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Frequency Reassignment Problem in Code Division Multiple Access Networks

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ABSTRACT

In this paper, we present a frequency reassignment problem (FRP) that arises when we add new base stations to resolve hot-spots or to expand the coverage of a code division multiple access (CDMA) network. For this problem, we develop an integer programming (IP) model along with some valid inequalities and preprocessing rules. Also, we develop an effective heuristic procedure that solves two sub-problems induced from the original problem in repetition. Computational results show that the proposed heuristic procedure finds a feasible solution of good quality within reasonable computation time. Also, the lower bound by-produced from the heuristic procedure is quite strong.

Keywords: Frequency Reassignment, Integer programming, Valid inequality, Heuristic.

1. INTRODUCTION

This paper deals with a new frequency reassignment problem (FRP) arising from the installation of new base stations (BSs) in a code division multiple access (CDMA) network. When we add new BSs to the current CDMA network in order to expand service area or to resolve hot-spots, we assign frequencies to new BSs. However, if we cannot find frequencies to assign to new BSs that do not incur any interference with the frequencies assigned to the existing BSs, we need to change the frequencies that are already assigned to the existing BSs in order to avoid (or to minimize) the interference between frequencies. Interference between frequencies decreases service quality, and reduces the capacity of a CDMA network (see [9]). Since all the BSs use the same frequency in a CDMA network, a pseudo noise

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(PN) code (corresponding to *frequency* in general term) is assigned to each BS such that mobile stations (MSs) can distinguish BSs by PN code. Thus, assigning (or reassigning) PN codes to BSs in a CDMA network is equivalent to assigning (or reassigning) frequencies to BSs in a frequency division multiple access (FDMA) based network. In this sense, we use more general term *frequency* instead of PN code. Below, a typical frequency reassignment process in a CDMA network is described (see [7]).

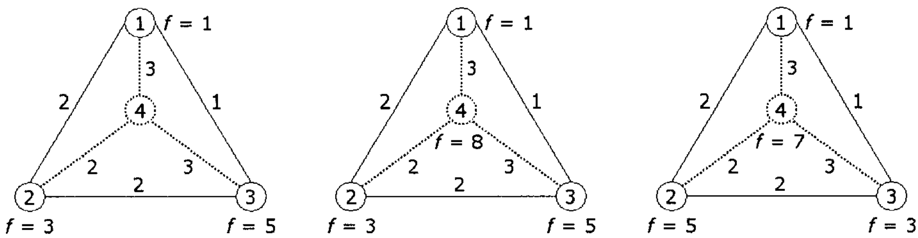
For a given BS, we block both hand-over requests from adjacent BSs and initial call set-up requests. Then, we gradually reduce the transmission power of a BS, during which most of the MSs connected to this BS are handed over to adjacent BSs. If there is no active MSs connected to this BS, we assign a new frequency to this BS. Then, we increase the transmission power of this BS to a target level gradually, and lift the block to initial call set-up and hand-over requests.

If we turn off transmission power of a BS abruptly, on-going calls covered by this BS will be forced to terminate. Also, if we increase transmission power of a BS drastically, the total transmission power of adjacent BSs may diverge over the power budget of a BS, which also forces unintended termination of on-going calls. That is, in a CDMA network, frequency reassignment process should be progressed slowly. As described, frequency reassignment at a BS may result in overloading of adjacent BSs. Also, some MSs may lose their connection to this BS. Thus, we need to minimize the number of frequency reassignments if necessary to obtain a frequency assignment table that avoids (or minimizes) interference between frequencies.

Now, let us consider some engineering constraints of frequency reassignments in a CDMA network. If a number of adjacent BSs change their frequencies in parallel, large geographical area may become out-of-service temporarily. Thus, we need to determine a frequency reassignment sequence for the BSs in need of frequency reassignments such that service failure over multiple adjacent BSs can be avoided. Also, during the frequency reassignment process, we may observe interference between new frequencies reassigned and the frequencies to be reassigned (but not reassigned). Thus, minimizing the interference that may occur in the middle of frequency reassignment process is also important when designing a frequency reassignment process, for example, determining a subset of BSs in need of frequency reassignments, frequency reassignment sequence and the frequencies to (re)assign to BSs.

Now, let us consider an example that illustrates the nature of the FRP. Suppose that we have a network with three BSs 1, 2 and 3 having initial frequencies 1, 3 and 5, respectively, and we add a new BS, indexed by 4. This is illustrated in

Figure 1(a), where the minimum distance between frequencies assigned to adjacent BSs is denoted by the number on links. Here, note that current frequency assignment is interference free. As shown in Figure 1(b), in order to obtain interference free frequency assignments, we can assign frequency 8 to BS 4 without the need to change any frequency. However, if we are allowed to use frequencies from 1 to 7, we have to reassign frequencies to some of the existing BSs. Figure 1(c) illustrates an example of frequency reassignment using the frequencies 1 ~ 7. Now, suppose that reassigning frequencies of BSs 2 and 3 in parallel is not allowed. Then, we have to change the frequencies assigned to BSs 2 and 3, respectively, in order or in reverse order. Based on the frequency assignment shown in Figure 1(c), we consider two alternative frequency reassignment processes. This is illustrated in Figure 2. As shown in Figure 2(a), we can change the frequency of BS 2 from 3 to 5 first. Then, we change the frequency of BS 3 from 5 to 3, and then assign frequency 7 to BS 4. Thus, the number of frequency reassignments is two. Also, note that at $t = 1$ both BSs 2 and 3 use the same frequency of 5, in which case MSs connected to either BS 2 or BS 3 may experience severe interference. Thus, we consider an alternative frequency reassignment process as shown in Figure 2(b). In this case, the minimum distance requirement to avoid interference is satisfied at all periods, while the number of frequency reassignments is three.



