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The Cost Impact of Information Delay in a Supply Chain*

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ABSTRACT

In this paper, the impact of information sharing, possibly with some delay, on costs in a simple supply chain in which there are two participants, a single retailer and a single manufacturer, is considered. When participants in the supply chain do not use fully integrated EDI, some delay associated with information sharing is inevitable. A mathematical model that allows us to quantify the cost incurred by the manufacturer in the supply chain under information sharing, possibly with some delay, vs. no information sharing is presented. From this model, some managerial implications are gleaned.

Keywords: Cost Impact, Information Delay, Supply Chain

1. INTRODUCTION

Maybe the biggest difference between traditional methods of supply chain management and recent developments in supply chain management is in the amount and type of information that is available at each stage of the supply chain. This has been made possible mainly by electronic data interchange (EDI) which again is supported by point-of-sale (POS) scanners, bar-coding technology, and online inventory and production control system (Buzzell and Ortmeyer [2]).

While EDI has caused considerable excitement in the press and among operations practitioners over the past several years, the majority of firms that use EDI do not actively use electronic transmission of data to enhance operations. In fact, many firms do not employ integrated EDI; rather, they manually re-key informa-

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tion received electronically (Bourland, Powell and Pyke [1]). In addition, information is often transmitted through other communication means, such as phones or faxes. In these cases, some delay in the sharing of information is inevitable.

There may also be some resistance to information sharing because the information to be shared is often sensitive one such as sales, inventory levels or production schedules. Therefore, companies may be afraid that they will give away competitive advantage or lose customers to other competitors (Hammond [6]). In this case, the participant in the system may be reluctant to give real time information and instead might share some delayed or outdated information instead.

From the preceding reasons, it can be seen that information sharing with some delay is not a rarity. Therefore, the questions of how costs in a supply chain depend on whether information is shared and how costs in a supply chain depend on how long information is delayed in case of information sharing are important ones from not only theoretical but also practical points of view. While much research has been done for answering the former question, little has been done for answering the latter one, as will be shown in Section 2. Intuitively, the sooner information is transmitted, the better because this should allow the participants to improve their forecasts promptly and to reduce the costs due to excess inventory and stockouts. Therefore, the actual value of information sharing will depend on the delay associated with information sharing and hence the pitfalls resulting from information sharing with some delay, need to be quantified, which is the goal of this paper.

The remainder of this paper is organized as follows: Section 2 reviews the previous research related to this paper. Section 3 develops a supply chain model and Section 4 shows major findings about the cost impact of information delay. Final remarks are addressed in Section 5.

2. LITERATURE REVIEW

Various issues in supply chain management have been considered by numerous researchers. Some of the issues that are relevant to this paper include the value of information and information sharing. Here, we briefly review some of the relevant previous literature on the value of information and information sharing in supply chain management.

Gavirneni, Kapuscinski and Tayur [5] consider a simple two stage supply chain with a single retailer and a single supplier, where the retailer follows an (s,

S) inventory policy and the supplier has a limited capacity. They consider three levels of information sharing: (i) no information sharing, (ii) the supplier is told the form of the retailer's inventory policy and the form of the customer demand distribution, and (iii) the supplier is told the form of the retailer's inventory policy the form of the customer demand distribution and the daily inventory levels at the retailer (from which the manufacturer can extract the demand information). They find that information sharing can significantly reduce the costs at the supplier, particularly when the capacity limit is not restrictive.

Lee, So and Tang [10] consider a two stage supply chain consisting of a single retailer and a single manufacturer. The demand process seen by the retailer is serially correlated and both the retailer and manufacturer know the exact form of the demand process (e.g., they both know the mean demand, the variance of the error terms and the correlation parameter). The retailer and manufacturer follow order-up-to inventory policies based on the most recently observed demand information. The authors find that sharing customer demand information can significantly reduce the costs at the manufacturer, particularly when the serial correlation is high.

Bourland, Powell and Pyke [1] consider how timely demand information can alter the behavior of two independently owned firms in a simple distribution chain. The demand process is i.i.d. They analyze the inventory decisions of a supplier that receives daily information on customer sales, which follow a stationary normal distribution. They quantify the value of timely demand information for the supplier when the ordering periods at the two locations are not synchronized. They find that the value of information increases with the service level at the supplier, supplier holding costs, demand variability, and offset time and that, in addition, the value of demand information increases as the length of the order cycle decreases. The authors also analyze the effects of information sharing on the customer, and show that, in general, with information sharing the supplier can provide better service to the customer in terms of a higher fill rate with an unchanged cycle service level.

Hariharan and Zipkin [7] consider a supplier who receives advance order information from his customers. Orders arrive at random with a due date, or a "demand lead time". They demonstrate that the information contained in the "demand lead times" improves system performance in the same way as a reduction in supply lead times.

DeCroix and Mookerjee [3] consider a periodic-review, finite horizon, stationary stochastic demand inventory system in which the manager has the opportunity each period to purchase information about demand in the upcoming period before deciding how much to order. They analyze the purchase and product replenishment decision for both perfect and imperfect demand information. They show that future demand information becomes less valuable at higher inventory levels and more valuable when there are more periods remaining to consider.

Recently, Kim and Ryan [9] consider an inventory model consisting of a single retailer and a single manufacturer where the retailer does not know the exact distribution of demand and thus must use some observed demand information to forecast demand and the manufacturer, as well, does not know the exact distribution of demand and thus must forecast demand. They present an extension of the basic newsvendor model to quantify the value of the observed demand information and the impact of suboptimal forecasting on the expected costs at the retailer Then they consider two cases for the manufacturer: (i) the retailer does not share customer demand information with the manufacturer and (ii) the retailer does share customer demand information instantaneously with the manufacturer. They show that having instantaneous access to customer demand information can reduce the costs at the manufacturer and that, when the manufacturer has a large number of previous orders placed by the retailer to use to forecast demand, the benefits of shared demand information are limited.

More recently, Kim [8] considers a supply chain in which there are two participants, a single retailer and a single manufacturer, and demand information by the retailer is shared instantaneously with the manufacturer. The demand process is a correlated one and either the retailer or manufacturer may not know the exact form of the demand process. He develops a mathematical model that allows us to quantify the cost incurred by each participant in the supply chain, when they implement inventory policies based on correct or incorrect assumptions about the demand process. From this model, the author shows that shared demand information is beneficial to the manufacturer only when the manufacturer understands the true nature of demand process and uses the demand information accordingly.

This paper addresses issues similar to those addressed in all of this previous research. However, this paper differs from all of this previous research with the exception of Bourland, Powell and Pyke [1] in that the information sharing is not assumed to be always instantaneous when information is shared. Unlike Bourland, Powell and Pyke [1], in this paper, serially correlated demand, which is a characteristic of most of today's consumer product industry (Lee, So and Tang [10] and Erkip, Nesim, Hausman, Warren and Nahmias [4]), is considered.

3. A SUPPLY CHAIN MODEL

3.1 Preliminaries

The supply chain model in this paper is somewhat similar to that addressed in Kim [8]. However, for the completeness of this paper, it is described fully in this subsection. Here, a supply chain consisting of a single retailer and a single manufacturer is considered where external demand is to occur at the retailer for a single item and a periodic review system is employed by each participant in the supply chain.

Tables summarizing the key notation used in this paper are provided at the end of this subsection. In this notation lower case letters are reserved for the retailer whereas upper case letters are reserved for the manufacturer. Let c and l denote the review period and lead time for the retailer, where l is a nonnegative integer multiple of c (i.e., $l \in \{0,1c,2c,\ldots\}$). Similarly, let C denote the review period for the manufacturer, where C is a positive integer multiple of the retailer's review period (i.e., $C = mc$ where $m \in \{1,2,3,\ldots\}$). Let L denote the lead time for the manufacturer, where L is a positive integer multiple of C (i.e. $L = MC = Mmc$ where $M \in \{1,2,3,\ldots\}$). Let $l' = l + c$ ($L' = L + C$) be the effective lead time for the retailer (manufacturer). Finally, let H and P denote the unit holding cost per item per period and shortage cost per item for the manufacturer. It is assumed that no fixed cost is incurred when an order is placed.

The underlying demand process faced by the retailer is an $AR(1)$ process. The approach presented here can be extended to an $AR(n)$, $n \in \{1,2,3,\ldots\}$, process. However, the analysis involved becomes much more complex. Since the purpose of this paper is to gain managerial insights, only the AR(1) process is considered. Let d_t be the demand faced by the retailer during period t. Then, d_t can be written as:

$$d_t = \mu + \rho d_{t-1} + \varepsilon_t , \tag{1}$$

where μ is a non-negative constant, ρ is a correlation parameter with $-1 < \rho < 1$, and the error terms, ε_t, are i.i.d. normal random variables with mean 0 and variance σ^2. It is assumed that μ is large enough relative to σ so that the probability of negative demand is negligible. Finally, it can be easily seen that if $\rho = 0$, Equation (1) implies that demands are i.i.d. with mean μ and variance

σ^2. It can easily be shown that, as $t \to \infty$, $E[d_t] = \frac{\mu}{1-\rho}$.

Next, the sequence of events within each review cycle is to be described. First the events occurring at the retailer is as follows. At the start of every review cycle at period t, where $t \in \{1, 1+c, 1+2c, \ldots\}$, the retailer observes the inventory level and the previous demands and calculates the order-up-to level, y_t, from which the retailer determines the order quantity, q_t, to place to the manufacturer. The retailer will receive the shipment of this order, placed at the start of period t, at the beginning of period $t+l$. Excess demand is backlogged.

Next, at the start of period t, where $t \in \{1, 1+c, 1+2c, \ldots\}$, the manufacturer receives and ships the required order quantity, q_t, to the retailer. In order to simplify the analysis, it is assumed that the manufacturer can always ship the entire order to the retailer. This assumption requires that, if the manufacturer does not have enough stock on hand to fill the order quantity, the manufacturer can always find an alternative source to borrow from, with some additional cost, P, per unit, and that the borrowed items are returned to the source when the next replenishment arrives, as if they were backlogged items. This assumption is required to obtain closed form expressions for the expected costs at the manufacturer in the supply chain.

Next, suppose the manufacturer places an order at the start of period t, where $t \in \{1, 1+C, 1+2C, \ldots\}$, right before the retailer orders, based on the inventory level and the previous demands from the retailer or, in case of information sharing, the demand information shared by the retailer. This order will arrive at the start of period, $t+L$. Note that the manufacturer could do better by ordering at the start of t right after the retailer orders, which would give the manufacturer more information on the retailer's order quantity, from which, in turn, the manufacturer could better determine his order quantity. However, because, in practice, there are many retailers served by one manufacturer and such a synchronization of ordering process is rarely expected to occur, the proposed model is used throughout the paper. Finally, the supplier from whom the manufacturer orders is assumed to have infinite capacity, so that the manufacturer's order is always satisfied after the fixed lead time, L.

In this model, it is assumed that the retailer and the manufacturer determine the order quantities in such a way that minimizes their own total expected holding and penalty costs over an infinite planning horizon, namely, long run average total inventory costs per period. Thus, the retailer and the manufacturer will use order-up-to inventory policies based on inventory position (on hand plus on order).

Showing the notations to be used in this paper ends this subsection. First, the notations for the retailer are summarized in Table 1.

