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Testing Harmonic Used Better than Aged in Expectation in Upper Tail(HUBAEUT) Class of Life Distributions Using Kernel Method

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Abstract. A new classes of life distribution , namely harmonic used better than aged in expectation in upper tail (HUBAEUT) is introduced. Testing exponentiality against this class is investigated using kernel method.The limiting null and nonnull distribution of the test statistics is normal and the null variance is calculated exactly. Selected critical values are tabulated for sample sizes of 5(1)40. Power of the test are estimated by simulation. the efficacies of the test statistics used for testing against HUBAEUT are calculated for som common alternatives and are compared to some other procedures. It is shown that proposed test is simple, has high relative effciency and power for some commonly used alternatives. The set of real data are used as an examples to elucidate the use of the proposed test statistics for practical reliability.

Key Words : *MHUBAEUT; Exponentiality; Kernel method; Asymptotic normality; Efficiency; Power; Asymptotic relative efficiency.*

1. INTRODUCTION

Let X be a nonnegative continuous random variable with distribution function $F(x)$, survival function $\overline{F} = 1 - F$. At age t, we define the random residual life by X_t with survival function $\overline{F_t} = \frac{\overline{F}(t+x)}{\overline{F}(t)}, x, t \geq 0$. Assume that X has a finite mean $\mu = E(X) = \int_0^\infty \overline{F}(u)du$. Some properties concerning the asympototic behavior of X_t as $t \to \infty$ will be used. Bhattacharjee(1982) gave the follwoing definition.

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Definition 1.1. If X is nonnegative random variable, its survivl function $\overline{F}$ is said to be finitly and positively smooth if a number $\gamma \in (0, \infty)$ exists such that:

$$\lim_{t \to \infty} \frac{\overline{F}(t+x)}{\overline{F}(t)} = e^{-x\gamma}, \tag{1.1}$$

Where γ is called the asymptotic decay coefficient of X. Denoting X_e be a random variable exponentialy distributed by mean $\frac{1}{\gamma}$, the following definitions implies that X_t converges to X_e in distribution written as $X_t \xrightarrow{d} X_e$. This property is usefule for discription of random life times of devices of unknown age.

Difinition 1.2. The distribution F is said to be used better than age UBA if for all $x, t \geq 0$

$$\int_0^\infty \overline{F}(x+t)du \geq \overline{F}(t)e^{-\gamma x}, \tag{1.2}$$

where γ is called is the asymptotic decay of X. From definition 1.2 we have the following definitions:

Definition 1.3. The distribution F is said to be harmonic used better than aged in expectation HUBAE if for all $x, t \geq 0$

$$\int_x^\infty \overline{F}(t)dt \geq \mu e^{-\gamma x}, \tag{1.3}$$

Definition 1.4 The distribution F is said to be harmonic used better than aged in expectation in upper tail HUBAEUT if for all $x, t \geq 0$

$$\int_x^\infty \nu(t)dt \geq \frac{\mu}{\gamma} e^{\gamma x}, \tag{1.4}$$

where $\nu(t) = \int_t^\infty \overline{F}(u)du$ and γ is asympototic decay of X.
We observe that the inequality of (1.4) is achieved when $F(x)$ has an exponential distribution with mean μ equal to the coefficient of the asymptotic decay γ, where the exponential distribution is the only which has the lack of memory property.
Alzaid (1994) showed that UBA class of life distribution is a subclass of the used better than age in expectation class (UBAE) and that if F is IHR (increasing hazard rate) ,then F is UBA. Similar implications between UBAE, NBUE and HNBUE were given by Di Crescenzo(1999). More recently, Willmot and Cai (2000) showed that the UBA class includes the DMRL class (decreasing mean residual life) while the UBAE includes the DVRL (decreasing variance residual life).Thus we have

$$\begin{array}{ccccccccccc} \text{IHR} & \subset & \text{DMRL} & \subset & \text{UBA} & \subset & \text{UBAE} & \subset & \text{HUBAE} & \subset & \text{HUBAEUT} \\ & & & & & & \cup & & & & \\ & & & & & & \text{DVRL} & & & & \end{array}$$

For definitions and details of the classes IHR, NBUE and DMRL, see Barlow and Prochan (1981) and for HNBUE see Klefsjo (1983) while for DVRL see Ab-Youssef (2004).

