

Reliable Replenishment Policy for Deteriorating Products under Day-terms Supplier Credit and Quantity Discounts for Freight Cost in a Supply Chain

– 공급사슬에서 신용거래와 수송비의 할인을 고려한 퇴화성제품의 신뢰성있는 재고보충정책 –

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Abstract

본 연구는 제조자(공급자)와 중간분배자로 구성된 공급사슬에서 시간에 따라 일정률로 퇴화하는 퇴화성 제품을 다루는 중간분배자의 신뢰성있는 재고보충정책을 분석하였다. 문제 분석을 위하여 제조자는 고객의 수요를 증대시키기 위한 수단으로 중간분배자로부터의 제품대금에 대하여 일정기간 동안 신용거래를 허용한다는 가정과 함께 수송량에 따라 할인되는 수송비를 고려하여 모형을 수립하였고, 중간분배자의 경제적 재고보충정책을 결정하기 위한 해법을 개발하였다.

Keywords : Reliable Replenishment Policy, Day-terms Credit, Freight Cost, Economies of Scale

1. Introduction

In the present competitive market, the relations between distributor (retailer) and manufacturer(supplier) have undergone significant changes with increasing emphasis on cooperation and information sharing. The main motivation behind such initiatives is to reduce transaction related costs and to capture a larger market share. In this regard, an effective supply chain network requires a cooperative relationship between the distributor and the manufacturer. A distributor and manufacturer together constitute a simple two-stage supply chain.

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2005년 12월접수; 2006년 2월 수정본 접수; 2006년 2월 게재 확정

In deriving the distributor's economic lot size, it is tacitly assumed that the distributor must pay for the items as soon as he receives them from a manufacturer. However, in today's business transactions, it is more and more common to see that the distributors are allowed some grace period before they settle the amount with the manufacturer. For a manufacturer (supplier) who offers trade credit, it is an effective means of price discrimination which circumvents antitrust measures and is also an efficient method to stimulate the demand of the product. For a distributor (retailer), it is an efficient method of bonding a supplier when the retailer is at the risk of receiving inferior quality goods or service and is also an effective means of reducing the cost of holding stocks. In this regard, a number of research papers appeared which deal with the economic lot sizing problem under a fixed credit period. Chapman *et al.*[1], Chung[3] and Goyal[4] analyzed the effects of trade credit on the optimal replenishment policy. A common assumption of the above researches is that the ordering cost contains a fixed cost alone. But, in many practical situations under supply chain, the order may be delivered in unit loads, i.e., trucks, containers, pallets, boxes, etc. and a quantity discount may occur in terms of the number of unit loads due to the economies of scale. In this regard, Lee[7] studied the economic order quantity model with set-up cost including a fixed cost and freight cost where the freight cost has a quantity discount. Also, Shinn *et al.*[8] analyzed the joint price and lot size determination problem under conditions of permissible delay in payments and quantity discounts for freight cost.

All the research works mentioned above implicitly assume that stock is depleted by customer's demand alone. This assumption is quite valid for products whose utility remains constant over time. However, there are numerous types of product whose utility does not remain constant over time. In this case, inventory is depleted not only by customer's demand but also by deterioration. Chu *et al.*[2], Hwang and Shinn[5], and Jaggi and Aggarwal[6] evaluated the effect of trade credit in determining the inventory policy of deteriorating products.

This paper deals with the distributor's reliable replenishment policy determination problem for an exponentially deteriorating product in a simple two-stage supply chain when the manufacturer permits delay in payments for an order of the product. It is also assumed that the ordering cost of the distributor contains not only a fixed cost but also a freight cost, which is a function of the lot-size.

2. Development of the Mathematical Model

In deriving the model, the following assumptions and notations are used:

- (i) The demand for the items is constant with time.
- (ii) Time period is infinite.
- (iii) Shortages are not allowed.
- (iv) Inventory is depleted not only by demand but also by deterioration with exponential distribution.
- (v) The distributor pays the freight cost for the transportation of the quantity purchased where the freight cost has a quantity discount.
- (vi) The manufacturer proposes a certain credit period and the purchasing cost of the items sold during the credit period is deposited in an interest bearing account with rate i . At the end of the credit period, the account is settled and the distributor starts paying for the interest charges on items in stock with rate r ($r \geq i$).

D : demand rate per year

H : unit stock holding cost per item per year excluding interest charges

tc : credit period by year set by the manufacturer

r : interest charges per \$ investment in stocks per year

i : interest rate which can be earned per \$ in a year

C : unit purchase cost in \$ per unit

S : fixed ordering cost in \$ per order

N_j : j th freight cost break quantity, $j = 1, 2, \dots, n$, where

$N_0 < N_1 < \dots < N_n < N_{n+1}$ with $N_0 = 0$ and $N_{n+1} = \infty$.