Table 1. Notation for Retailer

Notation	Description
c	review period
l	lead time
l'	effective lead time, $l' = l + c$
z	safety factor
$d_{t+k} \mid d_{t-1}$	demand in period $t+k$, $k \in \{0,1,2,...\}$, given the most recently observed demand, d_{t-1}
$d_t^{l'} \mid d_{t-1}$	actual and thus perceived total demand over the effective lead time, l', starting in period t, given the most recently observed demand, d_{t-1}
y_t	order-up-to level for period t for retailer
q_t	order quantity placed by retailer at the start of period t

Table 2. Notation for Manufacturer

Notation	Description
C	review period
L	lead time
L'	effective lead time, $L' = L + C$
H	holding cost per unit per unit time
P	penalty cost per unit associated with backlogged demand
Z	safety factor
$D_t^{L'}$	actual total demand over the effective lead time, L', starting in period t, faced by the manufacturer
$D_{IS_\delta,t}^{L'}$ $(D_{NI,t}^{L'})$	perceived total demand over the effective lead time, L', starting in period t, by manufacturer when information is shared with δ periods of delay (information is not shared)
$Y_{IS_\delta,t}$ $(Y_{NI,t})$	order-up-to level for period t for manufacturer when information is shared with δ periods of delay (information is not shared)
$Z_{IS_\delta,t}$ $(Z_{NI,t})$	standardized value of order-up-to level for period t for manufacturer when information is shared with δ periods of delay (information is not shared)
$\bar{Z}_{IS_\delta}$ $(\bar{Z}_{NI})$	expected value of $Z_{IS_\delta,t}$ $(Z_{NI,t})$
$\sigma^2_{Z_{IS_\delta}}$ $(\sigma^2_{Z_{NI}})$	variance of $Z_{IS_\delta,t}$ $(Z_{NI,t})$
INV_{IS_δ} (INV_{NI})	average inventory level per period for manufacturer when information is shared with δ periods of delay (information is not shared)
G_{IS_δ} (G_{NI})	long run average total inventory costs per period for manufacturer when information is shared with δ periods of delay (information is not shared)

The parameters for the manufacturer are defined in the same way as for the retailer, but with the lower case letters replaced with capital letters. The notation for the manufacturer is shown in Table 2. In this notation, IS (denoting Information Sharing) will be replaced by NI (denoting No Information Sharing) if information is not shared, e.g., $D^{L'}_{IS,t}$ will become $D^{L'}_{NI,t}$. In addition, in order to accommodate the case of information sharing with delay, a subscript to the right of IS is added, i.e., IS_δ is used, where δ, $\delta \in \{0,1,2,...\}$, is the number of periods of delay in the transmission of the demand information.

3.2 Order Quantity at the Retailer

Given the model outlined in Subsection 3.1, an approach for determining the order quantity at the retailer can now be described. It is assumed that the retailer knows the exact form of the demand process along with the relevant parameter values and takes advantage of this knowledge to determine the order-up-to level, y_t.

First, the actual and thus perceived total demand over the effective lead time for the retailer is described. Note the following relationship between d_{t+k}, $k \in \{0,1,2,...\}$, and d_{t-1} holds from Equation (1):

$$d_{t+k} \mid d_{t-1} = \left(\frac{1-\rho^{k+1}}{1-\rho} \right) \mu + \rho^{k+1} d_{t-1} + \sum_{u=0}^{k} \rho^{k-u} \varepsilon_{t+u} , \tag{2}$$

Equation (2) can be used to show that

$$d_t^{l'} \mid d_{t-1} = \sum_{u=0}^{l'-1} d_{t+u} \mid d_{t-1} = \theta(d_{t-1}) + \psi(\varepsilon_t, \varepsilon_{t+1}, ..., \varepsilon_{t+l'-1}) , \tag{3}$$

where $\theta(d_{t-1})$ is a linear function of d_{t-1}, the most recent demand observation, and $\psi(\varepsilon_t, \varepsilon_{t+1}, ..., \varepsilon_{t+l'-1})$ is a linear function of future unobserved error terms, $\varepsilon_t, \varepsilon_{t+1}, ..., \varepsilon_{t+l'-1}$. From Equation (3), it can be seen that the total demand over the effective lead time, $d_t^{l'} \mid d_{t-1}$, follows a normal distribution with mean $\theta(d_{t-1})$ and variance $V[\psi(\varepsilon_t, \varepsilon_{t+1}, ..., \varepsilon_{t+l'-1})]$. The detailed expressions for $\theta(d_{t-1})$ and $\psi(\varepsilon_t, \varepsilon_{t+1}, ..., \varepsilon_{t+l'-1})$ are provided in Appendix A.

Therefore, the order-up-to level for period t, denoted by y_t, is calculated as

$$y_t = \theta(d_{t-1}) + z\sqrt{V[\psi(\varepsilon_t, \varepsilon_{t+1}, ..., \varepsilon_{t+l'-1})]} , \tag{4}$$

where z, the safety factor, is a constant chosen to meet a desired service level.

Next, the retailer's order quantity for period t, which becomes the manufacturer's demand for that period, can be written as:

$$q_t = y_t - y_{t-c} + \sum_{u=0}^{c-1} d_{t-c+u} = \theta(d_{t-1}) - \theta(d_{t-c-1}) + \sum_{u=0}^{c-1} d_{t-c+u}. \quad (5)$$

In Equation (5), there lies a final difficulty with this model. The possibility exists that the retailer may not be able to raise the inventory level to the desired point in each review period. In other words, it is possible that $y_{t-c} - \sum_{u=0}^{c-1} d_{t-c+u} > y_t$. To handle this case, like other researchers, it is assumed that negative order quantities are allowed. If the mean of the total demand over the effective lead time is significantly larger than the standard deviation of the total demand over the effective lead time, then the probability that $q_t \le 0$ is very close to 0 for the model presented here, and thus this assumption will have little impact on our results.

It can be easily shown, from Equation (2) and Equation (5), that

$$q_t = \sum_{u=0}^{c-1}\left(\frac{1-\rho^c}{1-\rho}\right)\mu + \rho^c q_{t-c} - \sum_{u=0}^{c-1}\left(\frac{\rho^c - \rho^{l'+c-u}}{1-\rho}\right)\varepsilon_{t-2c+u} + \sum_{u=0}^{c-1}\left(\frac{1-\rho^{l'+c-u}}{1-\rho}\right)\varepsilon_{t-c+u}. \quad (6)$$

The detailed derivation of Equation (6) is provided in Appendix B.

Finally, as an example of this model, consider the case in which $c=1$, $l=0$ and $l'=1$. Then $y_t = \mu + \rho d_{t-1} + z\sigma$ and $q_t = y_t - y_{t-1} + d_{t-1} = (1+\rho)d_{t-1} - \rho d_{t-2}$.

3.3 Cost Analysis at the Manufacturer

Given the material in Subsection 3.2, an approach for determining the long run average total inventory costs per period at the manufacturer can now be described. First, note that the effective lead time demand distribution perceived by the manufacturer will be different depending on whether or not demand information is shared and, in case of information sharing, how much delay is involved. It is assumed that the manufacturer knows the form of the q_t, $t \in \{1, 1+c, 1+2c, \ldots\}$, and the exact form of the demand process along with the relevant parameter values. It is also assumed that, when information is shared, no matter how long the information is delayed, the manufacturer forecasts the effective lead time demand distribution only based on that shared demand information and not based on the order quantities placed by the retailer.

Next, note that the actual effective lead time demand faced by the manufacturer at the start of period t, $t \in \{1, 1+C, 1+2C, \ldots\}$, becomes:

$$D_t^{L'} = \sum_{u=0}^{m+mM-1} q_{t+cu} \mid d_{t-1}, d_{t-2}, \ldots, d_{t-c-1}, \tag{7}$$

where q_t is as given in Equation (5).

Next, it is easy to show that $D_t^{L'}$ follows a normal distribution, and can be represented as a linear function of the observed previous $c+1$ demands, d_{t-1}, $d_{t-2}, \ldots, d_{t-c-1}$, and a linear function of the future unobserved error terms, ε_t, $\varepsilon_{t-1}, \ldots, \varepsilon_{t+L'-c-1}$, as follows:

$$D_t^{L'} = \Theta(d_{t-1}, d_{t-2}, .., d_{t-c-1}) + \Psi(\varepsilon_t, \varepsilon_{t-1}, \ldots, \varepsilon_{t+L'-c-1}). \tag{8}$$

Therefore, the actual effective lead time demand has mean

$$E[D_t^{L'}] = \Theta(d_{t-1}, d_{t-2}, .., d_{t-c-1}),$$

and variance

$$V[D_t^{L'}] = V\left[\Psi(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1})\right],$$

where the detailed expressions for $\Theta(d_{t-1}, d_{t-2}, .., d_{t-c-1})$ and $\Psi(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1})$ are provided in Appendix C.

Next, the order-up-to level at the start of period t, $Y_{K,t}$, $K = IS_\delta, NI$, $\delta \in \{0, 1, 2, \ldots\}$, will be calculated as

$$Y_{K,t} = E[D_{K,t}^{L'}] + Z\sqrt{V[D_{K,t}^{L'}]} = \Theta_K(\circ) + Z\sqrt{V[\Psi_K(\bullet)]}, \tag{9}$$

where $D_{K,t}^{L'}$ is the random variable representing the perceived total demand over the effective lead time starting at time period t by the manufacturer, Z is a constant chosen to meet a desired service level and $\circ$ and $\bullet$ are the appropriate parameters under consideration, which will be shown afterwards. Note that the costs incurred by the manufacturer will be a function of this order-up-to level, shown in Equation (9), and the actual total demand over the effective lead time demand, shown in Equation (8).

Next, the long run average total inventory cost per period incurred by the manufacturer is considered. To calculate the expected holding cost per period, an expression for the average inventory level, given by Silver and Peterson [11], is used. Let the expected demand during the manufacturer's review period of length C, starting at period $t+L$, given d_{t+L-1}, be $E[D_{t+L}^{C}]$ and let the expected demand over the effective lead time, starting at period t, given d_{t-1}, $d_{t-2}, \ldots,$ d_{t-c-1}, be $E[D_t^{L'}]$. Then, for a periodic review inventory system with order-up-to level $Y_{K,t}$, the average inventory level over the C periods between $t+L$ and $t+L'$, denoted by $INV_{K,t+L}^{C}$, is approximated as follows:

$$INV_{K,t+L}^{C} = Y_{K,t} - E[D_t^{L'}] + \frac{E[D_{t+L}^{C}]}{2}.$$

Therefore, the expected average inventory level per period can be represented as follows:

$$INV_K = E\left[Y_{K,t} - E[D_t^{L'}]\right] + \frac{E\left[E[D_{t+L}^{C}]\right]}{2}, \tag{10}$$

where the outer expectations are taken over d_{t-1}, $d_{t-2}, \ldots,$ d_{t-c-1}, and d_{t+L-1}, respectively. Since, as $t \to \infty$, $E[d_t] = \frac{\mu}{1-\rho}$, it can be easily shown that, as $t \to \infty$, $\frac{E\left[E[D_{t+L}^{C}]\right]}{2} = \frac{C}{2}\left(\frac{\mu}{1-\rho}\right)$.

