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Testing NRBU Class of Life Distributions Using a Goodness of Fit Approach

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Abstract. In this paper, we present the U-Statistic test for testing exponentiality against new renewal better than used (NRBU) based on a goodness of fit approach. Selected critical values are tabulated for sample sizes n=5(1)30(10)50. The asymptotic Pitman relative efficiency relative to (NRBU) test given in the work of Mahmoud et all (2003) is studied. The power estimates of this test for some commonly used life distributions in reliability are also calculated. Some of real examples are given to elucidate the use of the proposed test statistic in the reliability analysis. The problem in case of right censored data is also handled.

Key words: *new renewal better than used (NRBU), U-statistic, goodness of fit testing, efficiency, relative efficiency, Monte Carlo methods, power and censored data.*

1. INTRODUCTION

Ever since the works of Barlow et all (1963) and Bryson and Siddiqui (1969), various classes of life distributions have been used to characterize positive or negative aging of engineering units, social and biological organs science, maintenance and biometrics.
Barlow and Prochan (1981) have shown the relation between some nations of aging such as the increasing failure rate (IFR), increasing failure rate average (IFRA), new better than used (NBU), new better than used in expectation (NBUE), and decreasing mean remaining life (DMRL). The problem of testing exponentiality against various classes of life distributions had a good deal of attention in the literature. For testing exponentiality against IFR, see Proschan and Pyke (1967), Barlow (1968), Bickel and Doksum (1969), Ahmed (1975) for the class IFR. See e.g. Ahmad (1975), Deshpand (1983), and Ahmad (1994) for the class IFRA. The work started by Hollander and Proschan (1972), Koul (1977), Kumazawa (1978) and

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Ahmad (1994) among others for NBU and extended to HNBUE by Hollander and Proschan (1975), Koul (1978) and Kanjo (1993). The procedures for testing against HNBUE are done by Rolski (1975), Klefsjo (1982), Ahmad (1995), Hendi et all (1998) and Ahmad et all (1999).

Let T be a nonnegative random variable with distribution function $F(t)$ where $F(t) = 0$ $t < 0$ while $F(0)$ may not be zero, survival function $\overline{F}(t)$, density function $f(t)$ and finite mean μ and variance σ^2.

Consider a device with life length T and life distribution F in the long run, the device is replaced instantly upon failure by a sequence of mutually independent devices. These devices are independent of the first unit and identically distributed with the same life distribution F. When the renewal of the system is continued in definitely the stationary life distribution of a device in operation at time t is given by,

$$W_F(t) = \mu^{-1}\int_0^t \overline{F}(u)du, \qquad \text{for } 0 \le t \le \infty,$$

Where $\mu = \mu_F = \int_0^\infty \overline{F}(x)dx \prec \infty$ is the mean of life distribution F.

Abouammoh et al (2000) introduced the NRBU, NRBUE and HNRBUE classes of life distributions and studied the relation between them.

On the other hand, there is a highly active literature dealing with nonparametric tests and their properties.

Abouammoh and Khalique (1998) investigated a test statistic for testing exponentiality versus NRBU based on total time on test (TTT)-transform empirically. The asymptotic *distribution of this test is established theoretically by El-arishy and Diab(1998)* .They showed that the distribution of the test statistic is asymptotically normal with mean zero and variance 1/48. Mahmoud et all (2002) investigated the test statistic for NRBU based on U-statistic. In (2003), Mahmoud et all studied two test statistics for NRBU and RNBU classes of life distributions as alternatives based on the moment inequalities.

Hendi and Abouammoh (2001) investigated two test statistics for testing exponentiality versus NBRUE, HNBRUE classes of life distribution based on U-statistic.

Finally, Montasser (2002) established the test statistic for testing exponentiality versus HNBRUE classes of life distribution based on U-test statistic

In fact, stochastic comparison between the random variable T with distribution F and its renewal random variable T_{W_F} with life distribution W_F, for which $W_F(0-) = 0$, density function w_F and renewal survival function $\overline{W_F}$, leads to the following Definition.