Testing exponentiality against the classes of life distribution has seen a good deal of attension. For testing against IHR, we refer to Barlow and Proschan(1981) and Ahmad (1994), among others. While testing against DMRL see Ahmad(1992) and testing against DVRl see Abu-yossef (2004). Testing against UBA and UBAE see Ahmad (2004). Finally Testing against HUBAE see Abu-Youssef and El-Batel (2005).

The plan of the rest of this paper is as follows: In section 2 a new test statistics based on a kernel method, for testing H_0: F is exponential against H_1: F is HUBAEUT and not exponential is given. this approches is based on defining a measure of departure from H_0 in favor of H_1 on pdf f(x) and then estimating this measure emperically. the emperical version of this measure require estimating f(x) and thus one may use the celberated kernel method. For background material on this method , we refer to the books by Scott(1992) and Jons and Wand (1995). Using kernel in reliability appears in early work of Wasston and Leadbetter (1994) and Ahmad (1976). While using kernel method for testing NBUC, NBUE, and HNBUE are given by Ahmad et al (1999,2003). Monte Carlo's null distribution critical points for sample sizes $n = 5(1)40$ is investigated in section 3. The Pitman asymptotic efficiency for comon alternatives is obtained in section 4. an example using data from Aboummoh et al (1994) in medical science is introduced in section 5.

2. TESTING HUBAEUT CLASS OF LIFE DISTRIBUTION

The test presented on a sample $X_1, X_2, \ldots, X_n$ from a population with distribution F , we wish to test the null hypothesis H_0 :$\overline{F}$ is exponential with mean μ against $\overline{F}$ is HUBAET and not exponential. Using the inequality (1.4),we may use the following as a measure of departure from H_0 in favor H_1

$$\delta_{hk} = \int_0^\infty [\int_x^\infty \nu(t)dt - \frac{\mu}{\gamma.} e^{-x\gamma}] f(x)dF(x), \tag{2.1}$$

since

$$\int_x^\infty \nu(t)dt = -x\nu(x) - \frac{x^2}{2}\overline{F}(x) + \frac{1}{2}\int_x^\infty u^2 dF(u).$$

Then

$$\delta_{hk} = \int_0^\infty [-x\nu(x) - \frac{x^2}{2}\overline{F}(x) + \frac{1}{2}\int_0^\infty I(U > x)u^2 dF(u) - \frac{\mu}{\gamma} e^{-x\gamma}] f(x)dF(x). \tag{2.2}$$

Note that under $H_0 : \delta_{hk} = 0$, while under $H_1 : \delta_{hk} > 0$. Thus to estimate δ_{ht} by $\hat{\delta}_{hk_n}$, let $X_1, X_2, \ldots, X_n$ be a random sample from F. Let $\overline{F}_n(x) = \frac{1}{n}\sum_{j=1}^n I(X_j > x)$

denote the emperical distribution of the random variable $\overline{F}(x), dF(x) = \frac{1}{n}, \hat{\gamma} = \frac{n}{\sum X_i}$ is the estimate of γ, μ is estimated by sample mean $\overline{X}$ and pdf $f(x)$ is estimated by $\hat{f}_n(x) = \frac{1}{na_n}\sum_j k(\frac{x-X_j}{a_n})$,were $k(.)$ be a known pdf symmetric and bounded with 0 mean and variance $\sigma_k^2 > 0$. Then $\hat{\delta}_{hk_n}$ is given by using (2.2) as

$$\hat{\delta}_{hk_n} = \int_0^\infty [-x\hat{\nu}_n(x) + \frac{x^2}{2}\hat{\overline{F}}_n(x) + \frac{1}{2}\int_0^\infty I(U > x)u^2 dF(u) - \frac{\hat{\mu}}{\hat{\gamma}}e^{-x\hat{\gamma}}]\hat{f}_n(x)dF_n(x), \tag{2.3}$$

