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MINIMAL BASICALLY DISCONNECTED COVERS OF PRODUCT SPACES

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Abstract. In this paper, we show that if the minimal basically disconnected cover ΛX_i of X_i is given by the space of fixed $\sigma Z(X)^\#$-ultrafilters on X_i $(i = 1, 2)$ and $\Lambda X_1 \times \Lambda X_2$ is a basically disconnected space, then $\Lambda X_1 \times \Lambda X_2$ is the minimal basically disconnected cover of $X_1 \times X_2$. Moreover, observing that the product space of a P-space and a countably locally weakly Lindelöf basically disconnected space is basically disconnected, we show that if X is a weakly Lindelöf almost P-space and Y is a countably locally weakly Lindelöf space, then $(\Lambda X \times \Lambda Y, \Lambda_X \times \Lambda_Y)$ is the minimal basically disconnected cover of $X \times Y$.

1. Introduction

It is known that minimal basically disconnected covers of some spaces are given by certain filter spaces. For example, Vermeer showed that the minimal basically disconnected cover ΛX of a compact space X is given by the Stone-space $S(\sigma Z(X)^\#)$ of $\sigma Z(X)^\#$ ([7]). In [4], we proved that the minimal basically disconnected cover ΛX of a locally weakly Lindelöf space X is characterized by the space of fixed $\sigma Z(X)^\#$-ultrafilters on X.

The purpose of this paper is to construct the minimal basically disconnected covers of some product spaces. First, we show that if the minimal basically disconnected cover ΛX_i of X_i is given by the space of fixed $\sigma Z(X)^\#$-ultrafilters on X_i $(i = 1, 2)$ and $\Lambda X_1 \times \Lambda X_2$ is a basically

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disconnected space, then $\Lambda X_1 \times \Lambda X_2$ is the minimal basically disconnected cover of $X_1 \times X_2$. The concept of a countably locally weakly Lindelöf space is introduced by Comfort, Hindman and Negrepontis ([2]). They showed that if X is a P-space and Y is a countably locally weakly Lindelöf basically disconnected space, then $X \times Y$ is basically disconnected. We show that the minimal basically disconnected cover ΛX of a countably locally weakly Lindelöf space X is given by the space of fixed $\sigma Z(X)^{\#}$-ultrafilters on X. Finally, we show that if X is a weakly Lindelöf almost P-space and Y is a countably locally weakly Lindelöf space, then $(\Lambda X \times \Lambda Y, \Lambda_X \times \Lambda_Y)$ is the minimal basically disconnected cover of $X \times Y$. All spaces in this paper are assumed to be Tychonoff spaces. For the terminologies, we refer to [6].

2. Minimal basically disconnected covers of product spaces

For any space X, the set $R(X)$ of all regular closed sets in X, when partially ordered by inclusion, becomes a complete Boolean algebra. The join, meet and complementation operations in $R(X)$ are defined as follows: if $A \in R(X)$ and $\{A_i \in R(X)\colon i \in I\}$, then $\bigvee\{A_i : i \in I\} = \mathrm{cl}_X(\bigcup\{A_i : i \in I\})$, $\bigwedge\{A_i : i \in I\} = \mathrm{cl}_X(\mathrm{int}_X(\bigcap\{A_i : i \in I\}))$, and $A' = \mathrm{cl}_X(X - A)$.

Recall that a map $f : Y \longrightarrow X$ is called *covering* if it is a perfect, continuous, irreducible map. It is well-known that for any covering map $f : Y \longrightarrow X$, the map $\phi : R(Y) \longrightarrow R(X)$, defined by $\phi(A) = f(A)$, is a Boolean algebra isomorphism and the inverse map ϕ^{-1} of ϕ is given by $\phi^{-1}(A) = \mathrm{cl}_Y(f^{-1}(\mathrm{int}_X(A))) = \mathrm{cl}_Y(\mathrm{int}_Y(f^{-1}(A)))$.