Definition 1.1: A random variable T or its distribution F is said to have new renewal better than used, denoted by NRBU property, if $T_t \le^{st} T_{W_F}$,where T_t is the conditional variable of T given t with distribution, $\overline{F_x}(t) = p(T \le t \mid T \ge x)$

This definition means that, T is NRBU, if

$$\overline{F_x}(t) \le \overline{W_F}(t), \qquad \forall x,t \ge 0 \qquad (1.1)$$

The corresponding dual class of life distribution is new renewal worse than used parent property , denoted by NRWU, and is define by reversing the inequality sign of (1.1).

The object of this work is to present test statistic for testing $H_0 : F$ is exponential against $H_1 : F$ is NRBU class of life distribution and not exponential with non constant failure rate. This test statistic is based on a goodness of fit approach using a random sample $X_1, X_2, \ldots, X_n$ from a continues life distribution F in the case of censored and non-censored data.

In contrast to goodness of fit problems, where the test statistic is based on a measure of departure from H_0 that depends on both H_0 and H_1, most tests in life testing setting, including those referenced above do not use the null distribution in devising the test statistics, this resulted in test statistics that are often difficult to work and require programming to calculated. Alternatively, we demonstrate in current work that in cooperating into the measure of departure from it can lead to simpler test statistics that are easy to work with are asymptotically equivalent in distribution to those cited above and may have equal or higher efficiency than the classical procedures. They also may have better finite sample behaviors.

In the following section we derive non parametric test for testing exponentiality against NRBU properties using goodness of fit approach.

2. TESTING EXPONENTIALITY AGAINST NRBU CLASS FOR NONCENSORED DATA

In this section, we derive a non parametric U-statistic for testing the null hypothesis H_0 :exp (- x), $x \geq 0$ vs. $H_1 : \overline{F}$ is NRBU. This test is based on a sample $X_1, X_2, \ldots, X_n$ from F.

2.1 Testing against NRBU Alternative

We propose in the following lemma a measure of departure from H_0.

Lemma 2.1 If F is NRBU then a measure of the deviation from the null hypothesis H_0 is $\delta_F > 0$, where

$$\delta_F = \mu \int_0^\infty t e^{-t} dF(t) - \left[\int_0^\infty e^{-t} dF(t) \right]^2 . \tag{2.1}$$

Proof. Clearly from (1.1) F is NRBU iff

$$\overline{F}(x)\overline{W}(t) - \overline{F}(x+t) \geq 0 .$$

Take the integral with respect to $F(x)$ and $F(t)$,

$$\delta_F = \int_0^\infty \int_0^\infty \overline{F}(x)W(t)dF_0(x)dF_0(t) - \mu \int_0^\infty \int_0^\infty \overline{F}(x+t)dF_0(x)dF_0(t).$$

With $W(t) = \int_t^\infty \overline{F}(u)du$, $\overline{W_F}(t) = \mu_F^{-1} \int_t^\infty \overline{F}(u)du$, and $dF_0(x) = e^{-x}$.

Thus the result follows.

2.2 Empirical test statistic for NRBU alternative.

Let $X_1, X_2, \ldots, X_n$ be a random sample from F. Let $dF_n(x) = \frac{1}{n}$ and $\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i$ be the estimates of $dF(x)$ and μ respectively.