(a) initial assignment (b) reassignment example 1 (c) reassignment example 2

Figure 1. Frequency reassignment example

Base Station	Frequency		
	$t = 0$	$t = 1$	$t = 2$
1	1	1	1
2	3	5*	5
3	5	5	3*
4	–	–	7

(a) reassignment process example 1

Base Station	Frequency			
	$t = 0$	$t = 1$	$t = 2$	$t = 3$
1	1	1	1	1
2	3	3	5*	5
3	5	7*	7	3*
4	–	–	–	7

(b) reassignment process example 2

Figure 2. Frequency reassignment process of Figure 1(c)

Although not presented in this example, we cannot find an interference free frequency assignment if we are allowed to use frequencies from 1 to 6. Thus, in this case, we need to consider reassigning frequencies, while allowing minimum interference unlike the work by Han [7] that seeks to find an interference free frequency assignment when frequency reassignment is completed.

On frequency reassignment problem, Han [7] firstly showed that this problem is NP-hard, and developed two integer programming (IP) formulations that do not allow any interference when frequency reassignment is completed. Unlike the frequency reassignment problem, there exists numerous studies on frequency assignment problem (FAP). Below, some of the distinguished research results on FAPs are summarized.

One of the generic FAP is to minimize the total number of frequencies needed to satisfy the minimum distance requirements for all pairs of adjacent BSs. This type of FAP was dealt by Hale [6], Gamst and Rave [5], Hao *et al.* [8] and Sung and Wong [12]. Hale [6] firstly showed that this problem can be expressed as a graph coloring problem.

Another type of FAP, referred to as Max-FAP, is to maximize the total number of frequencies assigned to BSs, while satisfying the minimum distance requirements for all pairs of adjacent BSs. Chang and Kim [2], Gamst and Rave [5], Marthar and Mattfeldt [10], Sung and Wong [12] and Tiourine *et al.* [13] dealt with the Max-FAP. In particular, Sung and Wong [12] improved the 'sequential packing' heuristic algorithm suggested by Gamst and Rave [5].

Hao *et al.* [8] also developed a tabu search algorithm to solve a FAP that minimizes the total interference between frequencies, referred to as MI(Minimum Interference)-FAP. For the MI-FAP, Tiourine *et al.* [13] developed an heuristic algorithm and compared the performance with a tabu search algorithm developed by Hao *et al.* [8]. Also, Tiourine *et al.* [13] obtained a tight lower bound by reformulation, and found an optimal solution using the branch-and-bound combined with some preprocessing routines. Besides, quite many research papers are well summarized in the work by Aardal *et al.* [1].

The remainder of this paper is organized as follows. In Section 2, we develop a mathematical formulation, and derive some valid inequalities. Also, we present a compact formulation by deriving some preprocessing rules. In Section 3, we develop an effective heuristic procedure based on a decomposition principle. Computational results are provided in Section 4, and Section 5 concludes this paper.

2. FORMULATION

Let us define some notations in order to formulate the problem FRP. Let N be a set of existing BSs, and let V be a set of new BSs. Also, let K be a set of frequencies, and let T be the maximum time periods, indexed by $t = 1,\dots, T$, allowed for frequency reassignments. Hereafter, we represent $t = 1,\dots, T$ by $t \leq T$ for notational convenience. Let $E = \{(i, j)\colon r(i, j) > 0$ for $i, j(> i) \in N \cup V\}$, where $r(i, j)$ is the minimum distance to avoid interference between frequencies assigned to BSs i and $j(> i) \in N \cup V$, respectively. In particular, we define $E(N) = \{(i, j)\colon r(i, j) > 0$ for $i, j(> i) \in N\}$. Also, let $A \subseteq E(N)$ be a set of pairs of adjacent BSs i and $j(> i) \in N$ such that reassigning frequencies simultaneously at BSs i and $j(> i)$ is not allowed Now, let us define decision variables and input parameters. Let $x_{tik} = 1$ if frequency $k \in K$ is assigned to BS $i \in N \cup V$ at t ($\leq T$), and 0 otherwise. Also, let $y_{ti} = 1$ if a new frequency is assigned to BS $i \in N$ at t ($\leq T$), and 0 otherwise. Let $v_t = 1$ if frequency reassignment is completed at t ($\leq T$), and 0 otherwise. And, let us denote the cost of reassigning the frequency of BS $i \in N$ by c_i. Let $u_{tij} = 1$ if the distance between frequencies assigned to BSs i and $j(> i) \in N \cup V$, respectively, is less than $r(i, j)$, and 0 otherwise. When $u_{tij} = 1$ for $t \leq T - 1$ and $(i, j) \in E$, interference cost p_{ij} occurs. Also, if $u_{tij} = 1$ for $t = T$ and $(i, j) \in E$, interference cost q_{ij} occurs. Here, note that interference cost $\boldsymbol{q}$ needs to be set quite large compared to interference cost $\boldsymbol{p}$ since the frequency assignment at $t = T$ influences on service quality continuously after the completion of frequency reassignment, while the influence of frequency assignment at $t \leq T - 1$ on service quality is temporal. Let $\{h_{ik}\}$ be the input vector indicating the initial frequency assigned to BS $i \in N$ at $t = 0$. That is, if frequency $k \in K$ is assigned to BS $i \in N$ at $t = 0$, we set $h_{ik} = 1$, and 0 otherwise.