LEMMA 2.1. *Let $f_i : X_i \longrightarrow Y_i$ ($i = 1, 2$) be a covering map. Then the map $f_1 \times f_2 : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$, defined by $(f_1 \times f_2)((x, y)) = (f_1(x), f_2(y))$, is a covering map.*

PROOF. The proof is straightforward. □

Let L be a complete Boolean algebra and M a sublattice of L. Then there is the smallest σ-complete Boolean subalgebra of L containing M, denoted by σM. For any space X, let $Z(X)^{\#} = \{\mathrm{cl}_X(\mathrm{int}_X(Z)) : Z$ is a zero-set in $X\}$. Then $Z(X)^{\#}$ is a sublattice of $R(X)$. For any spaces X, Y and $\mathcal{F} \subseteq 2^X$, let $\mathcal{F} \times Y = \{A \times Y : A \in \mathcal{F}\}$.

PROPOSITION 2.2. *For any spaces X and Y, we have $\sigma Z(X)^{\#} \times Y = \sigma(Z(X)^{\#} \times Y) \subseteq \sigma Z(X \times Y)^{\#}$.*

PROOF. Consider the map $\rho : R(X) \longrightarrow R(X) \times Y$, defined by $\rho(A) = A \times Y$. Then ρ is a Boolean algebra isomorphism and $\sigma Z(X)^{\#} \times Y$ is a σ-complete Boolean subalgebra of $R(X) \times Y$. Hence $\sigma Z(X)^{\#} \times Y$ is a σ-complete Boolean subalgebra of $R(X \times Y)$. Since $Z(X)^{\#} \times Y \subseteq \sigma Z(X)^{\#} \times Y$, $\sigma(Z(X)^{\#} \times Y) \subseteq \sigma Z(X)^{\#} \times Y$. Since $Z(X)^{\#} \times Y \subseteq \sigma(Z(X)^{\#} \times Y) \subseteq \sigma Z(X)^{\#} \times Y$, $Z(X)^{\#} \subseteq \rho^{-1}\ (\sigma(Z(X)^{\#} \times Y))$ and hence $\sigma Z(X)^{\#} \subseteq \rho^{-1}\ (\sigma(Z(X)^{\#} \times Y))$. So $\rho\ (\sigma Z(X)^{\#}) = \sigma Z(X)^{\#} \times Y \subseteq \sigma(Z(X)^{\#} \times Y)$. Thus $\sigma Z(X)^{\#} \times Y = \sigma(Z(X)^{\#} \times Y)$. Since $Z(X)^{\#} \times Y \subseteq Z(X \times Y)^{\#}$, we have $\sigma(Z(X)^{\#} \times Y) \subseteq \sigma Z(X \times Y)^{\#}$. □

DEFINITION 2.3. A space X is said to be a *basically disconnected space* if for any zero-set Z in X, $\mathrm{int}_X(Z)$ is closed in X.

DEFINITION 2.4. Let X be a space.

(a) A pair (Y, f) is called a *cover of* X if $f : Y \longrightarrow X$ is a covering map.

(b) A cover (Y, f) of X is called a *basically disconnected cover of* X if Y is a basically disconnected space.

(c) A basically disconnected cover (Y, f) of X is called a *minimal basically disconnected cover of* X if for any basically disconnected cover (Z, g) of X, there is a covering map $h : Z \longrightarrow Y$ with $f \circ h = g$.

Vermeer ([7]) showed that every space X has a minimal basically disconnected cover $(\Lambda X, \Lambda_X)$ and that if X is a compact space, then ΛX is the Stone-space of $\sigma Z(X)^{\#}$ and $\Lambda_X(\alpha) = \bigcap \alpha$ $(\alpha \in \Lambda X)$.