To calculate the expected shortage cost per period, first note that the expected number of stockouts between periods $t+L$ and $t+L'$ for a fixed value of $Y_{K,t}$ can be written as

$$\int_{Y_{K,t}}^{\infty} (D_t^{L'} - Y_{K,t})\, dF(D_t^{L'}), \tag{11}$$

where $F(D_t^{L'})$ is the cumulative distribution function (cdf) of demand for the L' periods starting at period t, given d_{t-1}, $d_{t-2}, \ldots,$ d_{t-c-1}, shown in Equation (8). Since $D_t^{L'}$ follows a normal distribution, Equation (11) can be simplified to:

$$\sqrt{V[D_t^{L'}]}\int_{Z_{K,t}}^{\infty} (X - Z_{K,t})\phi(X)\, dX, \tag{12}$$

where X is a standard normal random variable and $Z_{K,t} = \frac{Y_{K,t} - E[D_t^{L'}]}{\sqrt{V[D_t^{L'}]}}$ is the standardized value of the order-up-to level. Here $\phi(\bullet)$ is the probability distribution function (pdf) for the standard normal distribution. The mean and variance of the standardized value of the order-up-to level, $\bar{Z}_K$ and $\sigma^2_{Z_K}$, respectively, are defined as follows:

$$\bar{Z}_K = E[Z_{K,t}], \ \sigma^2_{Z_K} = V[Z_{K,t}]. \tag{13}$$

According to Zipkin [12], using a standard transformation, the above expression for the expected number of stockouts per review period can be further simplified to:

$$\sqrt{V[D_t^{L'}]}\left[\phi(Z_{K,t}) - Z_{K,t}(1 - \Phi(Z_{K,t}))\right],$$

where $\Phi(\bullet)$ is the cdf of standard normal distribution. When using this formula to calculate the expected stockouts, a difficulty occurs since $Z_{K,t}$ is a random variable due to the dependence of $Y_{K,t}$ on whether or not demand information is shared and, in case of information sharing, how much delay is involved. For this model, since $Z_{K,t}$ follows a normal distribution, we have:

$$E\left[\phi(Z_{K,t}) - Z_{K,t}(1 - \Phi(Z_{K,t}))\right] = h(\bar{Z}_K)(1 + \sigma^2_{Z_K}) - \bar{Z}_K(1 - H(\bar{Z}_K)), \tag{14}$$

where the expectation is taken over $Z_{K,t}$ and $h(\bullet)$ and $H(\bullet)$ are the pdf and cdf for a normal distribution with mean 0 and variance $1 + \sigma^2_{Z_K}$. For derivation, see Kim and Ryan [9].

Since $\sqrt{V[D_t^{L'}]}$ is a constant, regardless of the specific period t, $t \in \{1, 1+C, 1+2C, \ldots\}$, the long run average total inventory costs per period, denoted by G_K, given order-up-to level $Y_{K,t}$, can be written as follows:

$$G_K = H\ INV_K + \frac{P}{C}\sqrt{V[D_t^{L'}]}\left[h(\bar{Z}_K)(1 + \sigma^2_{Z_K}) - \bar{Z}_K(1 - H(\bar{Z}_K))\right]. \tag{15}$$

Note that, in Equation (15), the expected number of shortages during any review period is divided by the length of review period, C, to obtain the expected number of shortages per period. Note also that, in Equation (15), the terms that differ for $K = IS_\delta, NI$, $\delta \in \{0,1,2,...\}$, are the average inventory level (INV_K), the mean of the standardized order-up-to level ($\bar{Z}_K$) and the variance of the standardized order-up-to level ($\sigma^2_{Z_K}$). Therefore, in order to evaluate the long run average total inventory costs per period for manufacturer for each case, it is only necessary to evaluate these three quantities, i.e., Equation (10) and Equation (13)

Now it is ready to consider each of the cases described above. For each case, the expressions for $D^{L'}_{K,t}$, where $K = IS_\delta, NI$, $\delta \in \{0,1,2,...\}$, from which, in turn $Y_{K,t}$ (i.e., $\bar{Z}_K$ and $\sigma^2_{Z_K}$) and INV_K can be evaluated, will be found. Given these expressions, the long run average total inventory costs for each case can be calculated, as shown in Equation (15).

No Information Sharing When information on the actual customer demand is not shared, the manufacturer will elaborate on the retailer's previous order quantities to determine the relationship between the retailer's order quantities. Based on the relationship between q_{t+cu}, $u \in \{0,1,...,m+mM-1\}$, and q_{t-c}, the perceived total demand over the effective lead time by the manufacturer when there is no information sharing can be expressed as a function of the most recent order quantity, q_{t-c}, and already observed error terms, but unknown to the manufacturer, $\varepsilon_{t-2c}, \varepsilon_{t-2c+1}, ..., \varepsilon_{t-1}$, and unobserved error terms, $\varepsilon_t, \varepsilon_{t+1}, ..., \varepsilon_{t+L'-c-1}$, as follows:

$$D^{L'}_{NI,t} = \sum_{u=0}^{m+mM-1} q_{t+cu} \mid q_{t-c} = \Theta_{NI}(q_{t-c}) + \Psi_{NI}(\varepsilon_{t-2c}, \varepsilon_{t-2c+1}, ..., \varepsilon_{t+L'-c-1}), \quad (16)$$

where $\Theta_{NI}(q_{t-c})$ is a linear function of q_{t-c} and $\Psi_{NI}(\varepsilon_{t-2c}, \varepsilon_{t-2c+1}, ..., \varepsilon_{t+L'-c-1})$ is a linear function of error terms, $\varepsilon_{t-2c}, \varepsilon_{t-2c+1}, ..., \varepsilon_{t+L'-c-1}$.

Therefore, the perceived total demand over the effective lead time by the manufacturer follows a normal distribution with mean

$$E[D^{L'}_{NI,t}] = \Theta_{NI}(q_{t-c}),$$

and variance

$$V[D^{L'}_{NI,t}] = V\left[\Psi_{NI}(\varepsilon_{t-2c}, \varepsilon_{t-2c+1}, ..., \varepsilon_{t+L'-c-1})\right],$$

where the detailed expressions for $\Theta_{NI}(q_{t-c})$ and $\Psi_{NI}(\varepsilon_{t-2c}, \varepsilon_{t-2c+1}, ..., \varepsilon_{t+L'-c-1})$

are provided in Appendix D.

In this case, we have

$$INV_{NI} = Z\sqrt{V[D_{NI,t}^{L'}]} + \frac{C}{2}\left(\frac{\mu}{1-\rho}\right), \tag{17}$$

$$\bar{Z}_{NI} = E\left[\frac{Y_{NI,t} - E[D_t^{L'}]}{\sqrt{V[D_t^{L'}]}}\right] = \frac{Z\sqrt{V[D_{NI,t}^{L'}]}}{\sqrt{V[D_t^{L'}]}}, \tag{18}$$

and

$$\sigma_{Z_{NI}}^2 = \frac{V\left[E[D_{NI,t}^{L'}] - E[D_t^{L'}]\right]}{V[D_t^{L'}]}. \tag{19}$$

Finally, as an example of this model, consider the case in which $c = 1$, $l = 0$, $l' = 1$, $C = 1$, $L = 1$ and $L' = 2$, then, from the facts that $q_t = (1+\rho)d_{t-1} - \rho d_{t-2}$, shown in Equation (5) in Subsection 3.2 and that $d_{t-1+k} \mid d_{t-2+k} = \mu + \rho d_{t-2+k} + \varepsilon_{t-1+k}$, $k \in \{0,1\}$, shown in Equation (2) in Subsection 3.2, the following relationships hold between q_t and q_{t-1} and between q_{t+1} and q_{t-1}:

$$q_t \mid q_{t-1} = \mu + \rho q_{t-1} - \rho\varepsilon_{t-2} + (1+\rho)\varepsilon_{t-1},$$

and

$$q_{t+1} \mid q_{t-1} = (1+\rho)\mu + \rho^2 q_{t-1} - \rho^2\varepsilon_{t-2} + \rho^2\varepsilon_{t-1} + (1+\rho)\varepsilon_t .$$

Hence, $D_{NI,t}^{L'} = \sum_{u=0}^{1} q_{t+u} \mid q_{t-1} = q_t \mid q_{t-1} + q_{t+1} \mid q_{t-1} = (2+\rho)\mu + (\rho+\rho^2)q_{t-1} - (\rho+\rho^2)$ $\varepsilon_{t-2} + (1+\rho+\rho^2)\varepsilon_{t-1} + (1+\rho)\varepsilon_t$. Therefore, the following is obtained:

$$Y_{NI,t} = (2+\rho)\mu + (\rho+\rho^2)q_{t-1} + Z\sigma\sqrt{(\rho+\rho^2)^2 + (1+\rho+\rho^2)^2 + (1+\rho)^2} .$$

Note, in this case, we have $\Theta_{NI}(q_{t-1}) = (2+\rho)\mu + (\rho+\rho^2)q_{t-1}$ and $\Psi_{NI}(\varepsilon_{t-2}, \varepsilon_{t-1}, \varepsilon_t) = -(\rho+\rho^2)\varepsilon_{t-2} + (1+\rho+\rho^2)\varepsilon_{t-1} + (1+\rho)\varepsilon_t$.

Information Sharing With Delay When the information on the customer demand is shared with some number of periods delay δ , $\delta \in \{0,1,2,\ldots\}$, then the manufacturer will perceive the total demand over the effective lead time, given the demand information, $d_{t-1-\delta}$, $d_{t-1-\delta-1}$, …, as a function of shared demand information and some unknown error terms, as follows:

(i) When $0 \le \delta \le c$

Note that d_{t+k}, $k \in \{-\delta, -\delta+1, \ldots, L'-c-1\}$, are the demands that must be forecast, using Equation (2) in Subsection 3.2, with the most recent shared demand information, $d_{t-1-\delta}$, of the shared demand information, $d_{t-1-\delta}$, $d_{t-1-\delta-1}, \ldots$ Therefore, the perceived total demand over the effective lead time by the manufacturer when there is information sharing with δ periods of delay can be expressed as a function of the shared demand information, $d_{t-1-\delta}$, $d_{t-1-\delta-1}, \ldots, d_{t-1-c}$, already observed error terms, but unknown to the manufacturer, $\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t-1}$, and unobserved error terms, $\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1}$, as follows:

$$\begin{aligned} D_{IS_\delta,t}^{L'} &= \sum_{u=0}^{m+mM-1} q_{t+cu} \mid d_{t-1-\delta}, d_{t-1-\delta-1}, \ldots, d_{t-1-c} \\ &= \Theta_{IS_\delta}(d_{t-1-\delta}, d_{t-1-\delta-1}, \ldots, d_{t-1-c}) + \Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1}), \end{aligned} \tag{20}$$

where $\Theta_{IS_\delta}(d_{t-1-\delta}, d_{t-1-\delta-1}, \ldots, d_{t-1-c})$ is a linear function of $d_{t-1-\delta}$, $d_{t-1-\delta-1}, \ldots, d_{t-1-c}$, the shared demand information, and $\Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})$ is a linear function of error terms, $\varepsilon_{t-\delta}$, $\varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1}$.

Therefore, the perceived total demand over the effective lead time by the manufacturer follows a normal distribution with mean

$$E[D_{IS_\delta,t}^{L'}] = \Theta_{IS_\delta}(d_{t-1-\delta}, d_{t-1-\delta-1}, \ldots, d_{t-1-c}),$$

and variance

$$V[D_{IS_\delta,t}^{L'}] = V\left[\Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})\right],$$

where the detailed expressions for $\Theta_{IS_\delta}(d_{t-1-\delta}; d_{t-1-\delta-1}, \ldots, d_{t-1-c})$ and Ψ_{IS_δ} $(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})$ are provided in Appendix E.

(ii) When $\delta > c$

Note that d_{t+k}, $k \in \{-c-1, -c, \ldots, L'-c-1\}$, are the demands that must be forecast, again, using Equation (2) in Subsection 3.2, with the most recent shared demand information, $d_{t-1-\delta}$, of the shared demand information, $d_{t-1-\delta}, d_{t-1-\delta-1}, \ldots$ Therefore, the perceived total demand over the effective lead time by the manufacturer when there is information sharing with δ periods of delay can be expressed as a function of the most recent shared demand information, $d_{t-1-\delta}$, already observed error terms, but unknown to the manufacturer, $\varepsilon_{t-\delta}$, $\varepsilon_{t-\delta+1},\ldots,\varepsilon_{t-1}$, and unobserved error terms, $\varepsilon_t, \varepsilon_{t+1},\ldots,\varepsilon_{t+L'-c-1}$, as follows:

$$
\begin{aligned}
D_{IS_\delta,t}^{L'} &= \sum_{u=0}^{m+mM-1} q_{t+cu} \mid d_{t-1-\delta} \\
&= \Theta_{IS_\delta}(d_{t-1-\delta}) + \Psi_{IS_\delta}(\varepsilon_{t-\delta},\varepsilon_{t-\delta+1},\ldots,\varepsilon_{t+L'-c-1}),
\end{aligned} \tag{21}
$$

where $\Theta_{IS_\delta}(d_{t-1-\delta})$ is a linear function of $d_{t-1-\delta}$ and $\Psi_{IS_\delta}(\varepsilon_{t-\delta},\varepsilon_{t-\delta+1},\ldots,\varepsilon_{t+L'-c-1})$ is a linear function of error terms, $\varepsilon_{t-\delta}$, $\varepsilon_{t-\delta+1},\ldots,$ $\varepsilon_{t+L'-c-1}$.