Using the above notations, we can formulate the problem FRP as follows, denoted by **P1**.

P1: Minimize $\sum_{t \leq T}\sum_{i \in N} c_i y_{ti} + \sum_{t \leq T-1}\sum_{(i,j) \in E} p_{ij} u_{tij} + \sum_{(i,j) \in E} q_{ij} u_{Tij}$

Subject to

$\sum_{k \in K} x_{tik} = 1$	$t \leq T,\ i \in N,$	(1)
$\sum_{k \in K} x_{tik} = v_t$	$t \leq T,\ i \in V,$	(2)
$\sum_{t \leq T} v_t \geq 1$		(3)
$x_{tik} + x_{tjl} \leq 1 + u_{tij}$	$t \leq T,\ (i, j) \in E,\ k, l \in K\colon \lvert k - l \rvert < r(i, j),$	(4)
$y_{ti} \geq x_{tik} - x_{(t-1)ik}$	$t \leq T,\ i \in N,\ k \in K,$	(5)
$y_{ti} \geq x_{(t-1)ik} - x_{tik}$	$t \leq T,\ i \in N,\ k \in K,$	(6)
$y_{ti} + y_{tj} \leq 1$	$t \leq T,\ (i, j) \in A,$	(7)
$x_{tik} \in \{0,1\}$	$t \leq T,\ i \in N \cup V,\ k \in K,$	
$y_{ti} \in \{0,1\}$	$t \leq T,\ i \in N,$	

$u_{tij} \in \{0,1\}$ $\quad t \leq T, (i, j) \in E,$
$v_t \in \{0,1\}$ $\quad t \leq T,$

where $x_{0ik} = h_{ik}$ for $i \in N$ and $k \in K$.

Constraint (1) forces that a frequency should be assigned to each existing BS at all time periods. Constraint (2) forces that a frequency should be assigned to each new BS when frequency reassignment is completed. Constraint (3) forces that frequency reassignment should be completed within T time periods. Constraint (4) expresses the interference between frequencies assigned to a pair of adjacent BSs. Constraints (5) and (6) express the frequency reassignment at existing BSs. Constraint (7) prohibits simultaneous frequency reassignments for a pair of adjacent BSs in A.

Remark 1. Note that there exists an optimal solution satisfying that $v_T = 1$ and $v_t = 0$ for all $t \leq T - 1$, which implies that we don't have to consider any interference between frequencies assigned (or to reassign) to existing BSs and frequencies to assign to new BSs at $t \leq T - 1$. This enables us to delete constraint (3) from the **P1** which in turn enables us to set $x_{tik} = 0$ for all $t \leq T - 1$, $i \in V$ and $k \in K$, and to set $u_{tij} = 0$ for all $t \leq T - 1$ and $(i, j) \in E \setminus E(N)$. Then, we can rewrite the **P1** as follows, denoted by **P2**. □

P2: Minimize $\sum_{t \leq T} \sum_{i \in N} c_i y_{ti} + \sum_{t \leq T-1} \sum_{(i,j) \in E(N)} p_{ij} u_{tij} + \sum_{(i,j) \in E} q_{ij} u_{Tij}$

Subject to

$\sum_{k \in K} x_{tik} = 1$	$t \leq T, i \in N,$	(8)
$\sum_{k \in K} x_{Tik} = 1$	$i \in V,$	(9)
$x_{tik} + x_{tjl} \leq 1 + u_{tij}$	$t \leq T - 1, (i, j) \in E(N), k, l \in K: \lvert k - l \rvert < r(i, j),$	(10)
$x_{Tik} + x_{Tjl} \leq 1 + u_{Tij}$	$(i, j) \in E, k, l \in K: \lvert k - l \rvert < r(i, j),$	(11)
$y_{ti} \geq x_{tik} - x_{(t-1)ik}$	$t \leq T, i \in N, k \in K,$	(12)
$y_{ti} \geq x_{(t-1)ik} - x_{tik}$	$t \leq T, i \in N, k \in K,$	(13)
$y_{ti} + y_{tj} \leq 1$	$t \leq T, (i, j) \in A,$	(14)
$x_{tik} \in \{0,1\}$	$t \leq T, i \in N, k \in K,$	
$x_{Tik} \in \{0,1\}$	$i \leq V, k \in K,$	
$y_{ti} \in \{0,1\}$	$t \leq T, i \in N,$	
$u_{tij} \in \{0,1\}$	$t \leq T - 1, (i, j) \in E(N),$	
$u_{Tij} \in \{0,1\}$	$(i, j) \in E.$	

Next, we derive some valid inequalities based on the constraints (10), (11) and

(14) in order to enhance the lower bound of the **P2**.