F_j : freight cost for Q , $N_{j-1} < Q \leq N_j$, where $F_{j-1} < F_j$ and

$F_{j-1}/N_{j-1} > F_j/N_j$, $j = 1, 2, \dots, n$.

Q : distributor's lot size per order

T : time interval by year between successive orders

θ : positive number representing the stock deteriorating rate ($0 \leq \theta \leq 1$)

$I(t)$: stock level at time t

Note that the inequalities $F_{j-1} < F_j$ and $F_{j-1}/N_{j-1} > F_j/N_j$ are necessary to have some quantity discount in the freight cost for changing the order size from

N_{j-1} to N_j . Thus the cost for settling an order becomes $S + F_j$ for $N_{j-1} < Q \leq N_j$. Also, for the case of exponential deterioration, as stated by Hwang and Shinn[5], the rate at which inventory deteriorates will be propositional to on hand inventory, $I(t)$. Thus, the depletion rate of inventory at any time t is

$$\frac{dI(t)}{dt} = -\theta I(t) - D. \tag{1}$$

Observing that (1) is a first order linear differential equation, its solution is

$$I(t) = I(0)e^{-\theta t} - \frac{D}{\theta}(1 - e^{-\theta t}). \tag{2}$$

Equation (2) gives the inventory level at time t representing the combined effects of demand usage and exponential deterioration. Now, we determine the inventory loss caused by deterioration. Let $I^0(t)$ be the inventory level at time t where there were no deterioration. Then, the inventory loss caused by deterioration becomes

$$I^0(t) - I(t) = (I(0) - Dt) - \left(I(0)e^{-\theta t} - \frac{D}{\theta}(1 - e^{-\theta t}) \right) \tag{3}$$

$$= I(t)(e^{-\theta t} - 1) - Dt + \frac{D}{\theta}(e^{-\theta t} - 1). \tag{4}$$

Therefore, the quantity ordered per cycle becomes

$$Q = (I^0(T) - I(T)) + DT. \tag{5}$$

Note that because of the inventory carrying costs, it is clearly better off to have the inventory level reach zero just before reordering, i.e., $I(T) = 0$. With $I(T) = 0$, we have

$$Q = \frac{D}{\theta}(e^{\theta T} - 1). \tag{6}$$

And the stock level at time t is

$$I(t) = \frac{D}{\theta} (e^{\theta(T-t)} - 1), \quad 0 \leq t \leq T. \quad (7)$$

Now for the formulation of the distributor's total annual variable cost with respect to T , we consider the freight cost discount schedule, $N_{j-1} < Q \leq N_j$, $j = 1, 2, \dots, n$. By equation (6), if we denote $L_j = \frac{1}{\theta} \ln\left(\frac{\theta}{D} N_j + 1\right)$, then the inequality $N_{j-1} < Q \leq N_j$ can be rewritten as

$$L_{j-1} < T \leq L_j \text{ for } j = 1, 2, \dots, n. \quad (8)$$

Then, the total annual variable cost consists of the following elements.

1) Annual ordering cost = $\frac{S + F_j}{T}$ for $L_{j-1} < T \leq L_j$.

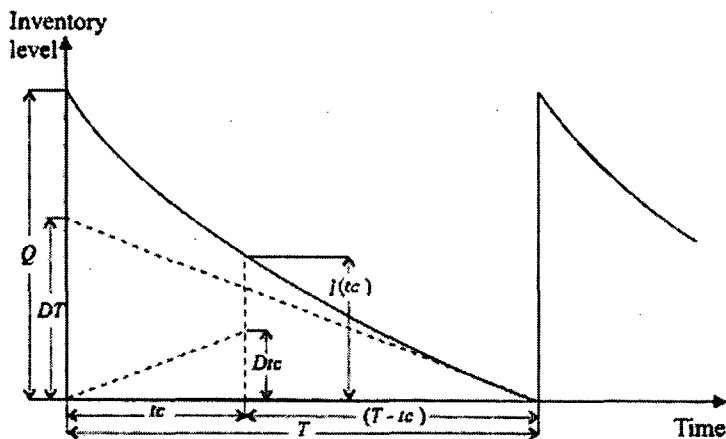
2) Annual purchasing cost = $\frac{CQ}{T} = \frac{CD(e^{\theta T} - 1)}{\theta T}$.

3) Annual inventory carrying cost = $\frac{H}{T} \int_0^T I(t) dt = \frac{HD(e^{\theta T} - \theta T - 1)}{\theta^2 T}$.

