International Journal of Concrete Structures and Materials
Vol.18, No.2E, pp.111~116, September 2006

# An Experimental Study on Interrelation of Influential Parameters on Unbonded Tendon Stress

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(Originally published in Korean version of ***Journal of KCI***, Vol.13, No.1, February 2001)

**Abstract:** The purpose of this study is to investigate the relations between unbonded tendon stress and its influential parameters, i.e. bonded reinforcement ratio, span/depth ratio, and loading type. To this end, the influence of such parameters was examined with twenty eight test results of previous studies. Afterwards, an experimental study was carried out with twenty one test specimens. The investigation of previous and current experiments revealed the followings; (1) The bonded reinforcement ratio and prestressing ratio were proved to be important variables on the unbonded tendon stress. (2) The ratio of span to depth and the type of loading affected the unbonded tendon stress partially although their effects varied with bonded reinforcement ratio. (3) AASHTO LRFD Code and Moon/Lim's design equations predicted the experimental results well with the safety margin.

**Keywords:** unbonded tendons, reinforcing index, loading type, span/depth ratio.

## 1. Introduction

Previous research results[1,2] report that the current ACI Code equation[3] evaluates the ultimate unbonded tendon stress very conservatively. In response to this, Moon/Lim[4-6] carried a series of experiments to propose a new equation to improve the current ACI Code equation so that it can evaluate the ultimate stress of unbonded tendons more appropriately.

First of all, they proposed a new design equation[6] based on a statistical analysis with previous experimental results. The experimental findings proved that such variables as effective prestressing ratio, quantity of reinforcing bars, quantity of tendons, span to depth ratio, concrete strength, type of loading, etc. influenced the ultimate stress of unbonded tendons. Furthermore, they performed an experimental research[5] to identify the factors, which made the ACI Code equation overestimate the ultimate stress of unbonded tendons. The experimental results indicated that the following cases had the ACI Code equation overestimate the increase in stress of unbonded tendons. They were 1) the reinforcement ratio approaching the maximum reinforcement ratio of $0.36\beta_1$, 2) large effective prestress, 3) the tendons being placed near neutral axis, and 4) the crack distribution zone being short. However, this series of research[3,4] attested the need for more inclusive study on such influential variables as reinforcement ratio, span to depth ratio, type of loading, etc.

Thus, this study aims to provide more rational data for evaluating the ultimate unbonded tendon stress by analyzing the effect of more specific variables on $\Delta f_{ps}$, ratio of the length of crack distribution zone, displacement, etc.

## 2. Analyses of previous research results

Fig. 1 shows the change in stress of unbonded tendons in response to the change in the quantity of reinforcing bars ($\rho$) based on the research results of references (4) and (5). The quantity of reinforcing bars used in the test specimens referenced in (4) followed the minimum reinforcing ratio of 0.4% as specified by the ACI Code. On the contrary, the test specimen referenced in number (5) employed the maximum reinforcement index of $0.36\beta_1$ as specified by the ACI Code. Overall, the test specimens had $\Delta f_{ps}$ decrease as the quantity of reinforcing bars increased. However, comparing the values computed from ACI Code equation with experimental values, the ACI Code underestimated the stress of tendons compared to the experimental values when the quantity of reinforcing bars was smaller and vice versa. This finding points to the fact that the quantity of reinforcing bars is an important variable that must be considered in the computation of ultimate stress of unbonded tendons. However, there is a point that requires a careful attention here. Up to now, the experiments on reinforcing bars involved only members of minimum steel bar ratio and maximum reinforcement ratio. Thus, this study attempts to carry out a more inclusive experiment on the influence of reinforcing bars.

Fig. 2 illustrates the change in unbonded tendon stress ($\Delta f_{ps}$) in response to the change in span to depth ratio ($L/d_p$). Reference (4) investigated the change in stress of tendons with test specimens of minimum reinforcement ratio of 0.4% under uniformly distributed loading. There are two distinctive properties being noticed.

First, the stress of the tendons gradually decreases as the span to depth ratio increases.

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Second, when the span to depth ratio is large, the increase in tendon stresses becomes very minimal.

