Run and Go Algorithm for Blind Equalization

자력복구 적응 채널등화기를 위한 Run and Go 알고리즘

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ABSTRACT

In this paper, we propose an adaptation strategy for blind equalizers, which combines a blind algorithm based on high order statistics and the decision directed LMS algorithm. In contrast to ″Stop-and-Go″ algorithm, where adaptation is stopped for unreliable signals, the proposed algorithm applies high order statistics (HOS) blind algorithm to the unreliable signals and applies DD-LMS for the reliable signals. The proposed algorithm, named ″Run-and-Go″ algorithm, inherits minimum MSE performance of DD-LMS and acquisition ability of blind algorithms. Furthermore, by updating the reliable signal region according to signal quality in each iteration, the convergence speed and acquisition ability is further improved.

요 약

이 논문은 high order statistics 에 기반을 둔 자력복구 알고리듬과 판정-LMS 를 조합하는 자력복구 적응 채널등화기를 제안한다. 저 신뢰도 신호에 대하여 적응을 멈추는 기존의 "Stop-and-Go" 알고리듬과는 달리 제안된 알고리듬은 저 신뢰도 신호에 대하여 자력복구 알고리듬 적응을 이용하기에 ″Run-and-Go″ 라고 명명하였으며 판정-LMS의 최소 MSE 성능과 자력복구 알고리듬의 신호 획득능력의 시너지 효과를 가진다. 제안된 알고리듬은 신뢰도 구간을 신호의 질에 따라 자동설정 함으로 수렴 속도와 신호 획득 능력을 각각 개선한다.

key Words : Adaptive Equalization, Blind Equalization, High order statistics, Bussgang algorithm.

Ⅰ. Introduction

In wireless digital broadcasting receivers, to meet increasing demand on high data throughput bind equalization is preferred. Most blind equalizers in digital receivers operate in two different modes, namely, acquisition and tracking.

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In acquisition mode, a blind algorithm based on high order statistics such as CMA is applied to update equalizer coefficients, while in tracking mode decision directed (DD) LMS algorithm is used. Although blind adaptation algorithms can equalize the received data without help of training signals regardless of initialization [1], they usually exhibit inferior MSE performance than DD-LMS algorithm. However, DD-LMS does not possess acquisition ability for the signals other than BPSK [2]. The transition between two different modes are not well

defined, and in most receivers, equalizers re-initialized once the receiver totally fails. Hence, in the presence of dynamic change of multipath channels, a smooth transition between two modes optimizing overall MSE performance is desired.

A straightforward method is to combine two algorithms linearly as done in Benveniste-Goursat Algorithm [3]. Another approach is to stop adaptation for the unreliable signals as in "Stop-and-Go" algorithm [4] preceeded by many variants, for example [5]. All these approaches fail to obtain both fast convergence and re/acquisition ability at the same time.

In this paper, we propose a blind adaptation algorithm named "Run-and-Go" to acquire the MMSE of DD-LMS in the absence of decision error and the (re)acquisition ability blind algorithms. First, we define an adjustable reliable region, parameterized by a single parameter spans [0,1], as the closeness to the detected symbols. Contrast to the "Stop-and-Go" algorithm, in the proposed method, hence named "Run-and-Go" Algorithm, a blind adaptation is applied instead of stopping adaptation for the unreliable region. Furthermore, the unreliable region is updated corresponding to the quality of the output signals measured by cluster variance. As the quality of signal improves, for example the blind portion of "Run-and-Go" equalizes the received signal, cluster variance increases and the unreliable signal region shrinks, and as the quality of signal deteriorates, for example channel varies abruptly and cluster variance increases, the unreliable region expands.

In section II we briefly review Bussgang-Type blind adaptation algorithms. We present the Run-and-Go algorithm with definition of reliable regions mainly for PAM signals. In section III reliable region optimization is considered based on minimization of the cluster variance. Section IV presents simulation results and section V concludes.