4) Annual capital opportunity cost(refer to Goyal[4]) :

(i) Case 1($tc \leq T$): (see Figure 1) As products are sold, the purchasing cost of the products is used to earn interest with annual rate i during the credit period tc . And the average number of products in inventory earning interest during time $(0, tc)$ is $\frac{Dtc}{2}$ and the interest earned per order becomes $\left(\frac{Dtc}{2}\right)tcCi$. When the account is settled, the products still in inventory have to be financed with annual rate r . Since the average number of products during time (tc, T) becomes $\frac{1}{(T - tc)} \int_{tc}^T I(t) dt$, the interest payable per order can be expressed as $Cr \int_{tc}^T I(t) dt$. Therefore,

$$\text{the annual capital opportunity cost} = \frac{Cr \int_{tc}^T I(t) dt - \frac{CiDtc^2}{2}}{T}$$



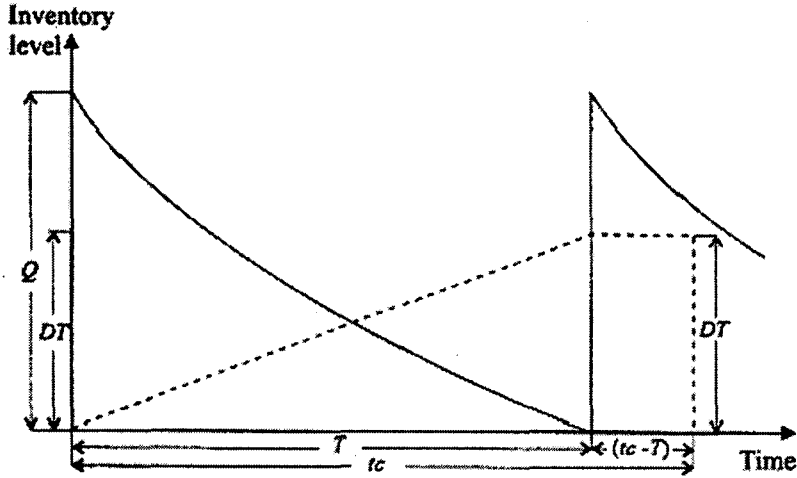
<Figure 1> Time-weighted inventory when $T \geq tc$

$$= \frac{1}{\theta^2 T} CrD(e^{\theta(T-tc)} - \theta(T-tc) - 1) - \frac{1}{2T} CiDtc^2.$$

(ii) Case 2 ($tc > T$): (see Figure 2) For the case of $tc > T$, all the purchasing cost of the products is used to earn interest with annual rate i during the credit period tc . The average number of products in inventory earning interest during time $(0, T)$ and (T, tc) become $\frac{DT}{2}$ and DT , respectively. Therefore,

$$\begin{aligned} \text{the annual capital opportunity cost} &= -\frac{\frac{DT}{2} TCi + DT(tc - T)Ci}{T} \\ &= \frac{CiDT}{2} - CiDtc. \end{aligned}$$

Therefore, depending on the relative size of tc to T , the distributor's total annual variable cost, $TC(T)$ has two different expressions as follows:



<Figure 2> Time-weighted inventory when $T \leq tc$

Case 1: $T \geq tc$

$$\begin{aligned}
 TC_{1,j}(T) = & \frac{S + F_j}{T} + \frac{CD(e^{\theta T} - 1)}{\theta T} + \frac{HD(e^{\theta T} - \theta T - 1)}{\theta^2 T} \\
 & + \left(\frac{CrD(e^{\theta(T-tc)} - \theta(T-tc) - 1)}{\theta^2 T} - \frac{CiDtc^2}{2T} \right) \quad (9) \\
 & , L_{j-1} < T \leq L_j, j = 1, 2, \dots, n.
 \end{aligned}$$

Case 2: $T < tc$

$$\begin{aligned}
 TC_{2,j}(T) = & \frac{S + F_j}{T} + \frac{CD(e^{\theta T} - 1)}{\theta T} + \frac{HD(e^{\theta T} - \theta T - 1)}{\theta^2 T} \\
 & + \left(\frac{CiDT}{2} - CiDtc \right) \quad (10) \\
 & , L_{j-1} < T \leq L_j, j = 1, 2, \dots, n.
 \end{aligned}$$