Then the estimate of δ_F in (2.1) is given by

$$\hat{\delta}_{F_n} = \hat{\mu}\int_0^\infty te^{-t}dF_n(t) - \left[\int_0^\infty e^{-t}dF_n(t)\right]^2. \tag{2.2}$$

And the empirical form of δ_F can be written as:

$$\hat{\delta}_{F_n} = \frac{1}{n^2}\sum_{j=1}^{n}\sum_{i=1}^{n}\left[X_i X_j e^{-X_j} - e^{-(X_i+X_j)}\right], \tag{2.3}$$

Set

$\Phi(X_1, X_2) = X_1 X_2 e^{-X_2} - e^{-(X_1+X_2)}$,

and define the symmetric Kernel

$$\psi(X_1, X_2) = \frac{1}{2!}\sum_R \Phi(X_{i1}, X_{i2})$$

Where the summation is over all arrangements of X_1, X_2, then $\hat{\delta}_{F_n}$ in (2.3) is equivalent to the U_n-statistic given by

$$U_n = \frac{1}{\binom{n}{2}}\sum_{i<j}\Psi(X_i, X_j). \tag{2.4}$$

Since the order of the Kernel in (2.4) is two, the procedure is very simple to calculate. The following Theorem summarizes the asymptotic normality of $\hat{\delta}_{F_n}$ or U_n.

Theorem 2.1 (i) As $n \to \infty$, U_n converges to δ_F with probability one.

(ii) As $n \to \infty$, $n^{\frac{1}{2}}(U_n - \delta_F)$ is asymptotically normal with mean 0 and variance σ^2

$$\sigma^2 = \text{var}\left\{Xe^{-X}\int_0^\infty udF(u) + X\int_0^\infty ue^{-u}dF(u) - 2e^{-X}\int_0^\infty e^{-u}dF(u)\right\} \tag{2.5}$$

(iii) Under H_0, the variance σ^2 reduces to,

$$\sigma_0^2 = Var\left[X e^{-X} + \frac{1}{4}X - e^{-X}\right] = \frac{67}{216}.$$

Proof. (i) Follows from Theorem A of Serfling [(1980), p190], while (ii), and (iii) using the standard Theorem of U-statistic cf. Lee (1989), we only need to evaluate the asymptotic variance σ^2 where,

$$\sigma^2 = \text{var}\left\{\sum_{i=1}^{2}\eta_i\right\}. \tag{2.6}$$

with $\eta_i = E(\phi(X_1, X_2 \mid X_i))$.

Table3.1 Critical values for the upper percentiles of $\hat{\delta}_{F_n}$.

n	%90	%95	%98	%99
5	.2900	.3586	.4384	.4923
6	.2719	.3419	.4102	.4502
7	.2423	.3087	.3821	.4311
8	.2396	.2975	.3608	.4009
9	.2231	.2830	.3521	.3884
10	.2155	.2765	.3344	.3791
11	.2071	.2616	.3192	.3581
12	.2012	.2485	.3075	.3456
13	.1917	.2398	.2935	.3267
14	.1795	.2317	.2852	.3204
15	.1793	.2286	.2791	.3222
16	.1766	.2196	.2729	.3054
17	.1645	.2119	.2602	.2899
18	.1581	.2009	.2541	.2886
19	.1565	.2040	.2536	.2898
20	.1507	.1934	.2374	.2646
21	.1495	.1890	.2326	.2622
22	.1440	.1849	.2292	.2548
23	.1435	.1826	.2276	.2543
24	.1373	.1829	.2234	.2466
25	.1390	.1774	.2202	.2505
26	.1379	.1784	.2187	.2500
27	.1356	.1714	.2151	.2408
28	.1258	.1662	.2044	.2294
29	.1263	.1640	.2035	.2268
30	.1274	.1612	.1996	.2239
40	.1067	.1355	.1741	.1916
50	.0952	.1245	.1525	.1722

Recall the definition of $\Phi(X_1, X_2)$, from above, so it is not difficult to show that

$$\eta_1 = X_1 \int_0^{\infty} ue^{-u} dF(u) - e^{-X_1} \int_0^{\infty} e^{-u} dF(u),$$

and $\eta_2 = X_2 e^{-X_2} \int_0^{\infty} u dF(u) - e^{-X_2} \int_0^{\infty} e^{-u} dF(u)$. Substitute in (2.6)

Hence (2.5) holds.