Remark 2. For $t \leq T - 1$, $(i, j) \in E(N)$ and $k \in K$, we obtain following valid inequalities by lifting the constraint (10).

$$x_{tik} + \sum_{f = \max\{1,\, k - (r(i,j) - 1)\},\ldots,\, \min\{|K|,\, k + (r(i,j) - 1)\}} x_{tjf} \leq 1 + u_{tij}, \tag{15a}$$

and

$$x_{tjk} + \sum_{f = \max\{1,\, k - (r(i,j) - 1)\},\ldots,\, \min\{|K|,\, k + (r(i,j) - 1)\}} x_{tif} \leq 1 + u_{tij}. \tag{15b}$$

Here, note that valid inequalities (15a) and (15b) dominate the constraints (10). Similarly, we can derive valid inequalities based on the constraint (11). For $(i, j) \in E$ and $k \in K$ at $t = T$, we can derive following valid inequalities.

$$x_{Tik} + \sum_{f = \max\{1,\, k - (r(i,j) - 1)\},\ldots,\, \min\{|K|,\, k + (r(i,j) - 1)\}} x_{Tjf} \leq 1 + u_{Tij}, \tag{16a}$$

and

$$x_{Tjk} + \sum_{f = \max\{1,\, k - (r(i,j) - 1)\},\ldots,\, \min\{|K|,\, k + (r(i,j) - 1)\}} x_{Tif} \leq 1 + u_{Tij}. \tag{16b}$$

Inequalities of type (15) and (16) were first investigated by Padberg [11] in the context of set packing problem, and were used by Fischetti *et al.* [4] for solving the well-known FAP. Also, note that valid inequalities (16a) and (16b) dominate the constraints (11). Now, let us consider constraint (14), from which we can derive a clique inequality as follows.

$$\sum_{i \in C} y_{ti} \leq 1 \qquad\qquad t \leq T, \tag{17}$$

where C is a clique of the graph $G(A)$ induced by edge set A. Obviously, clique inequality (17) dominates the constraint (14). Separating the inequality (17) amounts to finding a maximal clique in a graph $G(A)$. This problem is known to be NP-hard. Thus, we implement heuristic procedure for separating the inequality (17) from $G(A)$. Our approach is as follows. First, we detect a triangle subgraph $G(S)$, where $S \subseteq A$. Then, we expand $G(S)$ by adding a new BS $i \in N$ not in $G(S)$ until we cannot find any new BS to add. Then, we let $C = \{i \in G(S)\}$. □

3. HEURISTIC PROCEDURE

In this section, we develop an efficient heuristic procedure based on a decomposition principle. Let $\tilde{x}$ be a frequency assignment at $t = T$. If we let $T = 1$, we can delete index t from the **P2**. Also, if we delete constraint (14), the **P2** reduces to as follows, denoted by **P3**. This implies that we obtain $\tilde{x}$ by solving the **P3**.

P3: Minimize $\sum_{i \in N} \sum_{k \in K} c_i (1 - h_{ik}) x_{ik} + \sum_{(i,j) \in E} q_{ij} u_{ij}$

Subject to

$$\sum_{k \in K} x_{ik} = 1 \qquad i \in N \cup V,$$
$$x_{ik} + x_{jl} \le 1 + u_{ij} \qquad (i, j) \in E,\ k, l \in K\text{: } |k - l| < r(i, j),$$
$$x_{ik} \in \{0,1\} \qquad i \in N \cup V,\ k \in K,$$
$$u_{ij} \in \{0,1\} \qquad (i, j) \in E.$$

For a given $\tilde{x}$, we can define a set $\Omega = \{i \in N\text{: } \tilde{x}_{ik} \ne h_{ik} \text{ for some } k \in K\}$. Thus, to find a feasible solution to the **P2**, it is sufficient to determine at what time period t ($\le T$) to reassign frequencies assigned to the BSs in Ω considering constraint (14). Letting $\Omega(t) \subseteq \Omega$ be a set of BSs to reassign frequencies at t ($\le T$), any partition of Ω over t, $\{\Omega(t)\text{: } t \le T\}$, defines a feasible solution to the **P2** if $(i, j) \notin A$ for $i, j(> i) \in \Omega(t)$ and t ($\le T$).

Now, let us consider the outline of the proposed heuristic procedure. First, we find a frequency assignment at $t = T$, $\tilde{x}$, which automatically defines Ω. Then, we find a feasible solution $\{\Omega(t)\text{: } t \le T\}$ such that $\Omega(s) \cap \Omega(t) = \varnothing$ for all s and $t(> s) \le T$. That is, we change the initial frequency assigned to the BS in Ω only once. Then, we resume this process with an alternative feasible solution to the **P3**, $\tilde{x}$. The above process is repeated for a given time bound.