We recall that a space X is called *weakly Lindelöf* if for any open cover $\mathcal{U}$ of X, there is a countable subfamily $\mathcal{V}$ of $\mathcal{U}$ such that $\bigcup \mathcal{V}$ is dense in X and that a space X is called *locally weakly Lindelöf* if every element of X has a weakly Lindelöf neighborhood. In [4], we proved that the minimal basically disconnected cover $(\Lambda X, \Lambda_X)$ of a locally weakly Lindelöf space X is characterized as follows : $\Lambda X = \{\alpha : \alpha$ is a fixed $\sigma Z(X)^{\#}$-ultrafilter on $X\}$ with the topology generated by $\{\Lambda X - A^* : A \in \sigma Z(X)^{\#}\}$ and $\Lambda_X(\alpha) = \bigcap \alpha$, where $A^* = \{\alpha \in \Lambda X : A \in \alpha\}$. Hence ΛX is a subspace of $\Lambda \beta X$. If X is a space such that $\Lambda X = \{\alpha : \alpha$ is a fixed $\sigma Z(X)^{\#}$-ultrafilter on $X\}$, then for any $A \in \sigma Z(X)^{\#}$, $\Lambda_X(A^*) = A$ and so $A^* = \mathrm{cl}_{\Lambda X}(\Lambda_X^{-1}(\mathrm{int}_X(\mathrm{A})))$.

THEOREM 2.5. *Let X, Y be spaces such that $\Lambda X = \{\alpha : \alpha$ is a fixed $\sigma Z(X)^{\#}$-ultrafilter on $X\}$ and $\Lambda Y = \{\alpha : \alpha$ is a fixed $\sigma Z(Y)^{\#}$-ultrafilter on $Y\}$. If $\Lambda X \times \Lambda Y$ is a basically disconnected space, then $(\Lambda X \times \Lambda Y, \Lambda_X \times \Lambda_Y)$ is the minimal basically disconnected cover of $X \times Y$.*

PROOF. Let $K = \Lambda X \times \Lambda Y$, $g = \Lambda_X \times \Lambda_Y$, $W = \Lambda(X \times Y)$ and $h = \Lambda_{X\times Y}$. By Lemma 2.1, g is a covering map. Since K is basically disconnected, there is a covering map $f : K \longrightarrow W$ with $h \circ f = g$. Let $(\alpha,\beta) \neq (\gamma,\delta)$ in K. We may assume that $\alpha \neq \gamma$. Then there are A, $B \in \sigma Z(X)^{\#}$ such that $A \in \alpha$, $B \in \gamma$ and $A \wedge B = \emptyset$. By Proposition 2.2, $A \times Y$, $B \times Y \in \sigma Z(X \times Y)^{\#}$. Since h is a covering map, $\{\mathrm{cl}_W(h^{-1}(\mathrm{int}_{X\times Y}(Z))) : Z \in \sigma Z(X \times Y)^{\#}\} \subseteq \sigma Z(W)^{\#}$. Since W is basically disconnected, $\mathrm{cl}_W(h^{-1}(\mathrm{int}_{X\times Y}(A \times Y))) \bigcap \mathrm{cl}_W(h^{-1}(\mathrm{int}_{X\times Y}(B\times Y))) = \mathrm{cl}_W(h^{-1}(\mathrm{int}_{X\times Y}((A\cap B)\times Y))) = \emptyset$. Note that $h[f(\mathrm{cl}_{\Lambda X}(\Lambda_X^{-1}(\mathrm{int}_X(A))) \times \Lambda Y)] = g[\mathrm{cl}_{\Lambda X}(\Lambda_X^{-1}(\mathrm{int}_X(A))) \times \Lambda Y] = A \times Y = h(\mathrm{cl}_W(h^{-1}(\mathrm{int}_{X\times Y}(A \times Y))))$.

Hence $f[\mathrm{cl}_{\Lambda X}(\Lambda_X^{-1}(\mathrm{int}_X(A))) \times \Lambda Y] = \mathrm{cl}_W(h^{-1}(\mathrm{int}_{X\times Y}(A \times Y)))$. Since $(\alpha,\beta) \in A^* \times \Lambda Y = \mathrm{cl}_{\Lambda X}(\Lambda_X^{-1}(\mathrm{int}_X(A))) \times \Lambda Y$, $f((\alpha,\beta)) \in \mathrm{cl}_W(h^{-1}(\mathrm{int}_{X\times Y}(A \times Y)))$. Similarly $f((\gamma,\delta)) \in \mathrm{cl}_W(h^{-1}(\mathrm{int}_{X\times Y}(B \times Y)))$. So $f((\alpha,\beta)) \neq f((\gamma,\delta))$ and thus f is a homeomorphism. □