II. Run and Go Algorithm.

Let us consider a digital communication receiver system equipped with an adaptive equalizer (Figure 1).
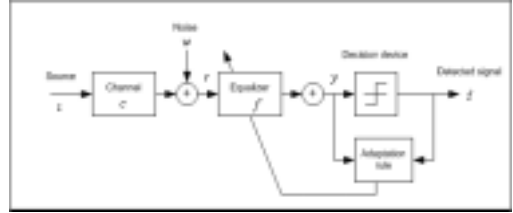

Figure 1. Communication system with an adaptive equalizer

Let r_k denote received signal at time k, f be equalizer of length N_f, and y_k denote equalizer output. We assume that f is a fractionally spaced equalizer, which can achieve perfect linear equalization (down-sampling is omitted in the Figure 1). The adaptation rule for f can be written in a general form as the following [6],

$$f_{n+1}(k) = f_n - \mu e(k) r*_k, \qquad (1)$$

where $e(k)$ denotes the error term. Blind algorithms based on high order statistics of the equalizer output that have the general form

$$e(k) = g_1(y_k) - g_2(y_k) \qquad (2)$$

where g_1 and g_2 are memory-less nonlinear functions, are called Bussgang-type. Bussgang-type algorithms include most popular blind algorithms such as CMA [7], where $e(k) = y_k |y_k|^2 - \gamma y_k$ and $\gamma = \dfrac{E(s^4)}{E(|s|^2)^2}$, Sato Algorithm [8], where $e(k) = y_k - \gamma \operatorname{sgn}(y_k)$ and $\gamma = \dfrac{E(s^2)}{E(|s|)}$. It is well known that many Bussgang-type adaptive algorithms perform well for the communication signals [9]. Especially, global convergence of CMA algorithm has been intensively studied [1]. DD-LMS is also a Bussgang-type algorithm ($e(k) = y_k - \widehat{y_k}$), but it

fails to globally converge, unlike CMA or Sato Algorithm [2].

Stop-and-Go algorithm is one of Bussgang-type algorithms which can be viewed as a combination of Sato Algorithm and DD-LMS,

$$e_{SG}(k) = \frac{1}{2} e_{dd} + \frac{1}{2} |e_{dd}| \mathrm{sgn}(e_s), \qquad (3)$$

where e_s denotes the error term of Sato Algorithm. It is devised in order to obtain global convergence of blind algorithm and, at the same time, maintain MSE performance of DD-LMS. As the name suggested, Stop-and-Go algorithm stops adaptation (i.e. $e_k = 0$) for the signal belongs to unreliable region which is defined as the following

$$\{y_k | \mathrm{sgn}(e_s(y_k)) \neq \mathrm{sgn}(e_{dd}(y_k))\} \qquad (4)$$

As a result, Sop-and-Go algorithm exhibits improved global convergency by sacrificing convergence speed.

Inspired by Stop-and-Go algorithm, we consider a blind algorithm which combines CMA or Sato Algorithm and DD-LMS in a way that the resulting algorithm applies CMA for the unreliable region instead of stopping, named "Run-and-Go." In order to develop this algorithm, fist, we define reliable region and unreliable region for the equalizer output. Without much loss of generality we define them for PAM constellations, since for a QAM constellation can be viewed as a pair of PAMs.

For $2M$-level PAM signals, let Γ be the radius of the constellations, i.e.

$$s_k \in \{(2M-1)\Gamma, \ldots, \Gamma, -\Gamma, \ldots, -(2M-1)\Gamma\} \qquad (5)$$

Recall that for $2M$-PAM the hard decision $\widehat{y_k}$ is given by

$$\widehat{y} = \arg{}_{2(k-1)\Gamma} \min |y - \{2(k-1)\Gamma\}| \qquad (6)$$

We partition the received signal set into two sets with respect to a parameter $\lambda \in [0,1]$ as the following,

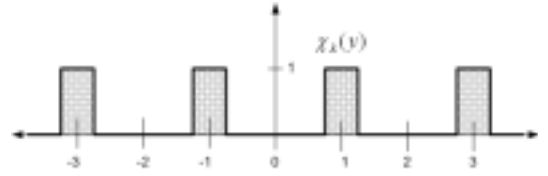$$R_\lambda = \{y_k \ | y_k - \widehat{y_k}| < (1-\lambda)\Gamma\}, \qquad (7)$$

Figure 2. Illustration of set partition for 4-PAM and $\lambda = 1/4$.