Remark 3. Note that the optimal objective value to the **P3** provides a lower bound to the **P2** since we obtain the **P3** from the **P2** by letting $T = 1$ and by deleting constraint (14). □

3.1 Initial Heuristic

Below, we describe an heuristic procedure to find an initial feasible solution $\{\Omega(t)\text{: } t \le T\}$ such that $\Omega(s) \cap \Omega(t) = \varnothing$ for all s and $t(> s)$.

Initialize. Let $\Omega(t) = \varnothing$ for $t \le T$, and let $t = 1$.
Step 1. Find a frequency assignment $\tilde{x}$ at $t = T$, and get Ω. If $\Omega = \varnothing$, stop. Otherwise, go to Step 2.
Step 2. If $\{(i, j) \in A\text{: } i, j\ (> i) \in \Omega\} = \varnothing$, stop. Otherwise, go to Step 3.
Step 3. Pick an arbitrary BS $i \in \Omega$. Then, let $\Omega(t) = \{i\}$ and $\Omega = \Omega \setminus \{i\}$. If $\Omega = \varnothing$, stop. Otherwise, go to Step 4.

Step 4. Pick an arbitrary BS $j \in \Omega$ such that $(i, j) \notin A$ for all $i \in \Omega(t)$. Then, let $\Omega(t) = \Omega(t) \cup \{j\}$, and let $\Omega = \Omega \setminus \{j\}$. If $\Omega = \varnothing$, stop. Otherwise, resume Step 4. If such a BS is not found, let $t = t + 1$ and go to Step 3.

Remark 4. For finding a frequency assignment $\tilde{x}$ at $t = T$ in Step 1, we solved the **P3** using a commercial optimization software, CPLEX Version 9.0 [3]. If the above heuristic procedure terminates at Step 1 or at Step 2, $\tilde{x}$ defines an optimal solution to the **P2** provided that we solved the **P3** optimally. However, if the above heuristic procedure terminates at Step 3 or at Step 4, we need to calculate the interference cost $\boldsymbol{p}$ incurred by the current partition $\{\Omega(t): t \leq T\}$. If no interference is observed at any time period $t \leq T - 1$, we see that the current partition $\{\Omega(t): t \leq T\}$ defines an optimal solution to the **P2** provided that we solved the **P3** optimally. Otherwise, we improve the initial solution, which is described in the following. □

3.2 Improving the Initial Solution

First, we seek to minimize interference cost $\boldsymbol{p}$ by finding an optimal partition of Ω over t, $\{\Omega(t): t \leq T\}$ for a given Ω. If we cannot reduce interference cost $\boldsymbol{p}$ even with the optimal partition of Ω, we generate an alternative frequency assignment $\tilde{x}$ at $t = T$, which defines a new set Ω. Then, we try to minimize interference cost $\boldsymbol{p}$ again based on a new Ω. This process is repeated for a given time bound. Another termination criterion is that we fail to find a new $\tilde{x}$. In Section 3.2.1, we describe a procedure that minimizes interference cost $\boldsymbol{p}$ based on an incumbent Ω. And, in Section 3.2.2, we describe a procedure that finds an alternative frequency assignment at $t = T$ in order to derive a new Ω.

3.2.1 Minimizing the interference cost

For a given Ω, minimizing the interference cost $\boldsymbol{p}$ is equivalent to optimally solving the following problem, denoted by **P4**.

P4: Minimize $\sum_{t \leq T-1} \sum_{i, j(\neq i) \in \Omega} p_{ij} u_{tij}$

Subject to

$x_{tif(i)} + x_{tig(i)} = 1 \qquad t \leq T - 1,\ i \in \Omega,$

$x_{tig(i)} + x_{tjf(j)} \leq u_{tij} + 1 \qquad t \leq T - 1,\ i, j(\neq i) \in \Omega\text{: } |f(i) - g(j)| < r(i, j),$

$y_{ti} = x_{(t-1)if(i)} + x_{tig(i)} - 1 \qquad t \leq T,\ i \in \Omega,$

$y_{ti} + y_{tj} \leq 1 \qquad t \leq T,\ i, j(> i) \in \Omega\text{: } (i, j) \in A,$

$\sum_{t \in T} y_{ti} = 1$ $\qquad i \in \Omega,$

$x_{tif(i)} \in \{0,1\}$ $\qquad t \leq T-1,\ i \in \Omega,$

$x_{tig(i)} \in \{0,1\}$ $\qquad t \leq T,\ i \in \Omega,$

$y_{ti} \in \{0,1\}$ $\qquad t \leq T,\ i \in \Omega,$

$u_{tij} \in \{0,1\}$ $\qquad t \leq T-1,\ i, j(\neq i) \in \Omega{:}\ |f(i) - g(j)| < r(i, j),$

where $f(i) = arg_{k \in K}\{h_{ik} = 1\}$ and $g(i) = arg_{k \in K}\{\tilde{x}_{ik} = 1\}$ for $i \in \Omega$.

Note that the **P4** minimizes interference cost $\boldsymbol{p}$ by determining an optimal time period $t \leq T$ to change the initial frequency $f(i)$ to a new frequency $g(i)$ for each BS $i \in \Omega$. Here, note that the **P4** can be obtained from the **P2** by

- setting $x_{Tik} = \tilde{x}_{ik}$ for $i \in N \cup V$, $k \in K$,
- setting $x_{tik} = \tilde{x}_{ik}$ for $t \leq T-1$, $i \in N \setminus \Omega$, $k \in K$, and
- adding $\sum_{t \leq T} y_{ti} = 1$ for $i \in \Omega$ to the **P2**.