Nevertheless, because the experiment of reference (4) has other variables as well as changes in the span to depth ratio, the possibility of interaction effect of these variables can not be excluded. Additionally, it was found that test specimens with the span to depth ratio below 15 must be considered also in order to examine the overall flow of the experimental results involving the span to depth ratio. Thus, more specific experiments focusing on the effect of changing the span to depth ratio only were carried out while keeping other variables constant.

Fig. 3 exhibits the result of experiments referenced in (4) and (5) as the relation between the change in the stress of tendons and the length ratio of crack distribution zone. As seen in the figure, the increase in stress of tendons tended to decrease as the length of crack distribution zone increases for the case of 1-point concentrated loading, and the increase in stress of tendons was not noticeable significantly for the case of uniformly distributed loading. This is considered due to the fact that a certain length of crack distribution zone is sufficiently secured due to the increase in maximum moment section for the case of uniformly distributed loading. On the contrary, the influence of reinforcing bars exerted greater for the case of 1-point concentrated loading. In other words, while the reinforcing bars have the role of distributing cracks of the member with unbonded tendons in general, the length of crack distribution zone increases as reinforcing bars take the role of distributing cracks for the case of larger quantity of reinforcing bars (mainly the test specimens in the B area of Fig. 3). However, although the length of crack distribution zone increases, the reason why the increase in stress of tendons decreases can be explained as follows. It is reasoned that, when the quantity of reinforcing bars increases, the stress transferred to the tendons is now shared by reinforcing bars in the equilibrium equation for the cross section to result in the decrease in the stress transferred to the tendons.

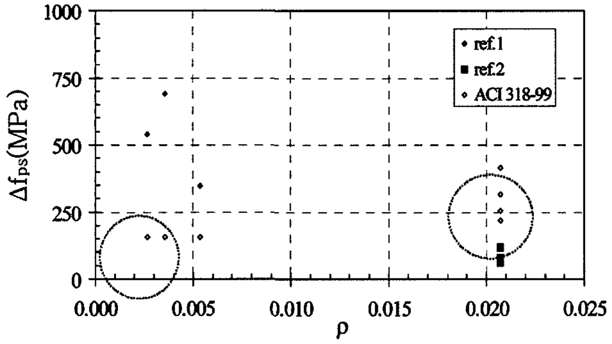


Fig. 1 Reinforcing ratio ρ vs. $\Delta f_{ps}$.

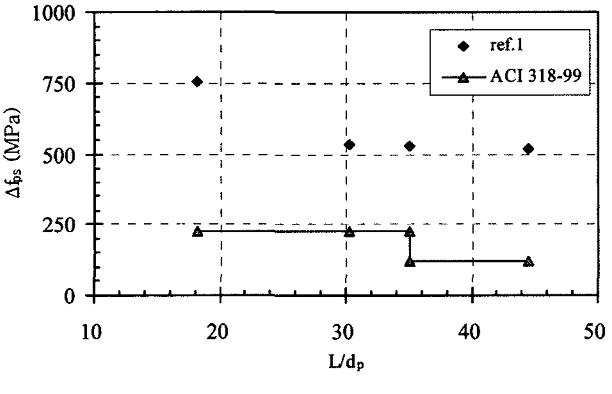


Fig. 2 Span/depth ratio $L/d_p$ vs. $\Delta f_{ps}$.

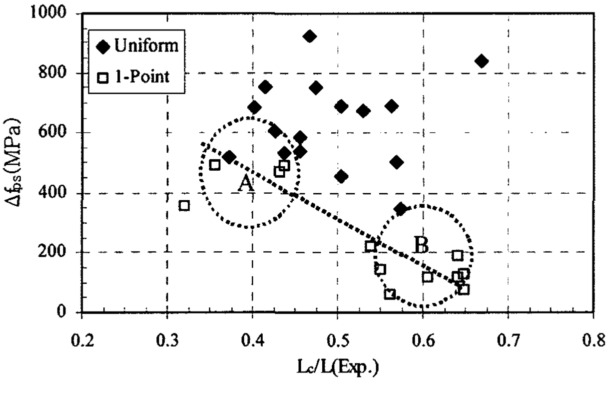


Fig. 3 Length ratio of crack distribution zone vs. $\Delta f_{ps}$.