Now, we define an error term parameterized by a parameter $\lambda \in [0,1]$ as the following

$$e_\lambda(k) = \begin{cases} y_k - \widehat{y_k} & \text{for } |y_k - \widehat{y_k}| \geq \Gamma \\ y_k(|y_k|^2 - \gamma) & \text{else} \end{cases} \qquad (8)$$

This error term blends CMA and DD-LMS. Notice that Run-and-Go algorithm is also Bussgang-type, by defining a characteristic function (Figure 2),

$$\chi_\lambda(y) = \begin{cases} 1 & y \in R_\lambda, \\ 0 & else \end{cases} \qquad (9)$$

Run-and-Go error term can be expressed as

$$\begin{aligned} e_\lambda(k) = (y_k - \widehat{y_k})\chi_\lambda(y_k) \\ + y_k(|y_k|^2 - \gamma)(1 - \chi_\lambda(y_k)) \end{aligned} \qquad (10)$$

We can also consider to blend Sato Algorithm and DD-LMS.

$$e_\lambda(k) = \begin{cases} y_k - \widehat{y_k} & \text{for } |y_k - \widehat{y_k}| \geq \Gamma \\ \gamma \mathrm{sgn}(y_k) - y_k & \text{else} \end{cases} \qquad (11)$$

Figure 3 illustrates the error function of CMA Run-and-Go algorithm for 4-PAM case. The piece-wise linear function is the error function of

DD-LMS, the qubic curve is that of CMA, and the bold line denotes the Run-and-Go error function for $\lambda=0.5$.

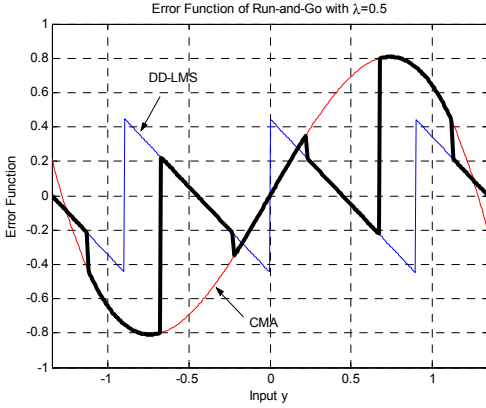Figure 3. Error Function of Run-and-Go with λ=0.5

Figure 3. Error Function of CMA Run-and-Go Algorithm for $\lambda=0.5$.

We also consider a variant of Run-and-Go algorithm, i.e. stopping adaptation for the reliable region.

$$e_\lambda(k)=\begin{cases} 0 & \text{for } |y_k-y_k|<\lambda\Gamma \\ y_k(|y_k|^2-\gamma) & \text{else} \end{cases} \qquad (12)$$

By observing the convergence of this "Run-and-Stop" algorithm, we can verify the convergence of Run and Go algorithm (Figure 4).

III. Reliable Region Optimization

We consider optimization method for λ corresponding to the signal quality. The signal quality is measured via Cluster Variance (CV). CV, denoted by G is given by

$$C=E|y_k-\widehat{y_k}|^2 \qquad (13)$$

CV at a given time k, C_k is ensemble average of the error square, but estimated by the following integration

$$C_{k+1}=(1-\rho)C_k+\rho|y_k-\widehat{y_k}|^2 \qquad (14)$$

In this optimization method, we are interested in converting adaptation mode to the DD-LMS as soon as possible and applying the blind algorithm when DD-LMS fails. Assuming $y_k-\widehat{y_k}$ is Gaussian for mild multipath channels, which is reasonable assumption due to the Central Limit Theorem, we have $P\{|y_k-\widehat{y_k}|<3\sigma\}=0.99$, where σ^2 is the variance of $y_k-\widehat{y_k}$ [10]. Hence, we declare open-eye when $3\sqrt{C}<\Gamma$, since C is an estimation of σ^2. Hence we set $\lambda=0$ for $3\sqrt{C}<\Gamma$. In the severe case ISI, we assume that $y_k-\widehat{y_k}$ distributes uniformly over $[-\Gamma,\Gamma]$. In such case $E|y_k-\widehat{y_k}|^2=\int_{-\Gamma}^{\Gamma}\frac{x^2}{2\Gamma}dx=\frac{1}{3}\Gamma^2$, and we declare closed eye for $3C>\Gamma^2$ and set $\lambda=1$. Consequently, we declare