If X is a weakly Lindelöf space, then every cozero-set in X is also weakly Lindelöf ([2]). Hence a space X is locally weakly Lindelöf if and only if every element x of X has a local base at x consisting of weakly Lindelöf open neighborhoods of x in X. So X is a locally weakly Lindelöf space if and only if for any collection $\{\mathcal{U}_i : i \in I\}$ of open covers of X and for any $x \in X$, there is a neighborhood G of x in X and for any $i \in I$, there is a countable subfamily $\mathcal{V}_i$ of $\mathcal{U}_i$ such that $G \subseteq \mathrm{cl}_X(\bigcup \mathcal{V}_i)$. Indeed, the neighborhood G of x may be chosen independent of the collection $\{\mathcal{U}_i : i \in I\}$.

DEFINITION 2.6. ([2]) A space X is called *countably locally weakly Lindelöf* if for any countable collection $\{\mathcal{U}_n : n \in N\}$ of open covers of X and for any $x \in X$, there is a neighborhood G of x in X and for any $n \in N$, there is a countable subfamily $\mathcal{V}_n$ of $\mathcal{U}_n$ such that $G \subseteq \mathrm{cl}_X(\bigcup \mathcal{V}_n)$.

Every locally weakly Lindelöf space is a countably locally weakly Lindelöf space but the converse need not be true ([2]).

Let X, Y be any spaces and $f : Y \longrightarrow X$ a continuous map. For any $U \subseteq X$, let $f_U : f^{-1}(U) \longrightarrow U$ denote the restriction and corestriction of f to $f^{-1}(U)$ and U, respectively. We can easily show that if X and Y are compact spaces and $f : Y \longrightarrow X$ is a covering map, then for any dense subspace U of X, $f_U : f^{-1}(U) \longrightarrow U$ is a covering map. For any space X, let $(\Lambda\beta X, \Lambda_\beta)$ denote the minimal basically disconnected cover of βX.

LEMMA 2.7. ([4]) *Let X be a space. If $\Lambda_\beta^{-1}(X)$ is a basically disconnected space, then $(\Lambda_\beta^{-1}(X), \Lambda_{\beta_X})$ is the minimal basically disconnected cover of X.*

Let Y be an extension of a space X. Then the map $\psi : R(Y) \longrightarrow R(X)$, defined by $\psi(A) = A \cap X$, is a Boolean algebra isomorphism. Hence for a σ-complete Boolean subalgebra $\mathcal{L}$ of $R(Y)$, $\mathcal{L}_X = \{A \cap X : A \in \mathcal{L}\}$ is a σ-complete Boolean subalgebra of $R(X)$.

THEOREM 2.8. (1) *Let $f : Y \longrightarrow X$ be a covering map and X a countably locally weakly Lindelöf space. Then Y is also countably locally weakly Lindelöf.*

(2) *Let X be a basically disconnected space and Y a countably locally weakly Lindelöf dense subspace of X. Then Y is a basically disconnected space.*

(3) *If X is a countably locally weakly Lindelöf space, then $(\Lambda_\beta^{-1}(X), \Lambda_{\beta_X})$ is the minimal basically disconnected cover of X.*

PROOF. (1) Let $\{\mathcal{U}_n : n \in N\}$ be a countable family of open covers of X and $y \in Y$. For any $n \in N$, let $\mathcal{B}_n = \{\bigcup \mathcal{U}'_n : \mathcal{U}'_n$ is a finite subfamily of $\mathcal{U}_n\}$. Then $\mathcal{B}_n$ is an open cover of Y that is closed under finite unions. Since f is a covering map, $\mathcal{B}'_n = \{X - f(Y - B) : B \in \mathcal{B}_n\}$ is an open cover of X ([6]). Since X is countably locally weakly Lindelöf, there is an open neighborhood G of $f(y)$ in X and for any $n \in N$, there is a countable subfamily $\mathcal{V}_n$ of $\mathcal{B}'_n$ such that $G \subseteq \mathrm{cl}_X(\bigcup \mathcal{V}_n)$. Let $\mathcal{V}_n = \{X - f(Y - B_{n_k}) : k \in N$ and $B_{n_k} \in \mathcal{B}_n\}$. Then $f^{-1}(G) \subseteq f^{-1}(\mathrm{cl}_X(\bigcup\{X - f(Y - B_{n_k}) : k \in N\}))$. Since $f^{-1}(G)$ is open in Y and f is a covering map, $f^{-1}(G) \subseteq \mathrm{cl}_Y(\mathrm{int}_Y(f^{-1}(\mathrm{cl}_X(\bigcup\{X - f(Y - B_{n_k}) : k \in N\})))) = \mathrm{cl}_Y(f^{-1}(\bigcup\{X - f(Y - B_{n_k}) : k \in N\}))$.