## 3. Experiment

The experiment was carried out for the parts that required

Table 1 Specimen lists.

| Spec. | b × h (mm) | Loading type | $A_{ps}$ ($\rho_p$) | $A_s$ ($\rho$) | $A_s'$ ($\rho'$) | L (mm) | $L/d_p$ |
|---|---|---|---|---|---|---|---|
| J-1 | 200 × 351 | 1-point | 2-Φ6<br>(0.00074) | 2-D13<br>(0.004) | 2-D10<br>(0.002) | 4,000 | 15 |
| J-2 | | | 4-Φ6<br>(0.00148) | | | | |
| J-3 | | | 6-Φ6<br>(0.00223) | | | | |
| J-4 | 200 × 351 | 2-point | 4-Φ6<br>(0.00148) | | | | 15 |
| J-5 | 334 × 225 | | | | | | 25 |
| J-6 | 468 × 172 | | | | | | 35 |
| J-7 | 600 × 141 | | | | | | 45 |
| K-1 | 200 × 351 | 1-point | 2-Φ6<br>(0.00074) | 4-D13<br>(0.008) | | | 15 |
| K-2 | | | 4-Φ6<br>(0.00148) | | | | |
| K-3 | | | 6-Φ6<br>(0.00223) | | | | |
| K-4 | 200 × 351 | 2-point | 4-Φ6<br>(0.00148) | | | | 15 |
| K-5 | 334 × 225 | | | | | | 25 |
| K-6 | 468 × 172 | | | | | | 35 |
| K-7 | 600 × 141 | | | | | | 45 |
| L-1 | 200 × 351 | 1-point | 2-Φ6<br>(0.00074) | 6-D13<br>(0.012) | | | 15 |
| L-2 | | | 4-Φ6<br>(0.00148) | | | | |
| L-3 | | | 6-Φ6<br>(0.00223) | | | | |
| L-4 | 200 × 351 | 2-point | 4-Φ6<br>(0.00148) | | | | 15 |
| L-5 | 334 × 225 | | | | | | 25 |
| L-6 | 468 × 172 | | | | | | 35 |
| L-7 | 600 × 141 | | | | | | 45 |

* Note : Straight tendon profile

$f_{pu}$ = 1,860 MPa grade
Φ6 = 0.1982 cm$^2$ (3-wire mono-strand)
$f_{se}$ = 0.6$f_{pu}$
$f_y$ = 420 MPa grade
D10 = 0.71 cm$^2$ (deformed bar)
D13 = 1.27 cm$^2$ (deformed bar)

further investigation based on the results of a series of previous experiments. The test specimens totaled twenty one as listed in Table 1. They are classified into three groups of such variables as loading type, quantity of tendons, and quantity of reinforcing bars.

As shown in Fig. 4, specimens are divided at the point of series 3 and 4 as the 1-point and 2-point loading type. The J, K, L series of specimens indicate the quantity of reinforcing bars and the numbers 1, 2, and 3 denote the quantity of tendons. Finally, the numbers 4, 5, 6, and 7 represent the ratio of span to depth. The detailed specification for the test specimen is shown in Fig. 5.

## 4. Discussion of experimental results

### 4.1 Influence of the quantity of tensile steel bars

Fig. 6 shows the distribution of the change in unbonded tendon stress in response to the reinforcing ratio of tensile reinforcing bars. Fig. 6(a) includes the experimental values of reference (4) and (5). As shown by the fitted dot-line, it was confirmed that the quantity of reinforcing bars influenced the change in unbonded tendon stress dominantly. Thus, only test specimens with tensile reinforcing bars (J, K, L-1, 2, 3) were chosen for a detailed examination of the influence of the reinforcing ratio as shown by Fig. 6(b). The general trend was that the increase in unbonded tendon stress slowed down as the reinforcing ratio of the tensile reinforcing bars increased. However, test specimens with larger quantity of tendons, i.e. J, K, L-3 series (shown by dotted circles), exhibited almost the same increase in the stress of the unbonded tendons even if the reinforcing ratio of the tensile reinforcing bars increased. It is reasoned that this observation is due to the more dominant effect of the quantity of tendons than the effect of reinforcing bars.