3. MONTE CARLO CRITICAL POINTS FOR $\hat{\delta}_{Fn}$ STATISTIC

We have, simulated upper percentiles of the Statistic $\hat{\delta}_{F_n}$, based on 5,000 replications from the standard exponential distribution of order 5(1)30(10)50. Computation is shown in Table 3.1

3.1 The power estimates

The power of the test statistic $\hat{\delta}_{F_n}$ is considered at significance level $\alpha = 0.05$ and for commonly used distributions in reliability modeling. These distributions are

(i) Pareto family, $\overline{F}(x) = (1 - \theta x)^{\frac{1}{\theta}}$ for $x \geq 0, \theta \geq 0$

(ii)Weibull family, $\overline{F}(x) = \exp(-x^{\theta})$ for $x \geq 0, \theta \geq 0$

(iii) Gamma family, $\overline{F}(x) = \frac{1}{\Gamma(\theta)} \int_x^{\infty} t^{\theta-1} e^{-t} dt$ for $x \geq 0, \theta \geq 0$

For the previous alternatives, the powers for the proposed test are tabulated in Table 3.2, using 5,000 replications for sample size n=10, 20 and 30, and parameter values θ =2, 3 and 4.

Table 3.2 Power estimates using $\alpha = .05$

Distribution	Parameter	Sample size		
	θ	10	20	30
Pareto	2	.998	1.000	1.000
	3	.999	1.000	1.000
	4	1.000	1.000	1.000
Weibull	2	1.000	1.000	1.000
	3	1.000	1.000	1.000
	4	1.000	1.000	1.000
Gamma	2	1.000	1.000	1.000
	3	1.000	1.000	1.000
	4	1.000	1.000	1.000

Table 3.2 shows that our test has a good power, specially in the case of weibull and gamma families, and the power is getting as smaller as NRBU approaches the exponential distribution.

4- ASYMPTOTIC RELATIVE EFFICIENCY (ARE).

We compare the asymptotic efficiency of our test relative to that tests of NRBU and RNBU classes based on U-statistic that proposed by Mahmoud (2003). We use the following alternatives:

(i) Weibull family: $\overline{F_1}(x) = \exp(-x^{\theta}), x > 0, \theta \geq 1$.

(ii) Linear failure rate family: $\overline{F_2}(x) = \exp(-x - \theta x^2/2), x > 0, \theta \geq 0$.

(iii) Makeham family: $\overline{F_3}(x) = \exp(-x + \theta(x + e^{-x} - 1)), x > 0, \theta \geq 0$.

(iv) Gamma family: $\overline{F_4}(x) = \int_x^{\infty} e^{-u} u^{\theta-1} du / \Gamma(\theta), x > 0, \theta \geq 0$.

Note that H_0 (the exponential) is attained at $\theta = 1$, in (i) and (iv), and is attained at $\theta = 0$ in (ii) and (iii).

Table 4.1 contains the asymptotic efficiencies of the NRBU test and the asymptotic relative efficiencies of the NRBU (RNBU) tests.

Table 4.1. Asymptotic relative efficiencies of $\hat{\delta}_{F_n}$ to Mahmoud et all (2003).

Efficiency	Weibull	Linear failure rate	Makeham	Gamma
$\frac{\mu'(\theta)}{\sigma(\theta_0)} = \frac{\frac{\partial \delta_F}{\partial \theta}\vert_{\theta=\theta_0}}{\sigma(\theta_0)} = \hat{\delta}_{F_n}$	.38499	.5673	.72324	1.4617
$\hat{\Delta}_{F_n}$ (Mahmoud et all (2003))	.3571	.7143	.0893	.1191
E($\hat{\delta}_{F_n}$, $\hat{\Delta}_{F_n}$)	1.0780	.7942	8.0989	12.2729