Note that for any $B \subseteq Y$, $f^{-1}(X - f(Y - B)) \subseteq B$. So $f^{-1}(G) \subseteq \mathrm{cl}_Y(f^{-1}(\bigcup\{X - f(Y - B_{n_k}) : k \in N\})) \subseteq \mathrm{cl}_Y(\bigcup\{B_{n_k} : k \in N\})$.

Since each B_{n_k} is a union of a finite subfamily of $\mathcal{U}_n$ and hence $\bigcup\{B_{n_k} : k \in N\}$ is a union of a countable subfamily of $\mathcal{U}_n$. Thus Y is a countably locally weakly Lindelöf space.

(2) Since X is a basically disconnected space, the set $B(X)$ of clopen sets in X is a σ-complete Boolean subalgebra of $R(X)$ ([7]) and hence $B(X)_Y = \{A \cap Y : A \in B(X)\}$ is also a σ-complete Boolean subalgebra of $R(Y)$. Moreover, $B(X)_Y$ is a base for Y.

Take any zero-set Z in Y. Suppose that $\mathrm{cl}_Y(Y - Z) \cap \mathrm{cl}_Y(\mathrm{int}_Y(Z)) \neq \emptyset$. Pick $y \in \mathrm{cl}_Y(Y - Z) \cap \mathrm{cl}_Y(\mathrm{int}_Y(\mathrm{Z}))$. Since $Y - Z$ is a cozero-set in Y, there is a sequence (A_n) of closed sets in Y such that $Y - Z = \bigcup\{A_n$

$: n \in N\} = \bigcup\{\text{int}_Y(A_n) : n \in N\}$. Since $(Y - Z) \cap \text{cl}_Y(\text{int}_Y(Z)) = \emptyset$, for any $n \in N$, $A_n \cap \text{cl}_Y(\text{int}_Y(Z)) = \emptyset$ and hence $(Y - A_n) \bigcup (Y - \text{cl}_Y(\text{int}_Y(Z))) = Y$. Since $B(X)_Y$ is a base for Y, for any $n \in N$, there is a subfamily $\mathcal{U}_n$ of $B(X)_Y$ such that $Y - A_n = \bigcup \mathcal{U}_n$ and hence $\mathcal{B}_n = \mathcal{U}_n \bigcup \{Y - \text{cl}_Y(\text{int}_Y(Z))\}$ is an open cover of Y. Since Y is countably locally weakly Lindelöf, there is a clopen neighborhood G of y in Y and for any $n \in N$, there is a countable subfamily $\mathcal{V}_n$ of $\mathcal{U}_n$ such that $G \subseteq \text{cl}_Y(\bigcup \mathcal{V}_n) \bigcup \text{cl}_Y(\text{int}_Y(Y - \text{cl}_Y(\text{int}_Y(Z)))) = \text{cl}_Y(\bigcup \mathcal{V}_n) \bigcup \text{cl}_Y(Y - Z)$. Since $\mathcal{V}_n$ is a countable subfamily of $B(X)_Y$ and X is basically disconnected, $\text{cl}_Y(\bigcup \mathcal{V}_n) \in B(X)_Y$. For any $n \in N$, let $D_n = \text{cl}_Y(\bigcup \mathcal{V}_n)$. Then for any $n \in N$, $G \subseteq D_n \bigcup \text{cl}_Y(Y - Z)$ and so $G \subseteq (\bigcap\{D_n : n \in N\}) \bigcup \text{cl}_Y(Y - Z)$. Thus $G \cap (\bigcup\{Y - D_n : n \in N\}) \cap \text{int}_Y(Z) = \emptyset$. Note that for any $n \in N$, $D_n \subseteq \text{cl}_Y(Y - A_n)$ and hence $\text{int}_Y(A_n) \subseteq Y - D_n$. So $Y - Z \subseteq \bigcup\{Y - D_n : n \in N\}$. Since $y \in \text{cl}_Y(Y - Z)$, $y \in \text{cl}_Y(\bigcup\{Y - D_n : n \in N\})$ and since for any $n \in N, Y - D_n \in B(X)_Y$, $\text{cl}_Y(\bigcup\{Y - D_n : n \in N\})$ is clopen in Y. Since $G \cap (\bigcup\{Y - D_n : n \in N\}) \cap \text{int}_Y(Z) = \emptyset$ and G, $\text{int}_Y(Z)$ are open in Y, $G \cap \text{cl}_Y(\bigcup\{Y - D_n : n \in N\})) \cap \text{int}_Y(Z) = \emptyset$ and since $\text{cl}_Y(\bigcup\{Y - D_n : n \in N\})$ is clopen in $Y, G \cap \text{cl}_Y(\bigcup\{Y - D_n : n \in N\})) \cap \text{cl}_Y(\text{int}_Y(Z)) = \emptyset$. This is a contradiction. So $\text{cl}_Y(Y - Z) \cap \text{cl}_Y(\text{int}_Y(Z)) = \emptyset$ and hence $\text{cl}_Y(\text{int}_Y(Z)) \subseteq Y - \text{cl}_Y(Y - Z) = \text{int}_Y(Z)$. Thus Y is a basically disconnected space.