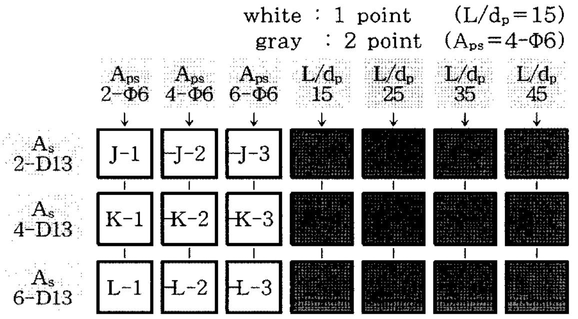


**Fig. 4** Specimen layout.

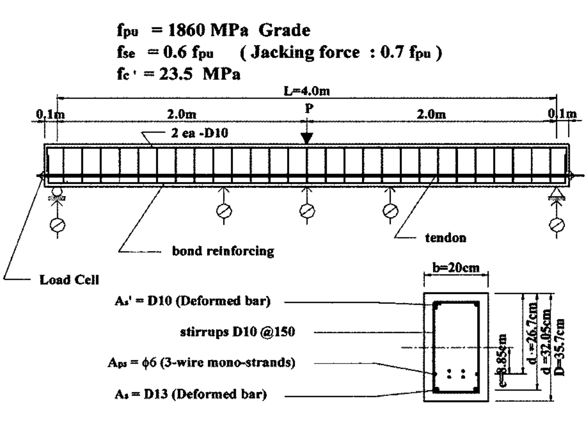


**Fig. 5** Typical specimen.

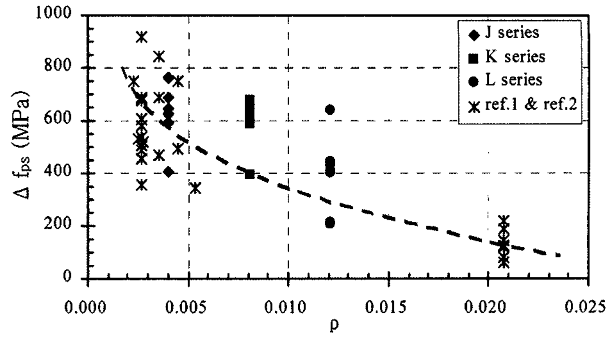


**Fig. 6**(a) Reinforcing ratio vs. $\Delta f_{ps}$.

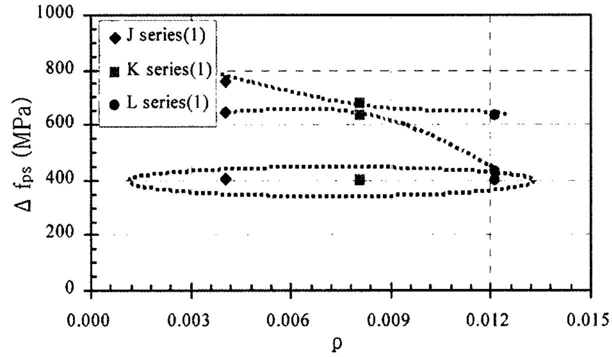


**Fig. 6**(b) Reinforcing ratio vs. $\Delta f_{ps}$.

**Table 2** Test results.

| Specimen | $f_{se}$ (kN) | $f_{ps}$ (kN) | $\Delta f_{ps}$ (kN) | $P_{cr}$ (kN) | $P_{max}$ (kN) | $P_{max}$ / $P_{cr}$ | $\delta_{max}$ (mm) | $L_c$ (mm) |
|---|---|---|---|---|---|---|---|---|
| J-1 | 22.12 | 37.23 | 15.11 | 22.38 | 54.08 | 2.42 | 97.2 | 164 |
| J-2 | 22.05 | 34.87 | 12.82 | 30.62 | 71.85 | 2.35 | 94.0 | 167 |
| J-3 | 22.17 | 30.21 | 8.04 | 36.28 | 73.03 | 2.01 | 43.2 | 141 |
| J-4 | 22.13 | 33.93 | 11.81 | 24.12 | 97.54 | 4.04 | 59.2 | 203 |
| J-5 | 22.19 | 35.76 | 13.57 | 29.81 | 56.73 | 1.90 | 123.4 | 224 |
| J-6 | 22.13 | 34.46 | 12.33 | 12.06 | 33.86 | 2.81 | 168.4 | 238 |
| J-7 | 22.11 | 35.70 | 13.59 | 12.55 | 27.67 | 2.20 | 194.4 | 265 |
| K-1 | 22.16 | 35.57 | 13.41 | 24.12 | 79.41 | 3.29 | 74.8 | 241 |
| K-2 | 22.13 | 34.65 | 12.53 | 29.81 | 92.95 | 3.12 | 90.6 | 224 |
| K-3 | 22.18 | 30.03 | 7.85 | 38.54 | 102.57 | 2.66 | 39.8 | 179 |
| K-4 | 22.16 | 34.78 | 12.62 | 46.28 | 134.9 | 2.91 | 59.4 | 242 |
| K-5 | 22.12 | 33.72 | 11.60 | 17.65 | 79.16 | 4.48 | 98.2 | 259 |
| K-6 | 22.14 | 34.15 | 12.00 | 13.14 | 49.76 | 3.79 | 150.2 | 280 |
| K-7 | 22.13 | 34.19 | 12.06 | 16.87 | 41.71 | 2.47 | 185.2 | 214 |
| L-1 | 22.13 | 34.74 | 12.61 | 32.85 | 103.94 | 3.16 | 71.0 | 264 |
| L-2 | 22.11 | 30.70 | 8.60 | 35.01 | 110.81 | 3.17 | 47.0 | 262 |
| L-3 | 26.88 | 34.78 | 7.89 | 39.22 | 129.66 | 3.31 | 40.0 | 267 |
| L-4 | 22.10 | 30.33 | 8.23 | 45.50 | 162.05 | 3.56 | 42.0 | 312 |