5. TESTING AGAINST NRBU CLASS FOR CENRORED DATA

In this section, a test statistic is proposed to test H_0 versus H_1 with randomly right censored samples. In the censoring model, we observe the pair $(Z_j, \delta_j), j = 1......n$ where $Z_j = \min(X_j, Y_j)$ and

$$\delta_j = \begin{cases} 1 & if \quad Z_j = X_j \quad (j^{th} observn\ is\ uncensored) \\ 0 & if \quad Z_j = Y_j \quad (j^{th} observn\ is\ censored) \end{cases},$$

where $X_1, X_2, ..., X_n$ denote their true lifetime from distribution F. And $Y_1, Y_2, ..., Y_n$ be (i.i.d.) according to distribution G. Also X's and Y's are independent. Let $Z(0) = 0 < Z(1) < Z(2) < ...... < Z(n)$ denote the ordered Z's and $\delta_{(j)}$ is the δ_j corresponding to $Z_{(j)}$ respectively.

Kaplan and Meier (1958) proposed the product limit estimator of $\overline{F}_n(X)$ as following.

$$\overline{F}_n(X) = 1 - F_n(X) = \prod_{[j: Z_{(j)} \le X]} \{(n-j)/(n-j+1)\}^{\delta_{(j)}}, \quad X \in [0, Z_{(n)}]. \tag{5.1}$$

Now for testing $H_0 : \delta_F = 0$, against $H_1 : \delta_F > 0$, using the random right censored data, we propose the following test statistic.

$$\hat{\delta}^C_{F_n} = \hat{\mu}\int_0^\infty te^{-t} dF_n(t) - \left[\int_0^\infty e^{-t} dF_n(t)\right]^2 \tag{5.2}$$

For computational purpose, $\hat{\delta}^C_{F_n}$ in (5.2) may be rewritten as

$$\hat{\delta}^C_{F_n} = \sum_{k=1}^{n} \prod_{m=1}^{k-1} c_m^{\delta_{(m)}} (Z_{(k)} - Z_{(k-1)}) \left[\sum_{j=1}^{n} X(j) e^{-X(j)} \left\{\prod_{p=1}^{j-2} c_p^{\delta_{(p)}} - \prod_{p=1}^{j-1} c_p^{\delta_{(p)}}\right\}\right]$$
$$- \left\{\sum_{i=1}^{n} e^{-X(j)} \left\{\prod_{m=1}^{i-2} c_m^{\delta_{(m)}} - \prod_{m=1}^{i-1} c_m^{\delta_{(m)}}\right\}\right\}^2, \tag{5.3}$$

where $dF_n(Z_j) = \overline{F}_n(Z_{j-1}) - \overline{F}_n(Z_j)$, $C_K = [n-k][n-k+1]^{-1}$, and

Table 5.1 gives the critical values percentiles of $\hat{\delta}^C_{F_n}$ for sample sizes n=5(1)50, 60(10)81, based on 5,000 replications.

Table 5.1 Critical values for percentiles of $\hat{\delta}^C_{F_n}$

n	%90	%95	%98	%99
5	.0803	.1095	.1403	.1578
6	.0844	.1101	.1439	.1641
7	.0877	.1165	.1481	.1675
8	.0873	.1140	.1433	.1588