3.2.2 Finding an alternative frequency assignment

We already have a frequency assignment at $t = T$. Thus, we try to find an alternative frequency assignment at $t = T$, which can be a basis for finding a new feasible solution to the **P2**. For the sake of notational convenience, let us denote the incumbent $\tilde{x}$ by w. Then, we can find a new $\tilde{x}$ by solving the **P3** after adding the following constraint to the **P3**:

$$\sum_{i \in N \cup V} \sum_{k \in K} w_{ik} x_{ik} \leq |N \cup V| - 1.$$

If the updated **P3** returns no feasible solution, or if the optimal objective value to the updated **P3** is greater than the total cost of the best solution to the **P2**, we terminate heuristic procedure. Otherwise, we minimize interference cost $\boldsymbol{p}$ based on a new $\tilde{x}$ by performing the procedure described in Section 3.2.1.

4. COMPUTATIONAL RESULTS

In order to evaluate the performance of the proposed heuristic procedure and the effectiveness of valid inequalities (15), (16) and (17), we generated test problems as follows.

Step 1. Generate $|N|$ + $|V|$ BSs at random on a rectangle with scale 1000 by

1000, and calculate the distance for all pairs of BSs.

Step 2. If the distance between a pair of BSs i and $j(> i) \in N \cup V$, d_{ij}, is greater than a threshold R, we set $r(i, j) = 0$, otherwise, we set $r(i, j) = \lceil R/d_{ij} \rceil$. In our test, we set $R = 300$ or 400 depending on the number of BSs.

Step 3. Find an interference free initial frequency assignment $\{h_{ik}\}$ for $i \in N$ and $k \in K$, using the following procedure.

> **Step 3.1** Pick an arbitrary BS and assign frequency of 1.
>
> **Step 3.2** Find a combination of BS and frequency $\{i, f\}$, where BS i has no assigned frequency and is adjacent to at least one of the BSs that are already assigned with frequencies, and where frequency f is the lowest index frequency that incurs no interference with adjacent BSs. Then, assign frequency f to BS i. If frequencies are assigned to all BSs, go to Step 4. Otherwise, resume Step 3.2.

Step 4. Let $|K|$ be the largest frequency index used in Step 3 multiplied by 0.8 or 0.9. For each BS, if the assigned frequency is less than or equal to $|K|$, let this BS belong to the set N. Otherwise, let this BS belong to the set V, and delete the initial frequency assigned to it.

Step 5. Let $A = \{(i, j) \in E(N)\colon d_{ij} < 0.7 \times R\}$. And, let c_i have a random integer value in the rage [10, 20] for $i \in N$. Also, let $p_{ij} = (c_i + c_j) \times r(i, j)$ for $(i, j) \in E(N)$ and $q_{ij} = 10 \times (c_i + c_j) \times r(i, j)$ for $(i, j) \in E$.

Note that frequency reassignment cost $\boldsymbol{c}$ is generated based on a uniform rule as described in Step 5. The reason is that, as described in Section 1 (Introduction), when frequency reassignment is conducted at a BS, all the MSs connected to this BS are handed over to adjacent BSs. Thus, we assume that frequency reassignment cost c_i can be set to be proportional to the average traffic load of each BS $i \in N$. In this paper, we set the average traffic load of a BS to be in the rage from 10 to 20, which can be scaled up (or down). Also, for a given $(i, j) \in E$, we generate interference costs $\boldsymbol{p}$ and $\boldsymbol{q}$ to be proportional to the sum of frequency reassignment costs, $c_i + c_j$, and to be proportional to the minimum distance requirement to avoid interference, $r(i, j)$. The rationality of this cost structure for $\boldsymbol{p}$ and $\boldsymbol{q}$ is that all the MSs connected either to BS i or to $j(> i)$ suffer from the same interference if two frequencies assigned to BSs i and $j(> i)$, respectively, are not separated enough since only one frequency is assigned to each BS.

The developed heuristic procedure was coded in Visual Basic 6.0 coupled with CPLEX 9.0 (see [3]), and was tested on Pentium IV PC (CPU: 2.8GHz, RAM: 512Mbytes). Computational results are presented in Tables 1 and 2, where we use the following notations.

- $P2_{LP}$: optimal objective value to the LP-relaxation of the formulation **P2**,
- $P2C_{LP}$: optimal objective value to the LP-relaxation of the formulation **P2** enhanced by the valid inequalities (15), (16) and (17) at the root node of branch-and-bound tree,
- P2C : optimal objective value to the formulation **P2** enhanced by the valid inequalities (15), (16) and (17), and
- *P3 : optimal objective value to the formulation* **P3**.

As stated in **Remark 2**, valid inequalities (15), (16) and (17) dominate the constraints (10), (11) and (12), respectively. Thus, we replaced the constraints (10), (11) and (12) by valid inequalities (15), (16) and (17), respectively, in order to obtain P2C (and $P2C_{LP}$).