(3) Suppose that X is a countably locally weakly Lindelöf space. Since $\Lambda_{\beta_X} : \Lambda_\beta^{-1}(X) \longrightarrow X$ is a covering map, by (2), $\Lambda_\beta^{-1}(X)$ is a countably locally weakly Lindelöf space. Since $\Lambda_\beta^{-1}(X)$ is dense in $\Lambda\beta X$, by (1), $\Lambda_\beta^{-1}(X)$ is a basically disconnected space. By Lemma 2.1, $(\Lambda_\beta^{-1}(X), \Lambda_{\beta_X})$ is the minimal basically disconnected cover of X. □

For any space X, the isomorphism $\psi : R(\beta X) \longrightarrow R(X)$ $(\psi(A) = A \cap X)$ induces a Boolean algebra isomorphism $\sigma Z(\beta X)^\# \longrightarrow \sigma Z(X)^\#$. Using this, we have the following:

COROLLARY 2.9. *For any countably locally weakly Lindelöf space X, $\Lambda X = \{\alpha : \alpha$ is a fixed $\sigma Z(X)^\#$-ultrafilter on $X\}$ with the topology generated by $\{\Lambda X - A^* : A \in \sigma Z(X)^\#\}$ and $\Lambda_X(\alpha) = \cap\alpha$.*

Recall that a space X is called a *P-space* (*an almost P-space*, resp.) if every zero-set in X is open (regular closed, resp.) in X. It is known that every weakly Lindelöf almost P-space is a basically disconnected space ([5]) and that if X is a P-space and Y is a countably locally weakly Lindelöf basically disconnected space, then $X \times Y$ is a basically disconnected space.

DEFINITION 2.10. Let $f : X \longrightarrow Y$ be a covering map. Then f is called *$Z^{\#}$-irreducible* if $f(Z(X)^{\#}) = Z(Y)^{\#}$.

If $f : X \longrightarrow Y$ is a covering map and Y is a weakly Lindelöf space, then f is $Z^{\#}$-irreducible and X is a weakly Lindelöf space ([3]). Using this, we have the following:

COROLLARY 2.11. *Let X be a weakly Lindelöf almost P-space and Y a countably locally weakly Lindelöf space. Then $(\Lambda X \times \Lambda Y, \Lambda_X \times \Lambda_Y)$ is the minimal basically disconnected cover of $X \times Y$.*

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