3. Determination of Optimal Policy

The problem is to find the distributor's economic replenishment cycle time T^* which minimizes $TC(T)$. Once T^* is found, the distributor's optimal lot size Q^* can be obtained by equation (6). Although the objective function can be differentiated, the resulting equation is mathematically intractable; that is, it is impossible to find the optimal solution in explicit form. Therefore, the model will

be solved approximately by using a truncated Taylor series expansion for the exponential term, i.e.,

$$e^{\theta T} \approx 1 + \theta T + \frac{1}{2} \theta^2 T^2 \tag{11}$$

which is a valid approximation for smaller values of θT . With the above approximation, the distributor's total annual variable cost can be rewritten as

$$TC_{1,j}(T) = \frac{S + F_j}{T} + CD + \frac{DT(H + \theta C)}{2} + \left(\frac{C(r - i)Dtc^2}{2T} + \frac{CrDT}{2} - CrDtc \right), \tag{12}$$

$$TC_{2,j}(T) = \frac{S + F_j}{T} + CD + \frac{DT(H + \theta C)}{2} + \left(\frac{CiDT}{2} - CiDtc \right). \tag{13}$$

Note that equation (11) is exact when $\theta = 0$ so that equation (9) and (10) reduce to the exact formulas equation (12) and (13) for non-deteriorating product.

For the normal condition ($r \geq i$) as stated by Goyal[4], $TC_{i,j}(T)$ is a convex function for every i and j . And so, there exists a unique value $T_{m,j}$, which maximizes $TC_{m,j}(T)$, $m = 1, 2$, and they are:

$$T_{1,j} = \sqrt{\frac{(2(S + F_j) + C(r - i)Dtc^2)}{D(H + \theta C + rC)}}, \tag{14}$$

$$T_{2,j} = \sqrt{\frac{2(S + F_j)}{D(H + \theta C + iC)}}. \tag{15}$$

Also, as shown by Shinn, Hwang and Park[8], $T_{m,j}$ and $TC_{m,j}(T)$ have the following two properties and two observations, one for Case 1 and the other for Case 2. Based on these observations, we only need to consider a finite number of candidate values of T in finding an optimal value .

Property 1. For m given, $T_{m,j} < T_{m,j+1}$, $j = 1, 2, \dots, n - 1$.

Property 2. For any T , $TC_{m,j}(T) < TC_{m,j+1}(T)$, $m = 1, 2$ and $j = 1, 2$,

..., $n-1$.

Observation 1. (for Case 1)

Suppose tc belongs to $(L_{a-1}, L_a]$ for some a . Let b be the largest index such that $T_{1,0} > L_b$. Also, let c be the larger value of a and b , and k ($k \geq c$) be the first index such that $T_{1,k} \leq L_k$, respectively.

- (i) If the index k ($\leq n$) exists and $T_{1,k} < tc$, then T^* must be less than tc .
- (ii) If the index k ($\leq n$) exists and $T_{1,k} \geq tc$, then we have to consider $T = L_c, L_{c+1}, \dots, L_{k-1}, T_{1,k}$ only as candidates for T^* .
- (iii) If $T_{1,j} > L_j$ for all $c \leq j \leq n$, then we have to consider $T = L_c, L_{c+1}, \dots, L_n$ as candidates for T^* .

Observation 2. (for Case 2)

Suppose tc belongs to $(L_{a-1}, L_a]$ for some a . Let b be the largest index such that $T_{2,0} > L_b$ and k ($k > b$) be the first index such that $T_{2,k} \leq L_k$, respectively.

- (i) If $L_b \geq tc$, L_{a-1} becomes the only candidate for T^* .
- (ii) If the index k ($\leq a$) exists and $T_{2,k} < tc$, then we have to consider $T = L_b, L_{b+1}, \dots, L_{k-1}, T_{2,k}$ as candidates for T^* .
- (iii) If $T_{2,j} > L_j$ for all $b \leq j < a$ and $T_{2,a} \geq tc$, then we have to consider $T = L_b, L_{b+1}, \dots, L_{a-1}$ as candidates for T^* .

4. Solution Algorithm and Numerical Example

4.1. Solution Algorithm

Based on the above observations, we develop the following solution procedure for determining T^* for the approximate model.

Step 1. This step identifies all the candidate values T_o of T satisfying $T_o \geq tc$.

- 1.1. Compute $T_{1,0} = \sqrt{\frac{(2S + C(r-i)Dtc^2)}{D(H + \theta C + rC)}}$ and find index b such that $T_{1,0} \in (L_b, L_{b+1}]$.

- 1.2. Find index a such that $tc \in (L_{a-1}, L_a]$ and let $c = \text{Max}[a, b]$.

1.3. Compute $T_{1,j}$ and find the first index k ($k \geq c$) such that $T_{1,k} \leq L_k$.

1.4. If the index k ($\leq n$) exists, then go to Step 1.5.

Otherwise, compute the total cost for $T_o = L_c, L_{c+1}, \dots, L_n$ and go to Step 2.