i.e

$$\hat{\delta}_{hk_n} = \frac{1}{n^3}\sum_i\sum_j\sum_k K(\frac{X_i - X_k}{a})\left\{(\frac{X_i^2}{2} + \frac{X_j^2}{2} - X_iX_j)I(X_j > X_i) - \frac{X_j}{\hat{\gamma}}e^{-X_i\hat{\gamma}}\right\}, \tag{2.4}$$

where

$$I(x_j > x_i) = \begin{cases} 1, & x_j > x_i \\ 0, & o.w \end{cases}$$

Let us rewrite (2.4) as

$$\hat{\delta}_{hk_n} = \frac{1}{n(n-1)(n-2)}\sum\sum\sum_{i\neq j\neq k} k(\frac{X_i - X_k}{a})\{\frac{(X_i^2}{2} + \frac{X_j^2}{2} - X_iX_j)I(X_j > X_i) - \frac{X_j}{\hat{\gamma}}e^{-X_i\hat{\gamma}}\}. \tag{2.5}$$

To make the test scale invariant, we take

$$\hat{\Delta}_{hk_n} = \frac{\hat{\delta}_{hk_k}}{\overline{X}^2}, \tag{2.6}$$

with measure of departure $\Delta_{hk_k} = \frac{\delta_{hk_n}}{\mu^2}$. Set

$$\phi(X_1, X_2, X_3) = K(\frac{X_1 - X_3}{a})\{[\frac{X_1^2}{2} + \frac{X^2}{2} - X_1X_2]I(X_2 > X_1) - \frac{X_2}{\hat{\gamma}}e^{-X_1\hat{\gamma}}\},$$

and define the symmetric kernel

$$\zeta(X_1, X_2, X_3) = \frac{1}{3}\sum_i \phi(X_{i1}, X_{i2}, X_{i3})$$

where the sum over all arrangment of (X_1, X_2 and X_3). Then δ_{hk} is equivalent to the U-statistic. Since Δ_{hk} and $\frac{\delta_{hk}}{\mu^3}$ have the same limiting distribution, we use $\sqrt{n}(\hat{\delta}_{hk_n} - \delta_{hk})$, the following theorem summarizes the large sample properties of $\hat{\delta}_{hk_n} or U_n$.

Theorem 1.1 If $na_a^4 \to 0$ as $n \to \infty$, if f has bounded second derivative and if $V(\psi_n(X_1)) < \infty$, where $\psi_n(X_1)$ is as given (2.13), then $\sqrt{n}(\hat{\delta}_{hk_n} - \delta_{hk})$ is asymptotically normal with mean 0 and variance $\lim_n V(\psi_n(X_1)$.
Under H_0, the variance = 0.42963
The following simple lemma is needed in the proof of Theorem 1.

Lemma 1.1 Let $\theta_n = E\hat{\delta}_{hk_n}$, then

$$\theta_n = \int_0^\infty E[\hat{f}_n(x)] \int_0^\infty f(x)[-x\nu(x) - \frac{x^2}{2}\overline{F}(x) + \frac{1}{2}\int_0^\infty u^2 I(U > x)dF(u) - \frac{\mu}{\gamma}e^{-x\gamma}]dF(x). \quad (2.7)$$

Proof. Note that $E\hat{f}_n(x) = \frac{1}{a}\int(\frac{x-y}{a})f(y)dy$. Set $g_n(x) = E\hat{f}_n(x)$, thus

$$E\hat{\delta}_{hk_n} = \theta_n = E[\phi_n(X_1, X_2, X_3)] \quad (2.8)$$

where

$$\phi(X_1, X_2, X_3) = k(\frac{X_1 - X_3}{a})\{(\frac{X_1^2}{2} + \frac{X_2^2}{2} - X_1X_2)I(X_2 > X_1) - \frac{X_2}{\gamma}e^{-x\gamma})\},$$

Hence

$$\theta_n = Eg_n(X_1)\{(\frac{X_1^2}{2} + \frac{X_2^2}{2} - X_1X_2)I(X_2 > X_1) - \frac{X_2}{\gamma}e^{-x\gamma})\}$$

$$= \int_0^\infty g_n(x)[-x\nu(x) - \frac{x^2}{2}\overline{F}(x) + \frac{1}{2}\int_0^\infty I(U > x)u^2 dF(u) - \frac{\mu}{\gamma}e^{-x\gamma}]f(x)dF(x). \quad (2.9)$$