Therefore, the perceived total demand over the effective lead time by the manufacturer follows a normal distribution with mean

$$E[D_{IS_\delta,t}^{L'}] = \Theta_{IS_\delta}(d_{t-1-\delta}),$$

and variance

$$V[D_{IS_\delta,t}^{L'}] = V\left[\Psi_{IS_\delta}(\varepsilon_{t-\delta},\varepsilon_{t-\delta+1},\ldots,\varepsilon_{t+L'-c-1})\right],$$

where the detailed expressions for $\Theta_{IS_\delta}(d_{t-1-\delta})$ and $\Psi_{IS_\delta}(\varepsilon_{t-\delta},\varepsilon_{t-\delta+1},\ldots,\varepsilon_{t+L'-c-1})$ are provided in Appendix F.

For both cases, we have

$$INV_{IS_\delta} = Z\sqrt{V[D_{IS_\delta,t}^{L'}]} + \frac{C}{2}\left(\frac{\mu}{1-\rho}\right), \tag{22}$$

$$\bar{Z}_{IS_\delta} = E\left[\frac{Y_{IS_\delta,t} - E[D_t^{L'}]}{\sqrt{V[D_t^{L'}]}}\right] = Z\frac{\sqrt{V[D_{IS_\delta,t}^{L'}]}}{\sqrt{V[D_t^{L'}]}}, \tag{23}$$

and

$$\sigma_{Z_{IS_\delta}}^2 = \frac{V\left[E[D_{IS_\delta,t}^{L'}] - E[D_t^{L'}]\right]}{V[D_t^{L'}]}. \tag{24}$$

Finally, as an example of this model, consider the case in which $c=1$, $l=0$, $l'=1$, $C=1$, $L=1$, $L'=2$ and $\delta=0$, then

$$
\begin{aligned}
D_{IS_0,t}^{L'} &= \sum_{u=0}^{1} q_{t+cu} \mid d_{t-1}, d_{t-2} \\
&= q_t \mid d_{t-1}, d_{t-2} + q_{t+1} \mid d_{t-1} \\
&= ((1+\rho)d_{t-1} - \rho d_{t-2}) \mid d_{t-1}, d_{t-2} + ((1+\rho)d_t - \rho d_{t-1}) \mid d_{t-1} \\
&= (1+\rho)\mu + (1+\rho+\rho^2)d_{t-1} - \rho d_{t-2} + (1+\rho)\varepsilon_t.
\end{aligned}
$$

Therefore, the following is obtained:

$$Y_{IS_0,t} = (1+\rho)\mu + (1+\rho+\rho^2)d_{t-1} - \rho d_{t-2} + Z\sigma(1+\rho).$$

Note, in this case, we have $\Theta_{IS_0}(d_{t-1}, d_{t-2}) = (1+\rho)\mu + (1+\rho+\rho^2)d_{t-1} - \rho d_{t-2}$, the same as $\Theta(d_{t-1}, d_{t-2})$, and $\Psi_{IS_0}(\varepsilon_t) = (1+\rho)\varepsilon_t$, the same as $\Psi(\varepsilon_t)$.

In the above example, if it were $\delta = 1$ instead of $\delta = 0$, then

$$\begin{aligned} D^{L'}_{IS_1,t} &= \sum_{u=0}^{1} q_{t+cu} \,|d_{t-2} \\ &= q_t \,|\, d_{t-2} + q_{t+1} \,|\, d_{t-2} \\ &= ((1+\rho)d_{t-1} - \rho d_{t-2}) \,|d_{t-2} + ((1+\rho)d_t - \rho d_{t-1}) \,|\, d_{t-2} \\ &= (2+2\rho+\rho^2)\mu + (1+\rho)\rho^2 d_{t-2} + (1+\rho+\rho^2)\varepsilon_{t-1} + (1+\rho)\varepsilon_t. \end{aligned}$$

Therefore, the following would be obtained:

$$Y_{IS_1,t} = (2+2\rho+\rho^2)\mu + (1+\rho)\rho^2 d_{t-2} + Z\sigma\sqrt{(1+\rho+\rho^2)^2 + (1+\rho)^2}.$$

Note, in this case, we have $\Theta_{IS_1}(d_{t-2}) = (2+2\rho+\rho^2)\mu + (1+\rho)\rho^2 d_{t-2}$ and $\Psi_{IS_1}(\varepsilon_{t-1}, \varepsilon_t) = (1+\rho+\rho^2)\varepsilon_{t-1} + (1+\rho)\varepsilon_t$.

4. BENEFITS OF INFORMATION SHARING

In this section, the model presented in Section 3 is used to analyze the value of information sharing, with some delay.

First, it may seem that the higher the expected value of the standardized value of the order-up-to level and/or the higher the variance of the standardized value of the order-up-to level, the higher the long run average total inventory costs per period for the manufacturer. To demonstrate this fact, the following proposition is provided.

Proposition 1. G_K, $K = IS_\delta, NI$, $\delta \in \{0,1,2,\ldots\}$, is an increasing function of both $\bar{Z}_K$ and $\sigma^2_{Z_K}$.

Proof. The proof of Proposition 1 is provided in Appendix G.

Next, it may seem that information sharing always leads to lower long run average total inventory costs per period for the manufacturer than no information sharing. This seemingly intuitive result is always true when information is shared with no delay. To demonstrate this fact, the following corollary is provided

Corollary 2. $G_{IS_0} < G_{NI}$

Proof. The proof of Corollary 2 is provided in Appendix H.

Next, we may suspect intuitively that, for the manufacturer, information sharing is always better than no information sharing, even when the information is shared with some delay. However, for the model presented here, there is some amount of delay after which there is no value of information sharing and after which the manufacturer may be better off with no information sharing. This is because the manufacturer always has another source of information on the customer demand – the retailer's orders. Therefore, the manufacturer may be able to obtain a better forecast for the actual total demand over the effective lead time demand, i.e., a forecast with a smaller variance, using the most recent orders placed by the retailer rather than using some delayed, i.e., too "old", demand information. To demonstrate this, the following two corollaries are provided.

Corollary 3. $G_{IS_{\delta+1}} > G_{IS_\delta}$ for all $\delta \in \{0,1,2,...\}$.

Proof. The proof of Corollary 3 is provided in Appendix I.

The above corollary states that the long run average total inventory costs per period for the manufacturer with information sharing are an increasing function of information delay, δ. From Proposition 1 and this corollary, the following corollary is obtained.

Corollary 4. There exists a level of information delay, δ', such that $G_{IS_\delta} > G_{NI}$ for all $\delta \geq \delta'$.

Proof. The proof of Corollary 4 is provided in Appendix J.

5. FINAL REMARKS

To evaluate the value of information sharing, possibly with some delay, a mathematical model has been developed in this paper. As mentioned in Section 1, in case of information sharing, very few participants of a supply chain operate under the ideal conditions, i.e., information sharing with no delay, assumed by most standard inventory models. Therefore, the question of how the actual costs at the manufacturer differ from the optimal ones, when there is some delay in the transmission of demand information, is an important one.

A brief summary of the key managerial insights gleaned in this paper is as follows:

- Shared demand information can be beneficial to the manufacturer if shared promptly; i.e., demand information reduces the manufacturer's long run average total inventory costs per period.
- The more quickly information is shared, the better; i.e., shorter delay helps reduce uncertainty associated with the perceived total demand over the effective lead time.
- If the delay is somewhat long, then information has no value. This is due to the fact that orders placed by the retailer contain some information on the most recent demands seen by the retailer. Therefore, the manufacturer is better off by extracting that recent information from the retailer's order quantities rather than using too old demand information blindly.

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APPENDIX A: Equation (3) in Subsection 3.2

By using Equation (2) in Subsection 3.2, we can write

$$\begin{aligned} d_t^{l'} \mid d_{t-1} &= \sum_{u=0}^{l'-1} d_{t+u} \mid d_{t-1} \\ &= \sum_{u=0}^{l'-1} \left[\left(\frac{1-\rho^{1+u}}{1-\rho} \right) \mu + \rho^{1+u} d_{t-1} + \sum_{v=0}^{u} \rho^{u-v} \varepsilon_{t+v} \right] \\ &= \theta(d_{t-1}) + \psi(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+l'-1}), \end{aligned}$$

where

$$\theta(d_{t-1}) = \sum_{u=0}^{l'-1} \left(\frac{1-\rho^{1+u}}{1-\rho} \right) \mu + \sum_{u=0}^{l'-1} \rho^{1+u} d_{t-1}, \tag{A.1}$$

and

$$\psi(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+l'-1}) = \sum_{u=0}^{l'-1} \sum_{v=0}^{u} \rho^{u-v} \varepsilon_{t+v} = \sum_{u=0}^{l'-1} \left(\frac{1-\rho^{l'-u}}{1-\rho} \right) \varepsilon_{t+u}. \tag{A.2}$$

APPENDIX B: Equation (6) in Subsection 3.2

By using Equation (5) in Subsection 3.2, we can write

$$q_t = \theta(d_{t-1} \mid d_{t-c-1}) - \theta(d_{t-c-1} \mid d_{t-2c-1}) + \sum_{u=0}^{c-1} d_{t-c+u} \mid d_{t-2c+u}.$$

Once again, by using the facts that $d_{t-c+k} \mid d_{t-2c+k} = \left(\frac{1-\rho^c}{1-\rho} \right) \mu + \rho^c d_{t-2c+k}$ $+ \sum_{u=0}^{c-1} \rho^{c-1-u} \varepsilon_{t-2c+k+1+u}$, $k \in \{-1, 0, \ldots, c-1\}$, shown in Equation (2) in Subsection 3.2, that $\theta(d_{t-1}) = \sum_{u=0}^{l'-1} \left(\frac{1-\rho^{1+u}}{1-\rho} \right) \mu + \sum_{u=0}^{l'-1} \rho^{1+u} d_{t-1}$, shown in Equation (A.1), and that $q_{t-c} = \theta(d_{t-c-1}) - \theta(d_{t-2c-1}) + \sum_{u=0}^{c-1} d_{t-2c+u}$, shown in Equation (5) in Subsection 3.2, in the above equation for q_t, and then rearranging and simplifying terms, we have

$$q_t = \sum_{u=0}^{c-1}\left(\frac{1-\rho^c}{1-\rho}\right)\mu + \rho^c q_{t-c} - \sum_{u=0}^{c-1}\left(\frac{\rho^c-\rho^{l'+c-u}}{1-\rho}\right)\varepsilon_{t-2c+u} + \sum_{u=0}^{c-1}\left(\frac{1-\rho^{l'+c-u}}{1-\rho}\right)\varepsilon_{t-c+u}\,. \quad \text{(B.1)}$$