1.5. If $T_{1,k} < tc$, then go to Step 2.

Otherwise, compute the total cost for $T_o = L_c, L_{c+1}, \dots, L_{k-1}, T_{1,k}$ and go to Step 2.

Step 2. This step identifies all the candidate values T_o of T satisfying $T_o < tc$.

2.1. Compute $T_{2,0} = \sqrt{\frac{2S}{D(H + \theta C + iC)}}$ and find index b such that $T_{2,0} \in (L_b, L_{b+1}]$.

2.2. If $L_b \geq tc$, then compute the total cost for $T_o = L_{a-1}$ and go to Step 3.

Otherwise, compute $T_{2,j}$ and find the first index k ($k > b$) such that $T_{2,k} \leq L_k$ and go to Step 2.3.

2.3. If the index k ($\leq a$) exists and $T_{2,k} < tc$, then compute the total cost for $T_o = L_b, L_{b+1}, \dots, L_{k-1}, T_{2,k}$ and go to Step 3.

Otherwise, compute the total cost for $T_o = L_b, L_{b+1}, \dots, L_{a-1}$ and go to Step 3.

Step 3. Select the distributor's economic replenishment time among T_o found in steps 1 and 2 which gives the minimum total cost.

4.2. Numerical Example

To illustrate the solution algorithm, the following problem is considered.

$D = 3223$, $H = \$0.1$, $tc = 0.3$, $r = 0.15$ (=15%), $i = 0.1$ (10%), $C = \$3$, $S = \$50$,
 $N_j = j \cdot 500$, $j = 1, 2, \dots, 10$, and $F_j = 10 \cdot j \cdot (1 - 0.02 \cdot (j - 1))$, $j = 1, 2, \dots, 10$
 and $\theta = 0.2$.

Then the distributor's economic replenishment cycle time, T^* , can be obtained through the following steps:

Step 1.

Step 1.1. Since $T_{1,0} = 0.197 \in (L_1, L_2]$, $b = 1$.

Step 1.2. Since $tc = 0.3 \in (L_1, L_2]$, $a = 2$ and $c = 2 = \max[a, b]$.

Step 1.3. Since $T_{1,2} = 0.222 \leq L_2 = 0.301$, $k = 2$.

Step 1.4. Since $k (= 2) < n (= 10)$, go to Step 1.5.

Step 1.5. Since $T_{1,2} = 0.222 < tc (= 0.3)$, go to Step 2.

Step 2.

Step 2.1. Since $T_{2,0} = 0.176 \in (L_1, L_2]$, $b = 1$.

Step 2.2. Since $L_1 (= 0.153) < tc (= 0.3)$, compute $T_{2,j}$, $j = 1, 2, \dots$. And since $T_{2,2} = 0.208 < L_2 (= 0.301)$, $k = 2$.

Step 2.3. Since $T_{2,2} (= 0.208) < tc (= 0.3)$, $T_o = L_1 (= 0.153)$, $T_{2,2} (= 0.208)$.

Step 3. Since $TC_{2,1}(L_1) = \$10017.65$ and $TC_{2,2}(T_{2,2}) = \$10048.74$, the distributor's economic replenishment cycle time becomes $L_1 (= 0.153)$.

5. Conclusions

In this paper, we have analyzed the distributor's reliable replenishment policy assuming that the freight cost has a quantity discount and the manufacturer provides a day-terms credit period for settling the amount the distributor owes to him in a supply chain. The ordering cost sometimes depends upon the ordering quantity, owing to discounts allowed by a shipping company for large order. In this regard, we think that the model presented in this paper may be more realistic for some real world problems in a supply chain. For the system presented, a mathematical model was developed. Recognizing that the model has a very complicated structure, a truncated Taylor series expansion is utilized to find a solution procedure and we can find the feasible solution by this procedure. To illustrate the validity of the procedure, an example problem was chosen and solved.

More work could be done to extend the results of this paper. For example, to recognize that a major reason for the manufacturer to offer a credit period to the distributor is to stimulate the demand of the product, the paper could be extended to the case of the product whose demand rate is a function of the distributor's retail price which in turn is affected by the length of the credit period.

6. References

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신 성 환 : 현재 한라대학교 공과대학 산업경영공학과 교수로 재직 중이며, 인하대학교를 졸업하고, 한국과학기술원 산업공학과에서 공학석사 및 공학박사학위를 받았다. 주요 관심 분야는 SCM, 물류관리, 생산관리 등이다.