frac{a^3}{3EI_a} + \frac{la}{3EI_l} + \frac{1}{k_1}\left(1+\frac{a}{l}\right)^2 + \frac{1}{k_2}\left(\frac{a}{l}\right)\right] \quad (13)$$

s.t.

$$0.300 \le x_1 \le 0.400$$
$$0.020 \le x_2 \le 0.040$$
$$0.058 \le x_3 \le 0.070$$

4.2 Results and discussion

Both the objectives G and Y_B belong to Class-1S design metrics. The designer's initial preferences are listed in Table 2. MATLAB is used to solve the optimization model formulated in equation (13). The initial Pareto design generated using physical programming is shown in Table 3.

We hope to improve upon the solution of the bending deflection, while the loss of the weight objective is considered. To simplify the optimization process, only one boundary of the preference region will be fuzzification, which will greatly influence the final optimal solution. In this section, the right boundary, in which the initial optimal solution of the bending deflection is located, is defined as a normal fuzzy number. Assume that $\delta_{22} = 0.14E\text{-}5$. The membership function formulated in equation (6) takes the form

$$\mu_{\tilde{g}_{22}}(g_2) = e^{-\left[\frac{g_i - (1.95E-5)}{0.14E-5}\right]^2} \quad (14)$$

The threshold λ is defined as 0.6 and the step is defined as 0.05. 17 initial candidate solutions are generated. The controllable region is regulated within ±10% of the initial optimal solution. When the threshold $\lambda = 0.75$, the optimum solutions are obtained, as shown in Table 3.

Compared to the initial design, it is obvious that both the design objectives are improved significantly. Considering the fuzzy factors of the designers' decision-making, the moderate deviations of the design objectives between IFPP and the normal physical programming are 4.3% and 6.0%, respectively. The relative proportions of design objectives are visualized in Figure 5. The values of the vertical axis present the boundaries of preference function from Desirable range to Unacceptable range, which are quantified in turn as 1, 2, 3, 4, 5. As seen in Figure 5, the solution

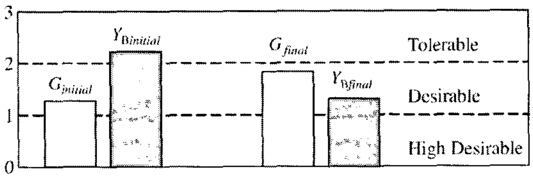

Fig. 5 Compare the initial optimum design and the final optimum design

Table 2 The initial region limits of designer's preferences

	Class type	g_{i5}	g_{i4}	g_{i3}	g_{i2}	g_{i1}
$f_1(x)$	1-S	16.0	15.0	14.0	13.0	12.0
$f_2(x)$	1-S	2.5E-5	2.2E-5	2.05E-5	1.95E-5	1.85E-5

Table 3 The results of design variables and design objective functions

	l_4 [m]	l_6 [m]	D_4 [m]	G [kg]	Y_B [m]
The initial value	0.368	0.028	0.070	14.104	2.020E-5
PP	0.350	0.020	0.066	12.284	1.973E-5
IFPP	0.393	0.020	0.064	12.806	1.881E-5

of bending deflection is improved from the tolerable area to the desirable area, while the weight objective is sacrificed to some extent, but still within the desirable area. Numerical analysis indicates that the applications of the fuzzy preference function can effectively control the optimization direction and scale of optimization objectives and express the designer's subjective intention more exactly.

5. Conclusions

This paper develops a new multi-objective optimization method, IFPP. In IFPP, fuzzy preference, introduced into physical programming, is used for adjusting the search direction and area, which matches the designers' intents exactly. Interactive decision-making enables the designer to control the optimization process to some extent, which can effectively improve the design efficiency and design result, thus avoiding time wasted in errant directions during the design process. The example illustrates that the proposed method can capture the fuzziness of the designer's preference structures, and conforms more exactly to engineering realities.

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