APPENDIX C: Equation (8) in Subsection 3.3

By using Equation (5) in Subsection 3.2, we can write

$$\begin{aligned} D_t^{L'} &= \sum_{u=0}^{m+mM-1} q_{t+cu} \mid d_{t-1}, d_{t-2}, \ldots, d_{t-c-1} \\ &= \sum_{u=0}^{m+mM-1}\left[\theta(d_{t+cu-1}) - \theta(d_{t+cu-c-1}) + \sum_{v=0}^{c-1} d_{t+cu-c+v}\right] \mid d_{t-1}, d_{t-2}, \ldots, d_{t-c-1} \\ &= \theta(d_{t+L'-c-1} \mid d_{t-1}) - \theta\left(d_{t-c-1}\right) + \sum_{u=0}^{c-1} d_{t-c+u} + \sum_{u=0}^{L'-c-1} d_{t+u} \mid d_{t-1}. \end{aligned}$$

Once again, by using the facts that $d_{t+k} \mid d_{t-1} = \left(\frac{1-\rho^{k+1}}{1-\rho}\right)\mu + \rho^{k+1} d_{t-1} + \sum_{u=0}^{k} \rho^{k-u}\varepsilon_{t+u}$, $k \in \{0,1,\ldots,L'-c-1\}$, shown in Equation (2) in Subsection 3.2, and that $\theta(d_{t-1}) = \sum_{u=0}^{l'-1}\left(\frac{1-\rho^{1+u}}{1-\rho}\right)\mu + \sum_{u=0}^{l'-1}\rho^{1+u} d_{t-1}$, shown in Equation (A.1), we have

$$D_t^{L'} = \Theta(d_{t-1}, d_{t-2}, \ldots, d_{t-c-1}) + \Psi(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1}),$$

where $\Theta(d_{t-1}, d_{t-2}, \ldots, d_{t-c-1})$ is

$$\sum_{u=0}^{l'-1}\rho^{1+u}\left(\frac{1-\rho^{L'-c}}{1-\rho}\right)\mu + \sum_{u=0}^{L'-c-1}\left(\frac{1-\rho^{1+u}}{1-\rho}\right)\mu - \sum_{u=0}^{l'-1}\rho^{1+u} d_{t-c-1} + \sum_{u=0}^{c-1} d_{t-c+u} + \sum_{u=0}^{l'-1}\rho^{1+u}\rho^{L'-c} d_{t-1} + \sum_{u=0}^{L'-c-1}\rho^{1+u} d_{t-1},$$

and $\Psi(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1})$ is

$$\sum_{u=0}^{l'-1}\rho^{1+u}\sum_{v=0}^{L'-c-1}\rho^{L'-c-1-v}\varepsilon_{t+v} + \sum_{u=0}^{L'-c-1}\sum_{v=0}^{u}\rho^{u-v}\varepsilon_{t+v}\,.$$

By simplifying and rearranging terms, we can rewrite $\Theta(d_{t-1}, d_{t-2}, \ldots, d_{t-c-1})$ as

$$\left[\left(\frac{1-\rho^{L'-c}}{1-\rho}\right)\sum_{u=0}^{l'-1}\rho^{1+u}+\sum_{u=0}^{L'-c-1}\left(\frac{1-\rho^{1+u}}{1-\rho}\right)\right]\mu-\sum_{u=0}^{l'-1}\rho^{1+u}d_{t-c-1}+\sum_{u=0}^{c-2}d_{t-c+u}+\left(1+\rho^{L'-c}\sum_{u=0}^{l'-1}\rho^{1+u}+\sum_{u=0}^{L'-c-1}\rho^{1+u}\right)d_{t-1}, \tag{C.1}$$

and $\Psi(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1})$ as

$$\sum_{u=0}^{L'-c-1}\left(\frac{1-\rho^{L'+l'-c-u}}{1-\rho}\right)\varepsilon_{t+u}. \tag{C.2}$$

APPENDIX D: Equation (16) in Subsection 3.3

Note the following relationship between q_{t+ck} and q_{t-c}, $k \in \{0,1,\ldots,m+Mm-1\}$, holds from the relationship between q_t and q_{t-c}, shown in Equation (B.1),

$$\begin{aligned} q_{t+ck} \mid q_{t-c} &= \sum_{u=0}^{k}\rho^{cu}\sum_{v=0}^{c-1}\left(\frac{1-\rho^{c}}{1-\rho}\right)\mu+\rho^{ck}q_{t-c} \\ &+\sum_{u=0}^{k}\rho^{c(k-u)}\left[\sum_{v=0}^{c-1}\left(\frac{\rho^{c}-\rho^{l'+c-v}}{1-\rho}\right)\varepsilon_{t+cu-2c+v}+\sum_{v=0}^{c-1}\left(\frac{1-\rho^{l'+c-v}}{1-\rho}\right)\varepsilon_{t+cu-c+v}\right]. \end{aligned}$$

Hence, we can write

$$D_{NI,t}^{L'} = \sum_{u=0}^{m+mM-1} q_{t+cu} \mid q_{t-c} = \Theta_{NI}(q_{t-c}) + \Psi_{NI}(\varepsilon_{t-2c}, \varepsilon_{t-2c+1}, \ldots, \varepsilon_{t+L'-c-1}),$$

where $\Theta_{NI}(q_{t-c})$ is

$$\sum_{u=0}^{m+mM-1}\sum_{v=0}^{u}\rho^{cv}\sum_{w=0}^{c-1}\left(\frac{1-\rho^{c}}{1-\rho}\right)\mu+\sum_{u=0}^{m+mM-1}\rho^{cu}q_{t-c}, \tag{D.1}$$

and $\Psi_{NI}(\varepsilon_{t-2c}, \varepsilon_{t-2c+1}, \ldots, \varepsilon_{t+L'-c-1})$ is

$$\sum_{u=0}^{m+mM-1}\sum_{v=0}^{u}\rho^{c(u-v)}\left[\sum_{w=0}^{c-1}-\left(\frac{\rho^{c}-\rho^{l'+c-w}}{1-\rho}\right)\varepsilon_{t+cv-2c+w}+\sum_{w=0}^{c-1}\left(\frac{1-\rho^{l'+c-w}}{1-\rho}\right)\varepsilon_{t+cv-c+w}\right].$$

By simplifying and rearranging terms, we can rewrite $\Psi_{NI}(\varepsilon_{t-2c}, \varepsilon_{t-2c+1}, \ldots,$

$\varepsilon_{t+L'-c-1})$ as

$$-\sum_{u=-2c}^{-c-1}\left(\frac{1-\rho^{L'c}}{1-\rho^{c}}\right)\left(\frac{\rho^{c}-\rho^{l'-c-u}}{1-\rho}\right)\varepsilon_{t+u}+\sum_{u=-c}^{-1}\left(\frac{1-\rho^{L'+l'-c-u}}{1-\rho}\right)\varepsilon_{t+u}+\sum_{u=0}^{L'-c-1}\left(\frac{1-\rho^{L'+l'-c-u}}{1-\rho}\right)\varepsilon_{t+u}. \quad (D.2)$$

Note that, since the third term in Equation (D.2) is identical to $\Psi(\varepsilon_t,\varepsilon_{t+1},\ldots,\varepsilon_{t+L'-c-1})$, shown in Equation (C.2), $\Psi_{NI}(\varepsilon_{t-2c},\varepsilon_{t-2c+1},\ldots,\varepsilon_{t+L'-c-1})-\Psi(\varepsilon_t,\varepsilon_{t+1},\ldots,\varepsilon_{t+L'-c-1})$, which is the sum of the first two terms in Equation (D.2), and which is a linear function of error terms other than ε_t, $\varepsilon_{t+1},\ldots,$ $\varepsilon_{t+L'-c-1}$, denotes additional source of variability, due to the fact that the manufacturer forecasts the actual total demand over the effective lead time using q_{t-c} instead of d_{t-1}, d_{t-2}, …, d_{t-c-1}.

APPENDIX E: Equation (20) in Subsection 3.3

Since d_{t+k}, $k\in\{-\delta,-\delta+1,\ldots,L'-c-1\}$, are the demands that must be forecast, using Equation (2) in Subsection 3.2, with the most recent shared demand observation, $d_{t-1-\delta}$, of the shared demand observations, $d_{t-1-\delta}$, $d_{t-1-\delta-1}$, …, we can write the perceived total demand over the effective lead time by the manufacturer when there is information sharing with δ periods of delay as

$$\begin{aligned}D_{IS_\delta,t}^{L'} &= \sum_{u=0}^{m+mM-1} q_{t+cu}\mid d_{t-1-\delta},d_{t-1-\delta-1},\ldots,d_{t-c-1}\\ &= \sum_{u=0}^{m+mM-1}\left[\theta(d_{t+cu-1})-\theta(d_{t+cu-c-1})+\sum_{v=0}^{c-1}d_{t+cu-c+v}\right]\mid d_{t-1-\delta},d_{t-1-\delta-1},\ldots,d_{t-c-1}\\ &= \theta(d_{t+L'-c-1}\mid d_{t-1-\delta})-\theta(d_{t-c-1})+\sum_{u=0}^{c-1-\delta}d_{t-c+u}+\sum_{u=0}^{L'-c-1+\delta}d_{t-\delta+u}\mid d_{t-1-\delta}.\end{aligned}$$

Using the facts that $d_{t+k}\mid d_{t-1-\delta}=\left(\frac{1-\rho^{k+1+\delta}}{1-\rho}\right)\mu+\rho^{k+1+\delta}d_{t-1-\delta}+\sum_{u=0}^{k+\delta}\rho^{k+\delta-u}\varepsilon_{t-\delta+u}$, $k\in\{-\delta,-\delta+1,\ldots,L'-c-1\}$, shown in Equation (2) in Subsection 3.2 and that $\theta(d_{t-1})=\sum_{u=0}^{l'-1}\left(\frac{1-\rho^{1+u}}{1-\rho}\right)\mu+\sum_{u=0}^{l'-1}\rho^{1+u}d_{t-1}$, shown in Equation (A.1), we have

$$D_{IS_\delta,t}^{L'} = \Theta_{IS_\delta}(d_{t-1-\delta}, d_{t-1-\delta-1}, \ldots, d_{t-1-c}) + \Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1}),$$

where $\Theta_{IS_\delta}(d_{t-1-\delta}, d_{t-1-\delta-1}, \ldots, d_{t-1-c})$ is

$$\sum_{u=0}^{l'-1} \rho^{1+u} \left(\frac{1-\rho^{L'-c+\delta}}{1-\rho} \right) \mu + \sum_{u=0}^{L'-c-1+\delta} \left(\frac{1-\rho^{1+u}}{1-\rho} \right) \mu - \sum_{u=0}^{l'-1} \rho^{1+u} d_{t-c-1}$$
$$+ \sum_{u=0}^{c-1-\delta} d_{t-c+u} + \sum_{u=0}^{l'-1} \rho^{1+u} \rho^{L'-c+\delta} d_{t-1-\delta} + \sum_{u=0}^{L'-c-1+\delta} \rho^{1+u} d_{t-1-\delta},$$

and $\Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})$ is

$$\sum_{u=0}^{l'-1} \rho^{1+u} \sum_{v=0}^{L'-c-1+\delta} \rho^{L'-c-1+\delta-v} \varepsilon_{t-\delta+v} + \sum_{u=0}^{L'-c-1+\delta} \sum_{v=0}^{u} \rho^{u-v} \varepsilon_{t-\delta+v}.$$

By simplifying and rearranging terms, we can rewrite $\Theta_{IS_\delta}(d_{t-1-\delta}, d_{t-1-\delta-1}, \ldots, d_{t-1-c})$ as

$$\left[\sum_{u=0}^{l'-1} \rho^{1+u} \left(\frac{1-\rho^{L'-c+\delta}}{1-\rho} \right) + \sum_{u=0}^{L'-c-1+\delta} \left(\frac{1-\rho^{1+u}}{1-\rho} \right) \right] \mu - \sum_{u=0}^{l'-1} \rho^{1+u} d_{t-c-1}$$
$$+ \sum_{u=0}^{c-1-\delta} d_{t-c+u} + \left(\sum_{u=0}^{l'-1} \rho^{1+u} \rho^{L'-c+\delta} + \sum_{u=0}^{L'-c-1+\delta} \rho^{1+u} \right) d_{t-1-\delta}, \tag{E.1}$$

and $\Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})$ as

$$\sum_{u=-\delta}^{-1} \left(\frac{1-\rho^{L'+l'-c-u}}{1-\rho} \right) \varepsilon_{t+u} + \sum_{u=0}^{L'-c-1} \left(\frac{1-\rho^{L'+l'-c-u}}{1-\rho} \right) \varepsilon_{t+u}. \tag{E.2}$$

Note that, since the second term in Equation (E.2) is identical to $\Psi(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1})$, shown in Equation (C.2), $\Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1}) - \Psi(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1})$, which is just the first term in Equation (E.2), and which is a linear function of error terms other than ε_t, $\varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1}$, denotes additional source of variability, due to the fact that the manufacturer forecasts the actual total demand over the effective lead time using $d_{t-1-\delta}$, $d_{t-1-\delta-1}, \ldots, d_{t-c-1}$ instead of d_{t-1}, $d_{t-2}, \ldots, d_{t-c-1}$.