| L-5 | 22.13 | 30.99 | 8.85 | 22.75 | 94.62 | 4.16 | 69.2 | 278 |
| L-6 | 22.14 | 26.38 | 4.23 | 13.24 | 58.69 | 4.43 | 56.2 | 280 |
| L-7 | 22.15 | 26.29 | 4.15 | 13.83 | 45.24 | 3.27 | 61.2 | 273 |

* Note
$f_{se}$ : Effective prestress /1ea
$f_{ps}$ : Ultimate tendon stress /1ea
$\Delta f_{ps}$ : Tendon stress increase /1ea
$P_{max}$ : Maximum load
$P_{cr}$ : Initial cracking load
$\delta_{max}$ : Maximum deflection
$L_c$ : Plastic hinge length

Thus, although the quantity of reinforcing bars influences the increase in unbonded tendon stress in general, its effect on the change in unbonded tendon stresses becomes relatively smaller due to the fact that the effect of the quantity of tendons is greater than the effect of the quantity of reinforcing bars when the quantity of tendons increases to result in greater reinforcing index.

### 4.2 Influence of the quantity of tendons

Fig. 7(a) illustrates the unbonded tendon stress in response to the quantity of tendons. Here, the relationship between the quantity of tendons and the increase in the stress of the tendons showed the tendency to disperse. This is reasoned that there are other variables besides the quantity of tendons in the specimens used in the graph to affect the stress of unbonded tendons.

Thus, excluding these interactive variables, the experimental results are remapped in Fig. 7(b). As the quantity of tendons increases, the change in the stress of tendons shows the tendency to decrease. This observation is construed due to the fact that the stress attributed to each tendon is reduced as the quantity of tendons increases.

### 4.3 Influence of span to depth ratio

Fig. 8 shows the increase in stress of tendons in response to span/depth ratio. The experiment was performed with the span/depth ratios of 15, 25, 35, and 45.