$$\begin{aligned} &\text{OpenEyes}\ :\ C<\frac{\Gamma^2}{9} \\ &\text{ClosedEyes}:\ C>\frac{\Gamma^2}{3} \end{aligned} \qquad (15)$$

Notice that $0\le C\le\Gamma^2$, since in the worst case we have $y_k=\pm\Gamma$.

We set λ such that $\lambda=0$ for open eyes and $\lambda=1$ for closed eyes, and for $\frac{\Gamma^2}{9}\le C\le\frac{\Gamma^2}{3}$ let λ varies linearly between the interval [0,1], corresponding to the estimate of G i.e.

$$\lambda=\min\left[\max\left[\frac{3}{2}\left(1-\frac{3}{\Gamma^2}C\right),0\right],1\right] \qquad (16)$$

Hence, the over-all adaptation rule for Run and Go algorithm using Open-Eye optimization is

$$f_{n+1}(k) = f_n + \mu e_\lambda(k) r*_k \qquad (17)$$

$$C_{n+1} = (1-\rho)C_n + \rho|y_k - \widehat{y_k}|^2$$

$$\lambda_n = \min\left[\max\left[\frac{3}{2}\left(1 - \frac{3}{\Gamma^2}C\right), 0\right], 1\right]$$

IV. Simulation Results

In this section we present simulation results of Run-and-Go algorithm. The source constellation considered here is 8-PAM signal (8-VSB for HDTV). The channel considered here is a fairly severe $T/2$-spaced multipath channel

$$c_1 = [0.1, 1, 0.1, 0.2, -0.1, -0.2, 0.5, 0.1]. \qquad (18)$$

under 30dB SNR.

Figure 4 shows CV trajectory curves of a CMA Run-and-Stop equalizer outputs for fixed λ's with 30dB SNR. Except $\lambda=1$ case, where the equalizer is updated via DD-LMS,, Run and Stop algorithm achieved open-eye constellation. Interestingly, $\lambda=0.2, 0.4, 0.6, 0.8$ performs better than CMA only case ($\lambda=0$).
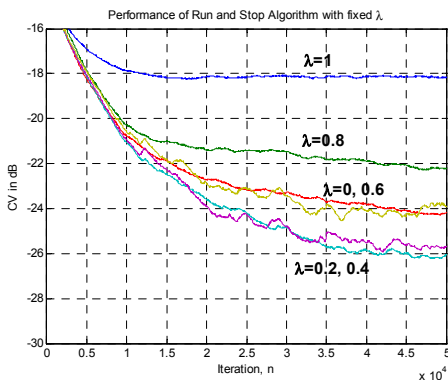

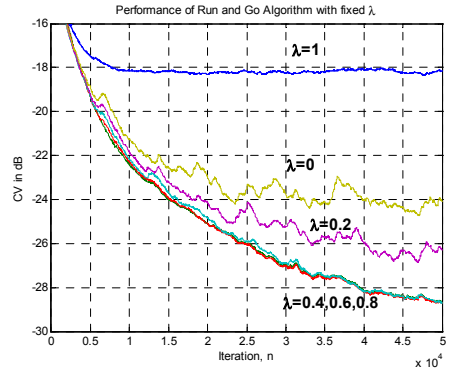Figure 4. Convergency performance of CMA Run-and-Stop Algorithm for various λ's.

Figure 5. Convergency performance of CMA Run-and-Go Algorithm for various λ's.