APPENDIX F: Equation (21) in Subsection 3.3

Since d_{t+k}, $k \in \{-c-1, -c, \ldots, L'-c-1\}$, are the demands that must be forecast, using Equation (2) in Subsection 3.2, with the most recent shared demand observation, $d_{t-1-\delta}$, of the shared demand observations, $d_{t-1-\delta}$, $d_{t-1-\delta-1}$, ..., we can write the perceived total demand over the effective lead time by the manufacturer when there is information sharing with δ periods of delay as

$$D_{IS_\delta,t}^{L'} = \sum_{u=0}^{m+mM-1} q_{t+cu} \mid d_{t-1-\delta}, d_{t-1-\delta-1}, \ldots, d_{t-c-1}$$
$$= \sum_{u=0}^{m+mM-1} \left[\theta(d_{t+cu-1}) - \theta(d_{t+cu-c-1}) + \sum_{v=0}^{c-1} d_{t+cu-c+v} \right] \mid d_{t-1-\delta}, d_{t-1-\delta-1}, \ldots, d_{t-c-1}$$
$$= \theta(d_{t+L'-c-1} \mid d_{t-1-\delta}) - \theta\left(d_{t-c-1} \mid d_{t-1-\delta}\right) + \sum_{u=0}^{c-1} d_{t-c+u} \mid d_{t-1-\delta} + \sum_{u=0}^{L'-c-1} d_{t+u} \mid d_{t-1-\delta}.$$

Using the facts that $d_{t+k} \mid d_{t-1-\delta} = \left(\dfrac{1-\rho^{k+1+\delta}}{1-\rho} \right) \mu + \rho^{k+1+\delta} d_{t-1-\delta} + \sum_{u=0}^{k+\delta} \rho^{k+\delta-u} \varepsilon_{t-\delta+u}$, $k \in \{ -\delta, -\delta+1, \ldots, L'-c-1\}$, shown in Equation (2) in Subsection 3.2 and that $\theta(d_{t-1}) = \sum_{u=0}^{l'-1} \left(\dfrac{1-\rho^{1+u}}{1-\rho} \right) \mu + \sum_{u=0}^{l'-1} \rho^{1+u} d_{t-1}$, shown in Equation (A.1), we have

$$D_{IS_\delta,t}^{L'} = \Theta_{IS_\delta}(d_{t-1-\delta}) + \Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1}),$$

where $\Theta_{IS_\delta}(d_{t-1-\delta})$ is

$$\sum_{u=0}^{l'-1} \rho^{1+u} \left[\left(\frac{1-\rho^{L'+\delta-c}}{1-\rho} \right) - \left(\frac{1-\rho^{\delta-c}}{1-\rho} \right) \right] \mu + \sum_{u=0}^{c-1} \left(\frac{1-\rho^{\delta-c+1+u}}{1-\rho} \right) \mu + \sum_{u=0}^{L'+\delta-c-1} \left(\frac{1-\rho^{1+u}}{1-\rho} \right) \mu$$
$$- \sum_{u=0}^{l'-1} \rho^{1+u} \left(\rho^{\delta-c} - \rho^{L'+\delta-c} \right) d_{t-1-\delta} + \sum_{u=0}^{c-1} \rho^{\delta-c+1+u} d_{t-1-\delta} + \sum_{u=0}^{L'+\delta-c-1} \rho^{1+u} d_{t-1-\delta},$$

and $\Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})$ is

$$\sum_{u=0}^{l'-1}\rho^{1+u}\sum_{v=0}^{L'+\delta-c-1}\rho^{L'+\delta-c-1-v}\varepsilon_{t-\delta+v}-\sum_{u=0}^{l'-1}\rho^{1+u}\sum_{v=0}^{\delta-c-1}\rho^{\delta-c-1-v}\varepsilon_{t-\delta+v}$$
$$+\sum_{u=0}^{c-1}\sum_{v=0}^{\delta-c+u}\rho^{\delta-c+u-v}\varepsilon_{t-\delta+v}+\sum_{u=0}^{L'+\delta-c-1}\sum_{v=0}^{u}\rho^{u-v}\varepsilon_{t-\delta+v}.$$

By simplifying and rearranging terms, we can rewrite $\Theta_{IS_\delta}(d_{t-1-\delta})$ as

$$\left\{\sum_{u=0}^{l'-1}\rho^{1+u}\left[\left(\frac{1-\rho^{L'+\delta-c}}{1-\rho}\right)-\left(\frac{1-\rho^{\delta-c}}{1-\rho}\right)\right]+\sum_{u=0}^{c-1}\left(\frac{1-\rho^{\delta-c+1+u}}{1-\rho}\right)+\sum_{u=0}^{L'+\delta-c-1}\left(\frac{1-\rho^{1+u}}{1-\rho}\right)\right\}\mu$$
$$+\left[\sum_{u=0}^{c-1}\rho^{\delta-c+1+u}+\sum_{u=0}^{L'+\delta-c-1}\rho^{1+u}-\sum_{u=0}^{l'-1}\rho^{1+u}\left(\rho^{\delta-c}-\rho^{L'+\delta-c}\right)\right]d_{t-1-\delta},\tag{F.1}$$

and $\Psi_{IS_\delta}(\varepsilon_{t-\delta},\varepsilon_{t-\delta+1},\ldots,\varepsilon_{t+L'-c-1})$ as

$$\sum_{u=-\delta}^{-c-1}\left[\left(\frac{\rho^{l'-c-u}-\rho^{L'+l'-c-u}}{1-\rho}\right)+\left(\frac{1-\rho^{-u}}{1-\rho}\right)\right]\varepsilon_{t+u}+\sum_{u=-c}^{-1}\left[\left(\frac{1-\rho^{L'+l'-c-u}}{1-\rho}\right)+\left(\frac{1-\rho^{-u}}{1-\rho}\right)\right]\varepsilon_{t+u}$$
$$+\sum_{u=0}^{L'-c-1}\left(\frac{1-\rho^{L'+l'-c-u}}{1-\rho}\right)\varepsilon_{t+u}.\tag{F.2}$$

Note that, since the third term in Equation (F.2) is identical to $\Psi(\varepsilon_t,\varepsilon_{t+1},\ldots,\varepsilon_{t+L'-c-1})$, shown in Equation (C.2), $\Psi_{IS_\delta}(\varepsilon_{t-\delta},\varepsilon_{t-\delta+1},\ldots,\varepsilon_{t+L'-c-1})-\Psi(\varepsilon_t,\varepsilon_{t+1},\ldots,\varepsilon_{t+L'-c-1})$, which is the sum of the first two terms in Equation (F.2), and which is a linear function of error terms other than ε_t, ε_{t+1}, …, $\varepsilon_{t+L'-c-1}$, denotes additional source of variability, due to the fact that the manufacturer forecasts the actual total demand over the effective lead time using $d_{t-1-\delta}$ instead of $d_{t-1}, d_{t-2},\ldots, d_{t-c-1}$.

APPENDIX G: Proof of Proposition 1 in Section 4

Note that, from Equation (15) in Subsection 3.3, G_K, $K=IS_\delta, NI$, $\delta\in\{0,1,2,\ldots\}$ can be represented as a function of $\bar{Z}_K$ and $\sigma^2_{Z_K}$. We start with proving that

$\frac{\bar{Z}_K}{\sqrt{1+\sigma^2_{Z_K}}} = Z$. From the expressions for $\bar{Z}_K$, shown in Equations (18) and (23) in Subsection 3.3, and from the expressions for $\sigma^2_{Z_K}$, shown in Equations (19) and (24) in Subsection 3.3, we have

$$\frac{\bar{Z}_K}{\sqrt{1+\sigma^2_{Z_K}}} = \frac{\dfrac{Z\sqrt{V[D^{L'}_{K,t}]}}{\sqrt{V[D^{L'}_t]}}}{\sqrt{1+\dfrac{V\left[E[D^{L'}_{K,t}]-E[D^{L'}_t]\right]}{V[D^{L'}_t]}}} = \frac{Z\sqrt{\dfrac{V[D^{L'}_{K,t}]}{V[D^{L'}_t]}}}{\sqrt{\dfrac{V[D^{L'}_t]+V\left[E[D^{L'}_{K,t}]-E[D^{L'}_t]\right]}{V[D^{L'}_t]}}}.$$

Therefore, in order to prove that $\frac{\bar{Z}_K}{\sqrt{1+\sigma^2_{Z_K}}} = Z$, it is sufficient to show that $V[D^{L'}_{K,t}] = V[D^{L'}_t] + V\left[E[D^{L'}_{K,t}]-E[D^{L'}_t]\right]$, which, in turn, implies that

$$V\left[\Psi_K(\bullet)\right] = V\left[\Psi(\varepsilon_t,\varepsilon_{t+1},\ldots,\varepsilon_{t+L'-c-1})\right] + V\left[\Theta_K(\circ)-\Theta(d_{t-1},d_{t-2},\ldots,d_{t-c-1})\right],$$

where $\circ$ and $\bullet$ are the appropriate parameters under consideration.
In Appendix D through Appendix F, after driving the expressions for $\Psi_K(\bullet)$, shown in Equations (D.2), (E.2) and (F.2), it has been mentioned that $\Psi_K(\bullet)$ can be represented as the sum of $\Psi(\varepsilon_t,\varepsilon_{t+1},\ldots,\varepsilon_{t+L'-c-1})$ and a linear function of error terms other than $\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1}$, which, can be written as $\Psi_K(\bullet)-\Psi(\varepsilon_t, \varepsilon_{t+1},\ldots,\varepsilon_{t+L'-c-1})$, denotes additional source of variability. Since the error terms, ε_t, are i.i.d. random variables, we have

$$V\left[\Psi_K(\bullet)-\Psi(\varepsilon_t,\varepsilon_{t+1},\ldots,\varepsilon_{t+L'-c-1})\right] = V\left[\Psi_K(\bullet)\right] - V\left[\Psi(\varepsilon_t,\varepsilon_{t+1},\ldots,\varepsilon_{t+L'-c-1})\right]. \quad \text{(G..1)}$$

Also note that the following equality holds

$$\Theta_K(\circ)+\Psi_K(\bullet) = \left[\Theta_K(\circ)+\Psi_K(\bullet)-\Psi(\varepsilon_t,\varepsilon_{t+1},\ldots,\varepsilon_{t+L'-c-1})\right] + \Psi(\varepsilon_t,\varepsilon_{t+1},\ldots,\varepsilon_{t+L'-c-1}),$$

and that the expression, $\Theta(d_{t-1},d_{t-2},\ldots,d_{t-c-1})+\Psi(\varepsilon_t,\varepsilon_{t+1},\ldots,\varepsilon_{t+L'-c-1})$, forecast the total demand over the effective lead time by manufacturer, as well as $\Theta_K(\circ)+\Psi_K(\bullet)$. Thus, the two expressions, $\left[\Theta_K(\circ)+\Psi_K(\bullet)-\Psi(\varepsilon_t,\varepsilon_{t+1},\ldots,\varepsilon_{t+L'-c-1})\right]$ and $\Theta(d_{t-1},d_{t-2},\ldots,d_{t-c-1})$, are statistically equivalent to each other, i.e., they have the same probability distribution. Therefore, we have

$$\Theta(d_{t-1},d_{t-2},..,d_{t-c-1}) = \Theta_K(\circ) + \Psi_K(\bullet) - \Psi(\varepsilon_t,\varepsilon_{t+1},...,\varepsilon_{t+L'-c-1}),$$

from which, in turn, we have

$$\begin{aligned} V\left[\Psi_K(\bullet) - \Psi(\varepsilon_t,\varepsilon_{t+1},...,\varepsilon_{t+L'-c-1})\right] &= V\left[\Theta(d_{t-1},d_{t-2},..,d_{t-c-1}) - \Theta_K(\circ)\right] \\ &= V\left[\Theta_K(\circ) - \Theta(d_{t-1},d_{t-2},..,d_{t-c-1})\right], \end{aligned} \quad \text{(G..2)}$$

where in the second step we have used the fact that $V[X] = V[-X]$, where X is a random variable.