9	.0800	.1018	.1296	.1498
10	.0823	.1029	.1296	.1521
11	.0786	.1009	.1305	.1492
12	.0775	.0996	.1251	.1451
13	.0727	.0914	.1100	.1278
14	.0735	.0922	.1158	.1311
15	.0703	.0870	.1088	.1239
16	.0678	.0856	.1044	.1220
17	.0655	.0798	.0968	.1148
18	.0647	.0795	.0989	.1162
19	.0629	.0784	.0961	.1120
20	.0623	.0754	.0951	.1080
21	.0585	.0728	.0887	.1012
22	.0591	.0720	.0875	.0980
23	.0576	.0696	.0837	.0959
24	.0576	.0691	.0838	.0946
25	.0558	.0690	.0842	.0943
26	.0550	.0666	.0794	.0924
27	.0522	.0623	.0767	.0859
28	.0520	.0626	.0755	.0854
29	.0507	.0621	.0744	.0839
30	.0508	.0603	.0718	.0845
31	.0506	.0614	.0747	.0840
32	.0486	.0577	.0711	.0796
33	.0482	.0579	.0700	.0797
34	.0470	.0562	.0662	.0746
35	.0462	.0546	.0645	.0728
36	.0461	.0546	.0666	.0734
37	.0449	.0538	.0648	.0737
38	.0450	.0534	.0649	.0722
39	.0442	.0527	.0645	.0715
40	.0436	.0510	.0594	.0658
41	.0428	.0503	.0598	.0675
42	.0418	.0493	.0594	.0663
43	.0424	.0496	.0585	.0669
44	.0410	.0480	.0567	.0630
45	.0404	.0470	.0554	.0609
46	.0404	.0476	.0557	.0617
47	.0392	.0461	.0550	.0619
48	.0392	.0458	.0542	.0604
49	.0385	.0448	.0534	.0596
50	.0383	.0450	.0539	.0593
60	.0352	.0398	.0468	.0503
70	.0326	.0377	.0436	.0486
81	.0302	.0343	.0393	.0431

5.1 Applications:

Example 1. Consider the data in Abouammoh et al (1994). These data represent 40 patients suffering from blood cancer from one of Ministry of Health Hospitals in Saudi Arabia and the ordered life time (in days) are recorded:
115, 181, 255, 418, 441, 461, 516, 739, 743, 789, 807, 865, 924, 983, 1024, 1062, 1063, 1165, 1191, 1222, 1222, 1251, 1277, 1290, 1357, 1369, 1408, 1455, 1478, 1549, 1578, 1578, 1599, 1603, 1605, 1696, 1735, 1799, 1815, 1852.

It was found that the test statistic $\hat{\delta}_{F_n}$ for the data set by using formula (2.3) $\hat{\delta}_{F_n}$ =3.32808E-047 that we reject H_1 which states that the set of data have NRBU property under significant level at 95% upper percentile.

Example 2. Using the data set given in Grubbs (1971), these data have been used in Shapiro et all (1995). The data set are the times between arrivals of 25 customers at a facility:
1.80, 2.89, 2.93, 3.03, 3.15, 3.43, 3.48, 3.57, 3.85, 3.92, 3.98, 4.06, 4.11, 4.13, 4.16, 4.23, 4.34, 4.37, 4.53, 4.62, 4.65, 4.84,4.91, 4.99, 5.17.

It was found that the test statistic for the data set, by using formula (2.3) is $\hat{\delta}_{F_n}$ =3.431166342E-001, which is less than the critical value in Table 3.1 Then we accept the null hypothesis of exponentiality and not NRBU property at 95% upper percentile.

Example 3. Consider the data Susarla and Vanryzin (1978), which represent 81 survival times (in weeks) of patients of melanoma. Out of these 46 represents non-censored data and the ordered values are:
13, 14, 19, 19, 20, 21, 23, 23, 25, 26, 26, 27, 27, 31, 32, 34, 34, 37, 38, 38, 40, 46, 50, 53, 54, 57, 58, 59, 60, 65, 65, 66, 70, 85, 90, 98, 102, 103, 110, 118, 124, 130, 136, 138, 141, 234.

The ordered censored observations are:
6, 21, 44, 50, 55, 67, 73, 80, 81, 86, 93, 100, 108, 114, 120, 124, 125, 129, 130, 132, 134, 140, 147, 148, 151, 152, 152, 158, 181, 190, 193, 194, 213, 215.

Now, taking into account the whole set of survival data (both censored and uncensored), and computing the statistic from (5.3) censored data, we get $\hat{\delta}^{C}_{F_n}$ =1.010297E-05, which is less than the critical value in Table 5.1 at %95 upper percentile , then , we reject H_1 which states that the set data have NRBU property.

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