Proof of Theorem 1.1 Note that

$$\sqrt{n}(\hat{\delta}_{hk_n} - \delta_{hk}) = \sqrt{n}(\hat{\delta}_{hk_n} - \theta_n) + \sqrt{n}(\theta_n - \delta_{hk}) \quad (2.10)$$

But

$$E\hat{f}_n(x) = \frac{1}{a}\int k(\frac{x-y}{a})f(y)dy = \int k(w)f(x - aw)dw \simeq f(x) + \frac{a^2}{2}f''(x)\sigma_k^2,$$

under the condition assumed on K. Hence

$$\theta_n \simeq \delta_{hk} + \frac{a^2}{2}\sigma_k^2\{\int_0^\infty f''(x)[-x\nu(x) - \frac{x^2}{2}\overline{F}(x) + \frac{1}{2}\int_0^\infty u^2 I(U > x)dF(u) - \frac{\mu}{\gamma}e^{x\gamma}]dF(x)\}. \quad (2.11)$$

Thus $\sqrt{n}(\theta_n - \delta_{hk}) = O(a^2\sqrt{n}) = 0(1)$ by assumptions. Note also $\hat{\delta}_{hk}$ is unbiased estimate of $\theta_n = E\hat{\delta}_{hk}$ and is asymptotically unbiased estimate of δ_{hk_n}. Next, note that

$$\begin{aligned} \sqrt{n}(\hat{\delta}_{hk_n} - \theta_n) &= \sqrt{n}(\frac{1}{n}\sum_{i=1}^{n}\psi_n(X_i)) + (n(n-1)(n-2)^{-1} \\ & \sum\sum_{i\neq j\neq l+k}\sum \xi_n(X_i, X_j, X_k) \end{aligned} \tag{2.12}$$

where

$$\begin{aligned} \psi_n(X_1) &= E[\phi_n(X_1, X_2, X_3)|X_1] + E[\phi_n(X_2, X_1, X_3)|X_1] \\ & +E[\phi_n(X_2, X_3, X_1)|X_1] - 3\theta_n \end{aligned} \tag{2.13}$$

and

$$\xi_n(X_1, X_2, X_3) = \phi_n(X_1, X_2, X_3) - \psi_n(X_1) - 3\psi\theta_n. \tag{2.14}$$

Now, by Layaponouff's central theorem, the first term in the right hand side of (2.14) is asymptotically normal if $L_n = \frac{E[\psi_n(X_1)]^{2+\delta}}{\sqrt{n}}[V(\psi_n(X_1))]^{1+\delta/2} \to 0$ as $n \to \infty$. Now using (2.12) it is easy to see for large n

$$\begin{aligned} E[\phi_n(X_1, X_2, X_3)|X_1] &= f(X_1)\{\frac{1}{2}X_1^2\overline{F}(x) + \frac{1}{2}\int_{X_1}^{\infty} u^2 dF(u) \\ & -X_1\int_{X_1}^{\infty} u dF(u) - \frac{\mu}{\gamma}e^{-\gamma x_1}\}, \end{aligned} \tag{2.15}$$

$$\begin{aligned} E[\phi_n(X_2, X_1, X_3)|X_1] &= \frac{1}{2}\int_0^{X_1} u^2 f^2(u)du + \frac{X_1^2}{2}\int_0^{X_1} f^2(u)du \\ & -X_1\int_0^{X_1} u f^2(u)du - \frac{X_1}{\gamma+2}, \end{aligned} \tag{2.16}$$

Observe that $E[\phi_n(X_2, X_3, X_1)|X_1]$ has the same representation as (2.15). Set $\eta(X_1)$ to be the sum of twice of right hand side of (2.15) plus that of (2.16).

Thus

$$\psi_n(X_1) = \eta(X_1) + O_p(a^2) \text{ say,} \tag{2.17}$$

Hence

$$V(\psi_n(X_1)) = Var(\eta_1(X_1)) + O(a^2),$$

and for $p > 2$,

$$E|\psi_n(X_1)|^p \leq C_p E|\eta(X_1)|^p = O(1).$$

Hence, $L_n \to 0$ as $n \to \infty$ provided that $na^4 \to 0$ as $n \to \infty$.