From the relationships, shown in Equation (G.1) and Equation (G.2), we have

$$V\left[\Psi_K(\bullet)\right] - V\left[\Psi(\varepsilon_t,\varepsilon_{t+1},...,\varepsilon_{t+L'-c-1})\right] = V\left[\Theta_K(\circ) - \Theta(d_{t-1},d_{t-2},..,d_{t-c-1})\right],$$

which is the desired result. As an example of statistical equivalence between $\left[\Theta_K(\circ) + \Psi_K(\bullet) - \Psi(\varepsilon_t,\varepsilon_{t+1},...,\varepsilon_{t+L'-c-1})\right]$ and $\Theta(d_{t-1},d_{t-2},..,d_{t-c-1})$, consider the case in which $c=1$, $l=0$, $l'=1$, $C=1$, $L=1$, $L'=2$, $K=NI$. From the examples in Subsection 3.3, we have $\left[\Theta_{NI}(q_{t-1}) + \Psi_{NI}(\varepsilon_{t-2},\varepsilon_{t-1},\varepsilon_t) - \Psi(\varepsilon_t)\right] = (2+\rho)\mu + (\rho+\rho^2)q_{t-1} - (\rho+\rho^2)\varepsilon_{t-2} + (1+\rho+\rho^2)\varepsilon_{t-1}$. Therefore, from the above expression for $\left[\Theta_{NI}(q_{t-1}) + \Psi_{NI}(\varepsilon_{t-2},\varepsilon_{t-1},\varepsilon_t) - \Psi(\varepsilon_t)\right]$, we have

$$\begin{aligned} &\left[\Theta_{NI}(q_{t-1}) + \Psi_{NI}(\varepsilon_{t-2},\varepsilon_{t-1},\varepsilon_t) - \Psi(\varepsilon_t)\right] \\ &\quad = (2+\rho)\mu + (\rho+\rho^2)q_{t-1} - (\rho+\rho^2)\varepsilon_{t-2} + (1+\rho+\rho^2)\varepsilon_{t-1} \\ &\quad = (2+\rho)\mu + (\rho+\rho^2)\left[(1+\rho)d_{t-2} - \rho d_{t-3}\right] - (\rho+\rho^2)\varepsilon_{t-2} + (1+\rho+\rho^2)\varepsilon_{t-1} \\ &\quad = (2+\rho)\mu + (\rho+\rho^2)\left(\mu + \rho d_{t-2} + \varepsilon_{t-2}\right) - (\rho+\rho^2)\varepsilon_{t-2} + (1+\rho+\rho^2)\varepsilon_{t-1} \\ &\quad = (2+\rho)\mu + (\rho+\rho^2)\left(\mu + \rho d_{t-2}\right) + (1+\rho+\rho^2)\varepsilon_{t-1} \\ &\quad = (2+\rho)\mu + (1+\rho+\rho^2)\left(\mu + \rho d_{t-2}\right) - \left(\mu + \rho d_{t-2}\right) + (1+\rho+\rho^2)\varepsilon_{t-1} \\ &\quad = (2+\rho)\mu + (1+\rho+\rho^2)\left(\mu + \rho d_{t-2} + \varepsilon_{t-1}\right) - \left(\mu + \rho d_{t-2}\right) \\ &\quad = (1+\rho)\mu + (1+\rho+\rho^2)d_{t-1} - \rho d_{t-2} \\ &\quad = \Theta(d_{t-1},d_{t-2}), \end{aligned}$$

where in the second step and in the third step we have used the fact that $q_{t-1} = (1+\rho)d_{t-2} - \rho d_{t-3}$ from Equation (5) in Subsection 3.2 and the fact that $d_{t-2} - \rho d_{t-3} = \mu + \varepsilon_{t-2}$ from Equation (2) in Subsection 3.2, respectively. Note also that we have

$$\begin{aligned} V\left[\Psi_{NI}(\varepsilon_{t-2},\varepsilon_{t-1},\varepsilon_t)-\Psi(\varepsilon_t)\right] \\ &= V\left[-(\rho+\rho^2)\varepsilon_{t-2}+(1+\rho+\rho^2)\varepsilon_{t-1}\right] \\ &= (\rho+\rho^2)^2+(1+\rho+\rho^2)^2 \\ &= \left[(\rho+\rho^2)^2+(1+\rho+\rho^2)^2+(1+\rho)^2\right]-(1+\rho)^2 \\ &= V\left[-(\rho+\rho^2)\varepsilon_{t-2}+(1+\rho+\rho^2)\varepsilon_{t-1}+(1+\rho)\varepsilon_t\right]-V\left[(1+\rho)\varepsilon_t\right] \\ &= V\left[\Psi_{NI}(\varepsilon_{t-2},\varepsilon_{t-1},\varepsilon_t)\right]-V\left[\Psi(\varepsilon_t)\right]. \end{aligned}$$

In the above example, if it were $K = IS_1$ instead of $K = NI$, from the examples in Subsection 3.3, we would have

$$\left[\Theta_{IS_1}(d_{t-2})+\Psi_{IS_1}(\varepsilon_{t-1},\varepsilon_t)-\Psi(\varepsilon_t)\right]=(2+2\rho+\rho^2)\mu+(1+\rho)\rho^2 d_{t-2}+(1+\rho+\rho^2)\varepsilon_{t-1}.$$

From the above expression for $\left[\Theta_{IS_1}(d_{t-2})-\Psi_{IS_1}(\varepsilon_{t-1},\varepsilon_t)+\Psi(\varepsilon_t)\right]$, the statistical equivalence between $\left[\Theta_{IS_1}(d_{t-2})-\Psi_{IS_1}(\varepsilon_{t-1},\varepsilon_t)+\Psi(\varepsilon_t)\right]$ and $\Theta(d_{t-1},d_{t-2})$, and from the expression for $\Psi_{IS_1}(\varepsilon_{t-1},\varepsilon_t)$, shown in Subsection 3.3, $V\left[\Psi_{IS_1}(\varepsilon_{t-1},\varepsilon_t)-\right.$ $\left.-\Psi(\varepsilon_t)\right]=V\left[\Psi_{IS_1}(\varepsilon_{t-1},\varepsilon_t)\right]-V\left[\Psi(\varepsilon_t)\right]$ can be easily established as well, which is left as an exercise for interested readers.

Next, note that, according to Kim [8], $\left[h(\bar{Z}_K)(1+\sigma^2_{Z_K})-\bar{Z}_K(1-H(\bar{Z}_K))\right]$ can be written as

$$\sqrt{1+\sigma^2_{Z_K}}\int_{\frac{\bar{Z}_K}{\sqrt{1+\sigma^2_{Z_K}}}}^{\infty}\left(X-\frac{\bar{Z}_K}{\sqrt{1+\sigma^2_{Z_K}}}\right)\phi(X)dX.$$

Therefore, we have

$$\begin{aligned} G_K &= H\ INV_K+\frac{P}{C}\sqrt{V[D_t^{L'}]}\left[h(\bar{Z}_K)(1+\sigma^2_{Z_K})-\bar{Z}_K(1-H(\bar{Z}_K))\right] \\ &= H\left[Z\sqrt{V[D_{K,t}^{L'}]}+\frac{C}{2}\left(\frac{\mu}{1-\rho}\right)\right]+\frac{P}{C}\sqrt{V[D_t^{L'}]}\sqrt{1+\sigma^2_{Z_K}}\int_{\frac{\bar{Z}_K}{\sqrt{1+\sigma^2_{Z_K}}}}^{\infty}\left(X-\frac{\bar{Z}_K}{\sqrt{1+\sigma^2_{Z_K}}}\right)\phi(X)dX \\ &= H\frac{C}{2}\left(\frac{\mu}{1-\rho}\right)+\sqrt{V[D_t^{L'}]}\left[H\bar{Z}_K+\frac{P}{C}\sqrt{1+\sigma^2_{Z_K}}\int_Z^{\infty}(X-Z)\phi(X)dX\right], \end{aligned}$$

where in the second step we have used the facts that $Z\sqrt{V[D_{K,t}^{L'}]} = \bar{Z}_K\sqrt{V[D_t^{L'}]}$ and that $\dfrac{\bar{Z}_K}{\sqrt{1+\sigma_{Z_K}^2}} = Z$.

Finally, from the above expression for G_K, it is clear that G_K is an increasing function of both $\bar{Z}_K$ and $\sigma_{Z_K}^2$.

APPENDIX H: Proof of Corollary 2 in Section 4

Since G_K is an increasing function of $\bar{Z}_K$ and $\sigma_{Z_K}^2$, shown in Appendix G, in order to prove the corollary, it is sufficient to show that

(i) $\bar{Z}_{NI} - \bar{Z}_{IS_0} > 0$.

(ii) $\sigma_{Z_{NI}}^2 - \sigma_{Z_{IS_0}}^2 > 0$.

(i) When $\delta = 0$, from the expression for $\Psi_{IS_\delta}\left(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1}\right)$, shown in Equation (E.2), $\Psi_{IS_0}\left(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1}\right)$ equals to $\Psi\left(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1}\right)$, which implies $\bar{Z}_{IS_0} = Z$. Next, from the expression for $\Psi_{NI}\left(\varepsilon_{t-2c}, \varepsilon_{t-2c+1}, \ldots, \varepsilon_{t+L'-c-1}\right)$, shown in Equation (D.2), and from the expression for $\Psi\left(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1}\right)$, shown in Equation (C.2), it is clear that $V[\Psi_{NI}(\varepsilon_{t-2c}, \varepsilon_{t-2c+1}, \ldots, \varepsilon_{t+L'-c-1})] - V[\Psi(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1})] = V[\Psi_{NI}(\varepsilon_{t-2c}, \varepsilon_{t-2c+1}, \ldots, \varepsilon_{t+L'-c-1}) - \Psi(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1})] > 0$, which implies $\bar{Z}_{NI} > Z$. Therefore, we have $\bar{Z}_{NI} - \bar{Z}_{IS_0} > 0$.