The experimental result revealed that J and K series specimens with relatively smaller quantity of tensile reinforcing bars ($\rho = 0.004$ and $0.008$) showed almost no change in the stress of tendons in response to the span/depth ratio. However, the L series specimens with relatively larger quantity of reinforcing bars ($\rho = 0.004$) exhibited the tendency that the unbonded tendon stress decreased gradually in response to increasing span to depth ratio. Additionally, the L series specimens had the stress of tendons reduced by about 50% as the span to depth ratio increased from 25 to 35. In other words, the response to span to depth ratio exhibited the similar result to ACI Code when the quantity of tensile reinforcing bars was relatively larger.

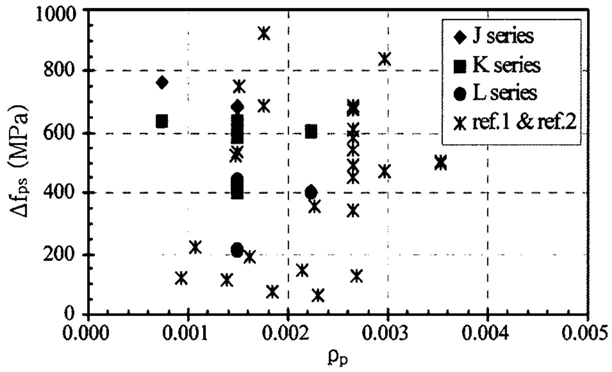


**Fig. 7**(a) Prestressing ratio vs. $\Delta f_{ps}$.

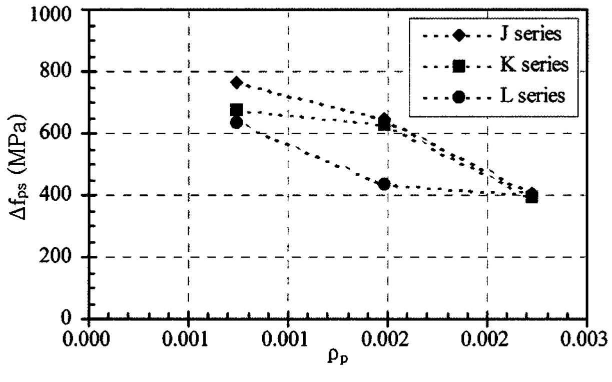


**Fig. 7**(b) Prestressing ratio vs. $\Delta f_{ps}$.

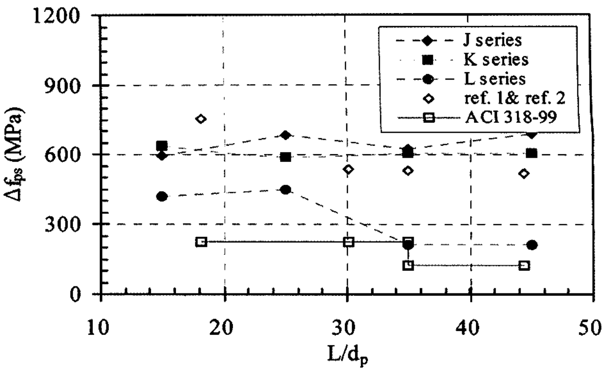


**Fig. 8** Span/depth ratio vs. $\Delta f_{ps}$.

### 4.4 Influence of loading type

Fig. 9 is a graph showing the effect of loading type on the change in stress of unbonded tendons. The test specimens had the quantity of tendons fixed at $\rho_p = 0.00148$ and were composed of 2 series with 1 point loading and 4 series with 2 point loading. As seen in the graph, unlike the experimental results reported in reference (4), both cases of 1 point loading and 2 point loading exhibited almost the same change in stress of unbonded tendons. It is reasoned that, because the experiment of reference (4) had the reinforcing ratio of $\rho = 0.0024 \sim 0.0048$ for the tensile reinforcing bars near the minimum reinforcement ratio for the reinforcing bars as specified by ACI Code, it had the influence of the loading type more significant than the influence of tensile reinforcing bars. Nevertheless, the test specimens of this study had the reinforcing ratio ($\rho$) of the tensile reinforcing bars between $0.004 \sim 0.012$, resulting in the influence of reinforcing ratio ($\rho$) of the tensile reinforcing bars greater than the influence of loading types. Moreover, J series specimens of the reinforcing ratio ($\rho$) of 0.004 for the tensile