Next, look at

$$E\left[\frac{\sqrt{n}}{n(n-1)(n-2)}\sum\sum_{i\neq j\neq k}\sum\sum \xi_n(X_i,X_j,X_k)\right]$$
$$= \frac{1}{n(n-1)^2(n-2)^2}\sum\sum_{i\neq j\neq k}\sum$$
$$E[\xi_n(X_i,X_j,X_k)\times\xi_n(X_i,X_j,X_k)]$$
$$= \frac{1}{(n-1)}E\xi_n^2(X_1,X_2,X_3)=O(na)^{-1}=O(1). \quad (2.18)$$

Under H_0, $\overline{F}(x)=e^{-x}$

$$\eta(X_1)=\frac{1}{8}-\frac{7}{12}X+\frac{1}{4}X^2-\frac{1}{8}e^{-2X}. \quad (2.19)$$

Thus $E_0[\eta(X_1)]=0$ and $\sigma_0^2=Var(\eta(X_1)=0.42963$ by direct calculation. The theorem is proved.

3. MONTE CARLO NULL DISTRIBUTION CRITICAL POINTS FOR $\hat{\Delta}_{ht_n}$ TEST

In pratice, simulated percentiles for small samples are commonly used by applied statisticians and reliability analyst. we have simulated the upper percentile points for 95%, 98%, 99%. Table 3.1 gives these percentile points of statistic $\hat{\Delta}_{hk_n}$ in (2.6) and the calculations are based on 5000 simulated samples of sizes $n=5(1)40$. The percentiles values change slowly as n incearses.

To use the above test, calculate $\sqrt{n}\hat{\Delta}_{ht_n}/\sigma_0^2$ and reject H_0 if this exceeds the normal variate value $Z_{1-\alpha}$.

4. ASYMPTOTIC RELATIVE EFFICIENCY (ARE)

We compare our test $\hat{\Delta}_{hk_n}$ in (2.6) to those of smaller classes such as DMRL, and UBAE . Here we choose the tests K^* and $\hat{\delta}_2$ are presented by Hollander and Prochan (1975) and Ahmad (2004) respectively for DMRL and UBAE classes of life distribution. The comparisons are achived by using Pitman asymptotic relative efficiency (PARE), which is defined as follows:
Let T_{1n} and T_{2n} be two statistics for testing H_o: $F_\theta\epsilon\{F_{\theta_x}\},\theta_n=\theta+\frac{c}{\sqrt{n}}$ with c an arbitrary constant, then PARE of T_{1n} relative to T_{2n} is defined by

$$e(T_{1n},T_{2n})=\frac{\mu_1'(\theta_o)}{\sigma_1(\theta_o)}/\frac{\mu_2'(\theta_o)}{\sigma_2(\theta_o)}$$

where $\mu_i'(\theta_o) = \lim_{n\to\infty} \frac{\partial}{\partial\theta} E(T_{in})_{\to\theta_o}$ and $\sigma_i^2(\theta_o) = \lim_{n\to\infty} VarE(T_{in}), i = 1, 2$. Two of the most commonly used alternatives (cf. Hollander and Proschan (1972)) are:

(i) Linear failure rate family : $\bar{F}_{1\theta} = e^{-x-\frac{\theta x^2}{2}}$, $x > 0, \theta > 0$

(ii) Makeham family : $\bar{F}_{2\theta} = e^{-x-\theta(x+e^{-x}-1)}$, $x > 0, \theta > 0$