(ii) When $\delta = 0$, from the expression for $\Theta_{IS_\delta}\left(d_{t-1-\delta}, d_{t-1-\delta-1}, \ldots, d_{t-1-c}\right)$, shown in Equation (E.1), $\Theta_{IS_0}\left(d_{t-1}, d_{t-2}, \ldots, d_{t-1-c}\right)$ equals to $\Theta\left(d_{t-1}, d_{t-2}, \ldots, d_{t-1-c}\right)$, which implies that $\sigma_{Z_{IS_0}}^2 = 0$. Next, in Appendix G, it has been shown that $V\left[\Theta_{NI}(d_{t-1}, d_{t-2}, \ldots, d_{t-1-c}) - \Theta(d_{t-1}, d_{t-2}, \ldots, d_{t-1-c})\right]$ equals to $V[\Psi_{NI}(\varepsilon_{t-2c}, \varepsilon_{t-2c+1}, \ldots, \varepsilon_{t+L'-c-1}) - \Psi(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1})]$ and that $V[\Psi_{NI}(\varepsilon_{t-2c}, \varepsilon_{t-2c+1}, \ldots, \varepsilon_{t+L'-c-1}) - \Psi(\varepsilon_t, \varepsilon_{t+1}, \ldots, \varepsilon_{t+L'-c-1})] > 0$, which implies that $\sigma_{Z_{NI}}^2 > 0$. Therefore, we have $\sigma_{Z_{NI}}^2 -$

$\sigma^2_{Z_{IS_0}} > 0$.

Finally, from the proofs for Part (i) and Part (ii), we have $G_{NI} - G_{IS_0} > 0$, which completes the proof.

APPENDIX I: Proof of Corollary 3 in Section 4

Since G_K is an increasing function of $\bar{Z}_K$ and $\sigma^2_{Z_K}$, shown in Appendix G, in order to prove the corollary, it is sufficient to show that

(i) $\bar{Z}_{IS_{\delta+1}} - \bar{Z}_{IS_\delta} > 0$, i.e., $\bar{Z}_{IS_\delta}$ is an increasing function of δ .

(ii) $\sigma^2_{Z_{IS_{\delta+1}}} - \sigma^2_{Z_{IS_\delta}} > 0$, i.e., $\sigma^2_{Z_{IS_\delta}}$ is an increasing function of δ .

(i) First from the expression for $\bar{Z}_{IS_\delta}$, shown in Equation (23) in Subsection 3.3, and from the expressions for $\Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})$, shown in Equations (E.2) and (F.2), we have $\bar{Z}_{IS_{\delta+1}} - \bar{Z}_{IS_\delta} > 0 \Leftrightarrow V\left[\Psi_{IS_{\delta+1}}(\varepsilon_{t-\delta-1}, \varepsilon_{t-\delta}, \ldots, \varepsilon_{t+L'-c-1})\right] - V\left[\Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})\right] > 0$. Next, from the expressions for $\Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})$, shown in Equations (E.2) and (F.2), it is clear that $V\left[\Psi_{IS_{\delta+1}}(\varepsilon_{t-\delta-1}, \varepsilon_{t-\delta}, \ldots, \varepsilon_{t+L'-c-1})\right] - V\left[\Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})\right]$ is equal to $V\left[\Psi_{IS_{\delta+1}}(\varepsilon_{t-\delta-1}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1}) - \Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})\right]$.

Note that $V\left[\Psi_{IS_{\delta+1}}(\varepsilon_{t-\delta-1}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1}) - \Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})\right]$ becomes:

(1) When $0 \le \delta \le c$, from the expression for $\Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})$, shown in Equation (E.2), we have

$$V\left[\Psi_{IS_{\delta+1}}(\varepsilon_{t-\delta-1}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1}) - \Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})\right] = V\left[\left(\frac{1-\rho^{L'+l'-c+\delta+1}}{1-\rho}\right)\varepsilon_{t-\delta-1}\right] > 0 ,$$

(2) When $\delta > c$, from the expression for $\Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \ldots, \varepsilon_{t+L'-c-1})$, shown in Equation (F.2), we have

$$V\Big[\Psi_{IS_{\delta+1}}(\varepsilon_{t-\delta-1},\varepsilon_{t-\delta+1},\dots,\varepsilon_{t+L'-c-1})-\Psi_{IS_{\delta}}(\varepsilon_{t-\delta},\varepsilon_{t-\delta+1},\dots,\varepsilon_{t+L'-c-1})\Big]$$
$$=V\left[\left[\left(\frac{\rho^{l'+\delta-c+1}-\rho^{L'+l'+\delta-c+1}}{1-\rho}\right)+\left(\frac{1-\rho^{\delta+1}}{1-\rho}\right)\right]\varepsilon_{t-\delta-1}\right]>0.$$

Therefore, we have $\bar{Z}_{IS_{\delta+1}}-\bar{Z}_{IS_{\delta}}>0$.

(ii) For convenience, only the case of $0\le\delta\le c$ is considered. However, an approach presented here can be applied to the case of $\delta>c$ as well. First from the expression for $\sigma^2_{Z_{IS_\delta}}$, shown in Equation (24) in Subsection 3.3, and from the expression for $\Theta_{IS_\delta}(d_{t-1-\delta},d_{t-1-\delta-1},\dots,d_{t-1-c})$, shown in Equations (E.1), we have $V\Big[\Theta_{IS_{\delta+1}}(d_{t-1-\delta-1},d_{t-1-\delta-2},\dots,d_{t-1-c})-\Theta(d_{t-1},d_{t-2},\dots,d_{t-1-c})\Big]$ $-V\big[\Theta\mid IS_\delta(d_{t-1-\delta},d_{t-1-\delta-1},\dots,d_{t-1-c})-\Theta(d_{t-1},d_{t-2},\dots,d_{t-1-c})\big]>0$, which implies $\sigma^2_{Z_{IS_{\delta+1}}}-\sigma^2_{Z_{IS_\delta}}>0$.
In Appendix G, we have proved

$$V\Big[\Theta_{IS_{\delta+1}}(d_{t-1-\delta-1},d_{t-1-\delta-2},\dots,d_{t-1-c})-\Theta(d_{t-1},d_{t-2},\dots,d_{t-1-c})\Big]$$
$$=V\Big[\Psi_{IS_{\delta+1}}(\varepsilon_{t-\delta-1},\varepsilon_{t-\delta},\dots,\varepsilon_{t+L'-c-1})-\Psi(\varepsilon_{t},\varepsilon_{t+1},\dots,\varepsilon_{t+L'-c-1})\Big],$$

and

$$V\Big[\Theta_{IS_{\delta}}(d_{t-1-\delta},d_{t-1-\delta-1},\dots,d_{t-1-c})-\Theta(d_{t-1},d_{t-2},\dots,d_{t-1-c})\Big]$$
$$=V\Big[\Psi_{IS_{\delta}}(\varepsilon_{t-\delta},\varepsilon_{t-\delta+1},\dots,\varepsilon_{t+L'-c-1})-\Psi(\varepsilon_{t},\varepsilon_{t+1},\dots,\varepsilon_{t+L'-c-1})\Big].$$

Therefore, in order to prove Part (ii), it is sufficient to show that

$$V\Big[\Psi_{IS_{\delta+1}}(\varepsilon_{t-\delta-1},\varepsilon_{t-\delta},\dots,\varepsilon_{t+L'-c-1})-\Psi(\varepsilon_{t},\varepsilon_{t+1},\dots,\varepsilon_{t+L'-c-1})\Big]$$
$$-V\Big[\Psi_{IS_{\delta}}(\varepsilon_{t-\delta},\varepsilon_{t-\delta+1},\dots,\varepsilon_{t+L'-c-1})-\Psi(\varepsilon_{t},\varepsilon_{t+1},\dots,\varepsilon_{t+L'-c-1})\Big]>0.$$

Note that, from the expressions for $\Psi_{IS_\delta}(\varepsilon_{t-\delta},\varepsilon_{t-\delta+1},\dots,\varepsilon_{t+L'-c-1})$, shown in Equation (E.2), $V\Big[\Psi_{IS_{\delta+1}}(\varepsilon_{t-\delta-1},\varepsilon_{t-\delta},\dots,\varepsilon_{t+L'-c-1})-\Psi(\varepsilon_{t},\varepsilon_{t+1},\dots,\varepsilon_{t+L'-c-1})\Big]-V\Big[\Psi_{IS_{\delta}}(\varepsilon_{t-\delta},\varepsilon_{t-\delta+1},\dots,\varepsilon_{t+L'-c-1})-\Psi(\varepsilon_{t},\varepsilon_{t+1},\dots,\varepsilon_{t+L'-c-1})\Big]$ can be simplified to $V\Big[\Psi_{IS_{\delta+1}}(\varepsilon_{t-\delta-1},\varepsilon_{t-\delta},\dots,\varepsilon_{t+L'-c-1})$ $-\Psi_{IS_{\delta}}(\varepsilon_{t-\delta},\varepsilon_{t-\delta+1},\dots,\varepsilon_{t+L'-c-1})\Big]$, which has already been shown to be positive in Part (i). Therefore, we have $\sigma^2_{Z_{IS_{\delta+1}}} - \sigma^2_{Z_{IS_\delta}} > 0$.

Finally, from the proofs for Part (i) and Part (ii), we have $G_{IS_{\delta+1}} > G_{IS_\delta}$, which completes the proof.

APPENDIX J: Proof of Corollary 4 in Section 4

When $\delta > c$, from the expressions for $\Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \dots, \varepsilon_{t+L'-c-1})$, shown in Equation (F.2), we have

$$V\left[\Psi_{IS_\delta}\left(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \dots, \varepsilon_{t+L'-c-1}\right)\right] = V\left[\Psi_{IS_{\delta-1}}\left(\varepsilon_{t-\delta+1}, \dots, \varepsilon_{t+L'-c-1}\right) + \left[\left(\frac{\rho^{l'+\delta-c} - \rho^{L'+l'+\delta-c}}{1-\rho}\right) + \left(\frac{1-\rho^\delta}{1-\rho}\right)\right]\varepsilon_{t-\delta}\right]$$

$$= V\left[\Psi_{IS_{\delta-1}}\left(\varepsilon_{t-\delta+1}, \dots, \varepsilon_{t+L'-c-1}\right)\right] + V\left[\left[\left(\frac{\rho^{l'+\delta-c} - \rho^{L'+l'+\delta-c}}{1-\rho}\right) + \left(\frac{1-\rho^\delta}{1-\rho}\right)\right]\varepsilon_{t-\delta}\right]$$

Since

$$V\left[\left[\left(\frac{\rho^{l'+\delta-c} - \rho^{L'+l'+\delta-c}}{1-\rho}\right) + \left(\frac{1-\rho^\delta}{1-\rho}\right)\right]\varepsilon_{t-\delta}\right] = \left[\left(\frac{\rho^{l'+\delta-c} - \rho^{L'+l'+\delta-c}}{1-\rho}\right) + \left(\frac{1-\rho^\delta}{1-\rho}\right)\right]^2 \sigma^2,$$

we have

$$\lim_{\delta\to\infty} V\left[\left[\left(\frac{\rho^{l'+\delta-c} - \rho^{L'+l'+\delta-c}}{1-\rho}\right) + \left(\frac{1-\rho^\delta}{1-\rho}\right)\right]\varepsilon_{t-\delta}\right] = \left(\frac{1}{1-\rho}\right)^2 \sigma^2 \neq 0.$$

Therefore, we have, as $\delta \to \infty$, $V\left[\Psi_{IS_\delta}(\varepsilon_{t-\delta}, \varepsilon_{t-\delta+1}, \dots, \varepsilon_{t+L'-c-1})\right] \to \infty$, which implies $G_{IS_\delta} \to \infty$. In addition, since $G_{NI} > G_{IS_0}$, shown in Proposition 1 in Section 4, and G_{IS_δ} is an increasing function of δ, shown in Proposition 2 in Section 4, there must be a specific value of δ, say δ', after which $G_{NI} < G_{IS_\delta}$ for all $\delta > \delta'$. It may seem strange that, as $\delta \to \infty$, we have $G_{IS_\delta} \to \infty$. Note that this result is derived from the assumption that, in case of information sharing, the manufacturer uses only the shared customer demand information to calculate the order-up-to level.