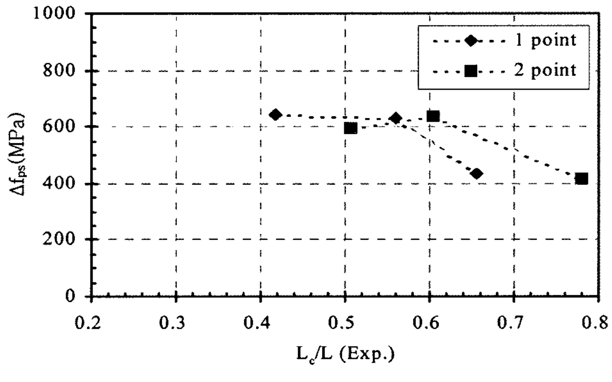
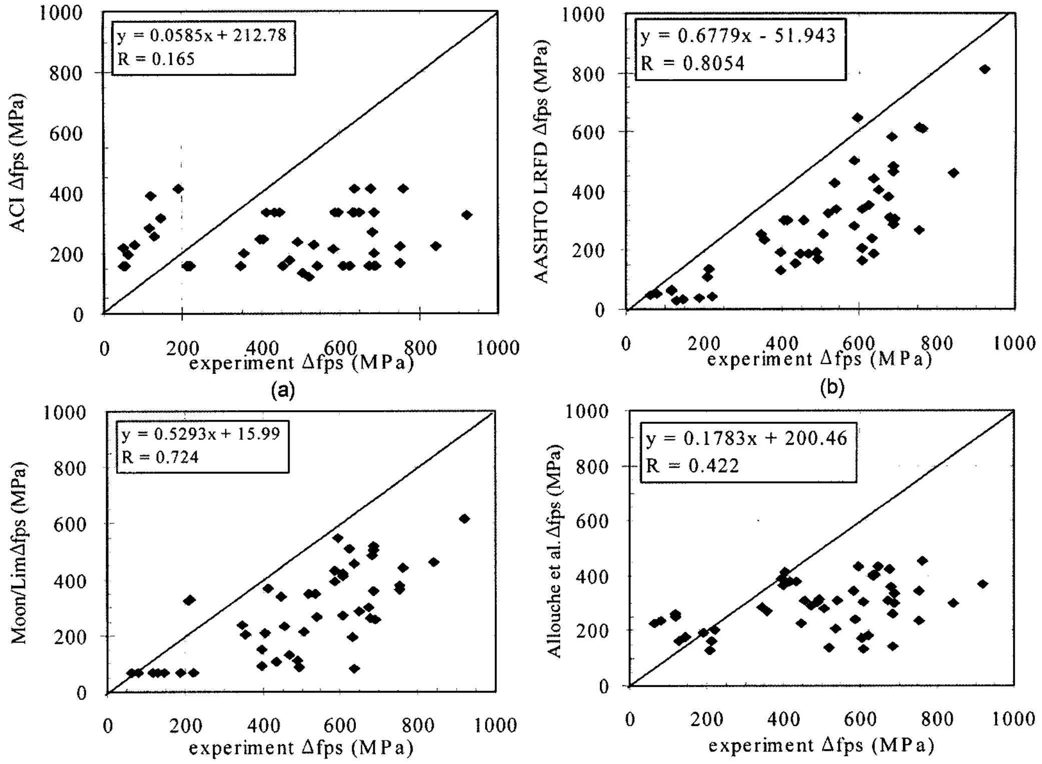


**Fig. 9** Effects of loading types.



(a) (b) (c) (d)

**Fig. 10** (a) ACI 318-99 Code, (b) AASHTO LRFD code, (c) Moon/Lim's equation, (d) Allouche et al.'s equation.

re-bars among the test specimens of this study had slightly different increase in the stress depending on the 1- point and 2-point loading types. Thus, the reason that the test specimens, which had larger quantity of tensile reinforcing bars, showed marginal influence of loading type is construed due to the action of reinforcing bars distributing the cracks mainly.