Table 3.1 Critical Values of $\hat{\Delta}_{hu_n}$

n	95%	98%	99%
5	0.1202	0.2478	0.4105
6	0.1151	0.2244	0.3100
7	0.1106	0.1976	0.2787
8	0.0916	0.1592	0.2288
9	0.0937	0.1595	0.2265
10	0.0869	0.1505	0.2108
11	0.0911	0.1463	0.1998
12	0.0827	0.1356	0.1766
13	0.0773	0.1218	0.1661
14	0.0753	0.1179	0.1502
15	0.0762	0.1192	0.1553
16	0.073	0.1101	0.1369
17	0.0733	0.1095	0.1422
18	0.0717	0.107	0.1387
19	0.068	0.1039	0.1369
20	0.0663	0.0999	0.1259
21	0.0665	0.102	0.1281
22	0.0644	0.0964	0.1208
23	0.0615	0.0904	0.1131
24	0.0622	0.0907	0.1123
25	0.0614	0.087	0.1064
26	0.0583	0.0858	0.1053
27	0.0593	0.088	0.1045
28	0.0615	0.0886	0.1088
29	0.0581	0.0846	0.1046
30	0.0557	0.0796	0.0969
31	0.0551	0.0789	0.0996
32	0.0571	0.0813	0.1018
33	0.0562	0.0803	0.1002
34	0.0519	0.0729	0.0892
35	0.0533	0.0766	0.0922
36	0.0527	0.0735	0.0893
37	0.0496	0.069	0.0871
38	0.0508	0.0720	0.0871
39	0.0497	0.0706	0.0846
40	0.0503	0.0702	0.0854

The null hypothesis is at $\theta = 0$ for linear failure rate and Makeham families respectively. Direct calculations of PAE of K^*, $\hat{\delta}_2$ and $\hat{\Delta}_{hk_n}$ are summarized in Table 4.1.

Table 4.1 Calculations of PAE

Distribution	K^*	$\hat{\delta}_2$	$\hat{\Delta}_{hk_n}$
F_1 Linear failure rate	0.806	0.630	1.4126
F_2 Makeham	0.289	0.385	0.51915

The efficiencies in Table 4.1 show clearly our U-statistic ($\hat{\Delta}_{hk_n}$ perform well for F_1andF_2.

In Tble 4.2 we give PARE's of $\hat{\Delta}_{hk_n}$ with respect to V^* and $\hat{\delta}_2$ whose PAE are mentioned in Table 4.1.

Table 4.2 PARE of $\hat{\Delta}_{hk_n}$ with respect toV^* and $\hat{\delta}_2$

Distribution	$e_{F_i}(\hat{\Delta}_{hk_n}, V^*)$	$e_{F_i}(\hat{\Delta}_{hk_n}, \hat{\delta}_2)$
F_1 Linear failure rate	1.75	2.24
F_2 Makham	1.79	1.34

It is clear that from Table 4.2 the statistic $\hat{\Delta}_{hk_n}$ perform well for $\overline{F}_1$ and $\overline{F}_2$ respectively. Hence our test, which deals the much larger HUBAE is better and also simpler. Finally, the power of the test statistics $\hat{\Delta}_{ht_n}$ is considerd for 95% percentile in Table 3.1 for two of the most commonly used alternatives [see Hollander and Proschan (1975)], they are

(i) Linear failure rate : $\bar{F}_\theta = e^{-x-\frac{\theta x^2}{2}}, \quad x > 0, \theta > 0$

(ii) Weibull : $\bar{F}_\theta = e^{-x^\theta}, \quad x \geq 0, \theta > 0$

Table 4.3 Power estimate of $\hat{\Delta}_{E_{hk}}$

Distribution	θ	Sample Size		
		n=10	n=20	n=30
F_1	1	0.973	0.988	0.997
Linear failure	2	0.982	0.995	0.999
rate	3	0.985	0.997	0.999
F_2	1	0.943	0.936	0.942
Weibull	2	0.955	0.955	0.956
	3	0.951	0.955	0.969

5. NUMERICAL EXAMPLE

Consider the data in Abouammoh et al (1994). These data represent 40 patients suffering from blood cancer from one of the Ministry of Health Hospital in Saudi Arabia and the ordered life times (in days) are 115, 181, 255, 418, 441, 461, 516, 739, 743, 789, 807, 865, 924, 983, 1024, 1062, 1063, 1169, 1191, 1222, 1222, 1251, 1277, 1290, 1357, 1369, 1408, 1455, 1478, 1549, 1578, 1578, 1599, 1603, 1604, 1696, 1735, 1799, 1815, 1852.

Using equation (2.6), the value of test statistics, based on the above data is $\hat{\Delta}_{hu_n} = -3.04 \times 10^{-7}$. This value leads to reject H_1 at the upper percentile 95% see Table 3.1. Therefore the data has not HUBAEUT Property.

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