In Tables 1 and 2, we report computational results for 30 test problems, respectively. For each test problem shown in Table 1, we assumed that only 1 ~ 4 new BSs are added, while we assumed that 4 ~ 11 new BSs are added for the test problem in Table 2. In this paper, we consider 40 BSs at maximum including new BSs. Considering that we face with the frequency reassignment problem addressed in this paper when we add new BSs to resolve hot-spots at a small geographical area or to expand the coverage, it seems that the size of our test problem having 40 BSs at maximum is not small at all.

We terminated the CPLEX optimization procedure and the heuristic procedure after 10,000 seconds and after 1,000 seconds, respectively, except 6 test problems numbered by 24, 25, 27-30 in Table 2. Computation time is presented in parenthesis. The mark "NA" represents that CPLEX optimization procedure failed to find an initial feasible solution in 10,000 seconds. From Tables 1 and 2, we see that valid inequalities (15), (16) and (17) significantly improves the lower bound of the formulation **P2**. However, note that the **P2C** enhanced by valid inequalities (15)-(17) is not always useful for finding a better feasible solution within 10,000 seconds. It is quite interesting since the valid inequalities (15)-(17) are not added to the **P2**, but replaced the constraints (10)-(12), respectively. Also, we see that the proposed heuristic procedure finds a feasible solution of good quality to the most test problems within 1,000 seconds. In particular, the heuristic procedure finds an optimal solution to 28 test problems out of 60 test problems, which is indicated by asterisk mark (*) at the fifth column "Opt" of Tables 1 and 2. The marks "b", "w" and "e" indicate that the heuristic procedure found a "better", "worse" and "equally good" feasible solution, respectively, compared with that obtained by directly solving the **P2** or the **P2** enhanced by valid inequalities (15)-(17). Another observation is that the optimal objective value to the **P3** provides a very tight lower bound.

Table 1. Computational results of test problems (|V| = 1 ~ 4)

No	Size	Lower bound	Upper bound	Opt
	$\|N\|,\|V\|,\|E\|,\|K\|,T$	$P2_{LP}$, $P2C_{LP}$, P3	P2, P2C, Heuristic	
1	19, 1, 76, 17, 9	48, 115, 578(2)	598(10K), 598(10K), 598(21)	*
2	19, 1, 75, 13, 9	85, 217, 770(1)	770(10K), 770(10K), 770(1)	*
3	18, 2, 51, 8, 9	86, 314, 637(1)	637(183), 637(106), 637(1)	*
4	19, 1, 62, 15, 9	50, 106, 219(1)	219(4490), 219(746), 219(1)	*
5	18, 2, 58, 11, 9	107, 362, 965(2)	1026(10K), 1026(6.7K), 1072(1K)	w
6	18, 2, 50, 10, 9	107, 256, 1003(1)	1100(10K), 1100(7.3K), 1121(1K)	w
7	19, 1, 85, 16, 9	55, 113, 339(1)	750(10K), 750(10K), 387(1K)	b
8	19, 1, 96, 16, 9	62, 152, 402(1)	940(10K), 402(10K), 402(1)	*
9	16, 4, 74, 12, 8	116, 279, 1246(11)	1350(10K), 1350(10K), 1442(1K)	w
10	19, 1, 69, 14, 9	63, 130, 530(2)	530(6.9K), 530(6.6K), 530(2)	*
11	29, 1, 158, 15, 14	109, 251, 387(1)	1190(10K), 1450(10K), 410(1K)	b
12	27, 3, 164, 17, 14	159, 392, 993(28)	3020(10K), NA(10K), 1002(1K)	b
13	28, 2, 136, 19, 14	68, 177, 290(1)	841(10K), 1020(10K), 311(1K)	b
14	28, 2, 168, 17, 14	72, 237, 627(25)	1410(10K), 3181(10K), 632(1K)	b
15	29, 1, 181, 17, 14	69, 154, 492(6)	900(10K), 1150(10K), 580(133)	b
16	28, 2, 143, 14, 14	137, 313, 826(6)	1690(10K), 2982(10K), 826(6)	*
17	27, 3, 154, 17, 13	97, 252, 695(19)	1364(10K), 3894(10K), 773(1K)	b
18	28, 2, 141, 20, 14	103, 285, 783(19)	1831(10K), NA(10K), 863(1K)	b
19	29, 1, 131, 17, 14	47, 121, 395(1)	395(10K), 1335(10K), 395(1)	*
20	29, 1, 157, 18, 14	41, 132, 264(1)	300(10K), 810(10K), 264(1)	*
21	37, 3, 276, 19, 18	135, 289, 676(90)	4170(10K), NA(10K), 831(1K)	b
22	38, 2, 308, 24, 19	79, 246, 373(48)	1141(10K), 901(10K), 513(1K)	b
23	39, 1, 266, 23, 19	89, 156, 172(1)	172(3626), 172(81), 172(1)	*
24	37, 3, 288, 20, 18	151, 329, 898(121)	NA(10K), NA(10K), 920(1K)	b
25	38, 2, 302, 22, 19	76, 257, 576(23)	2210(10K), NA(10K), 590(1K)	b
26	38, 2, 267, 20, 19	50, 152, 1000(224)	1000(10K), 1740(10K), 1000(224)	*
27	37, 3, 271, 20, 18	175, 374, 768(15)	2740(10K), 2700(10K), 921(1K)	b
28	38, 2, 308, 26, 19	87, 140, 376(9)	2280(10K), 1346(10K), 399(1K)	b
29	38, 2, 287, 23, 19	82, 215, 576(20)	2170(10K), 2089(10K), 829(1K)	b
30	38, 2, 233, 20, 19	94, 255, 395(6)	485(10K), 1205(10K), 443(1K)	b