However, the research of Harajli and Kanj[7)] reported that the loading type is a variable of little importance in the increase of stress of unbonded tendons. A careful examination of their research contents revealed that the quantity of tensile reinforcing bars in their test specimens was $\rho = 0.0048 \sim 0.0103$. Thus, analyzing their research results in the context of the findings of this research, it can be explained that their research had used the quantity of reinforcing bars significantly greater, resulting in their finding of insignificant influence of loading type.

## 5. Comparison of design equations

A comparison study of the experimental results of reference (4) and (5) besides the experimental results of this study with existing design equations is presented in Fig. 10. The design equations chosen for this comparison study are ACI Code equation[3)] eq. (1), AASHTO LRFD Code equation[8)] eq. (2), Moon/Lim's design equation[6)] eq. (3), and Allouche et al.'s design equation[2)] eq. (4). Among these design equations, the design equation of Allouche et al. is a recently proposed design equation[2)], that has not been cited in previous studies. It is a design equation developed by revising the Canadian design Code equation. This design equation considers the loading type and the effect of plastic hinge on patterned loading to evaluate the unbonded tendon stress appropriately.

The result of this comparison study of the values computed from the above design equations with the experimental values indicates that the AASHTO LRFD design equation depicted in Fig. 10(b) and Moon/Lim's design equation depicted in Fig. 10(c) show relatively satisfactory values close to the experimental values.

$$f_{ps} = f_{se} + 69 + \frac{1.4 f_c'}{k\rho_p}$$
$$k = 100(L/d_p \leq 35)$$
$$k = 300(L/d_p > 35) \qquad (1)$$

$$f_{ps} = f_{se} + \Omega_u E_{ps} \varepsilon_{cu}\left(\frac{d_p}{c} - 1\right)\frac{L_1}{L_2}$$
$$f_{ps} \leq 0.94 f_{py}$$
$$\Omega_u = \frac{1.5}{L/d_p} \text{ (1 - point loading)}$$
$$\Omega_u = \frac{3.0}{L/d_p} \text{ (2 - point or } \textit{uniform loading}) \qquad (2)$$

$$f_{ps} = 70 + 0.8 f_{se} + \frac{1}{15}\frac{(A_s' - A_s) f_y}{A_{ps}} + 6.5\sqrt{\frac{d_s f_c'}{d_p \rho_p}\left(\frac{1}{f} + \frac{d_p}{L}\right)}$$
$$f_{se} + 70 \leq f_{ps} \leq f_{py} \qquad (3)$$

$$f_{ps} = f_{se} + \frac{8,000}{l_e'}(d_p - c_y)\left[1 + \left(\frac{c_y}{d_p}\right)^2\right]$$
$$f_{se} + 70 \leq f_{ps} \leq f_{py}$$
$$c_y = \frac{A_{ps} f_{py} + A_s f_y}{\alpha_1 \beta_1 f_c' b_w} \qquad (4)$$

## 6. Conclusions

The following conclusions are derived based on the experimental results of this study.

1) The quantity (reinforcement ratio) of reinforcing bars is an important variable in influencing the increase of stress of unbonded tendons. However, when the quantity of tendons approaches the maximum reinforcing index of $0.36\beta_1$, the effect of the quantity of reinforcing bars was marginal due to the greater influence of the quantity of tendons.

2) It was determined that the span to depth ratio was influential to the increase in the tendon stress only when it approached the maximum reinforcement index of $0.36\beta_1$. Thus, it is desirable to consider the change in unbonded tendon stress with respect to the span to depth ratio in its relation to the reinforcement ratio.

3) It was found that the loading type was influential to the increase in unbonded tendon stress only when the reinforcement index was relatively smaller.

4) Among the design equations to compute the unbonded tendon stress, AASHTO LRFD Code equation and Moon/ Lim's equation were found to evaluate it better than other alternatives.

## Acknowledgements

The authors would like to thank KISWIRE Ltd., Seoul, Korea for the provision of unbonded tendons for the experimental purpose.

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