Table 2. Computational results of test problems ($|V|$ = 4 ~ 11)

No	Size	Lower bound	Upper bound	Opt
	$\|N\|,\|V\|,\|E\|,\|K\|,T$	$P2_{LP}$, $P2C_{LP}$, P3	P2, P2C, Heuristic	
1	11, 9, 48, 4, 5	406, 2160, 7072(1)	7072(176), 7072(2) , 7072(1)	*
2	13, 7, 57, 5, 6	348, 968, 6095(5)	6095(837), 6095(30), 6095(5)	*
3	17, 3, 57, 10, 8	98, 320, 1084(1)	1084(395), 1084(318), 1084(1)	*
4	15, 5, 43, 6, 7	197, 582, 4136(1)	4136(103), 4136(18), 4136(1)	*
5	12, 8, 48, 5, 6	370, 1176, 6802(1)	6802(277), 6802(4), 6802(1)	*
6	14, 6, 42, 5, 7	218, 744, 6618(1)	6618(425), 6618(14), 6618(1)	*
7	16, 4, 45, 7, 8	166, 509, 2698(1)	2698(237), 2698(8), 2698(1)	*
8	12, 8, 54, 4, 6	375, 3799, 9351(1)	9351(119), 9351(2), 9351(1)	*
9	11, 9, 46, 5, 5	310, 1215, 7573(1)	7573(318), 7573(7), 7573(1)	*
10	13, 7, 40, 5, 6	218, 630, 3435(1)	3435(37), 3435(3), 3435(1)	*
11	22, 8, 102, 7, 11	353, 1079, 5391(80)	6338(10K), 5994(10K), 5500(211)	b
12	25, 5, 91, 9, 12	248, 753, 2560(62)	3088(10K), 2861(10K), 2560(62)	*
13	25, 5, 95, 8, 12	255, 683, 4211(57)	4633(10K), 4211(10K), 4211(57)	*
14	28, 2, 104, 13, 14	81, 201, 1963(13)	2240(10K), 1963(10K), 1963(13)	*
15	25, 5, 106, 10, 12	184, 476, 2556(71)	3076(10K), 3096(10K), 3010(169)	b
16	25, 5, 117, 12, 12	169, 419, 1805(83)	3021(10K), 2612(10K), 2001(92)	b
17	20, 10, 97, 7, 10	352, 1124, 6610(471)	7795(10K), 6610(10K), 6610(471)	*
18	27, 3, 106, 11, 13	171, 484, 2383(18)	2383(10K), 2738(10K), 2383(18)	*
19	24, 6, 103, 8, 12	249, 703, 4887(26)	5713(10K), 6019(10K), 5101(1K)	*
20	26, 4, 92, 11, 13	152, 418, 1985(9)	2553(10K), 2569(10K), 2105(1K)	*
21	34, 6, 183, 13, 17	170, 500, 1776(606)	5326(10k), 7698(10k), 1908(1K)	b
22	31, 9, 153, 8, 15	435, 1114, 6420(575)	8897(10k), 9277(10k), 6899(1K)	b
23	36, 4, 131, 10, 18	202, 366, 1932(20)	2693(10k), 2000(10k), 2000(66)	e
24	33, 7, 140, 10, 16	360, 979, 4396(4855)	7173(10k), 7071(10k), 5017(4855)	b
25	35, 5, 158, 10, 17	332, 797, 3825(1134)	5345(10k), 4220(10k), 4220(1134)	e
26	36, 4, 160, 13, 18	155, 353, 1809(111)	3495(10k), 4612(10k), 2001(319)	b
27	29, 11, 139, 9, 14	339, 949, 5983(6327)	7436(10K), 6206(10K), 6117(6327)	b
28	36, 4, 175, 15, 18	97, 342, 3344(2762)	5060(10K), 6830(10K), 3533(2762)	b
29	36, 4, 179, 13, 18	182, 419, 2645(5758)	3290(10K), 2930(10K), 2930(5758)	e
30	29, 11, 145, 8, 14	513, 1336, 9128(4322)	11094(10K), 10920(10K), 9310(4322)	b

5. CONCLUSION

In this paper, we addressed a new frequency reassignment problem arising from the installation of new BSs in a CDMA network. And, we developed mathematical formulations for this problem along with some valid inequalities. For solving large size problems, we developed a heuristic procedure that solves two sub-problems induced from the original problem in repetition. Computational results show that the developed heuristic procedure finds a feasible solution of good quality within reasonable time bound.

Further research task is to develop a meta-heuristic procedure in order to handle larger problem instances. Also, as an extension of this study, frequency reassignment for the global system for mobile communications (GSM) based telecommunication system, allocating multiple frequencies to each BS, should be followed.

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