Figure 5 presents CV trajectories of a CMA Run-and-Go equalizer outputs for the same λ's considered in the previous simulation. The CMA-only case ($\lambda=0$) and DD-LMS only case ($\lambda=1$) show the same CV performance (about -24dB and -18dB, respectively), but other λ's achieve better performance than Run-and-Stop case. These two simulation results show the convergence performance of Run-and-Go algorithm for various fixed λ's.

Now, in the following simulation we consider adaptation of λ as well. We consider a severe multipath channel,

$$c_2 = [-0.1, 0.2, -0.3, 1, -0.3, 0.2, 0.6, \qquad (19)$$
$$-0.2, 0.1, -0.2, 0.1, -0.1, 0.8, 0.2],$$

and apply a decision feedback equalizer. For $\lambda=1$ the CV trajectory shows that DD-LMS cannot equalize the received data. For $\lambda=0$, Run-and-Go with CMA adaptation barely achieves open-eye with unsatisfactory MSE performance. The Run-and-Go with $\lambda=0.6$ achieves the minimum MSE, but converges slowly. The adaptive λ Run-and-Go achieves fast convergence of CMA and at the same time MMSE of $\lambda=0.6$ case.
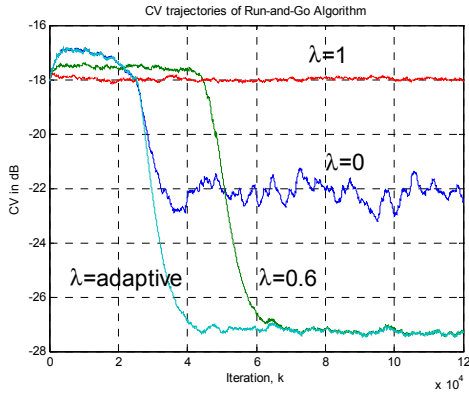
Figure 6. Convergency performance of CMA Run-and-Go Algorithm with adaptive λ.

In order test the self-configuration ability of Run-and-Go algorithm, during transmission under channel c_1 in (18), we replace the multipath channel c_1 with a second channel defined as

$$c_3 = [-0.15, 1, -0.1, 0.2, 0.6, -0.2, 0.1, 0.2] \qquad (19)$$
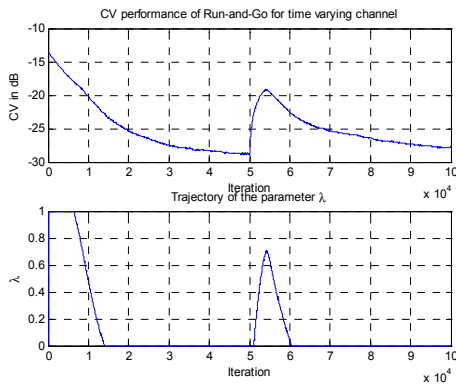
abruptly at the 50,000th sample.

Figure 7. CV and λ Trajectories of CMA Run-and-Go Algorithm in the presence of time-varying multipath.
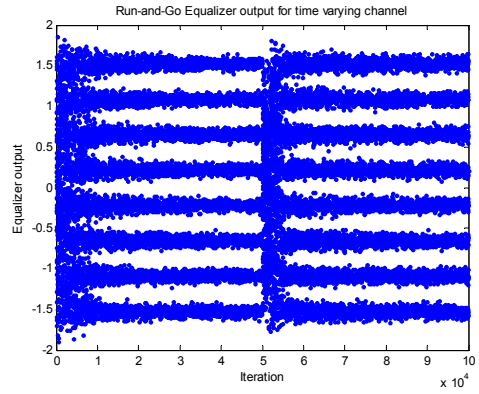

Figure 8. Equalizer output of Run-and-Go algorithm

Figure 7 shows the CV and λ trajectories of CMA Run-and-Go equalizer from the cold-start and re-acquisition, as Run-and-Go changes from CMA mode to DD-LMS mode. In this simulation, the λ corresponding to the signal quality is given by open-eye optimization method. Notice that for the CV over -18dB, which is $10\log_{10}(\Gamma^2/3)$ with $\Gamma = 0.2182$ for the normalized 8-PAM signal, λ is clipped to 1 and for the CV under -23dB ($10\log_{10}(\Gamma^2/3)$) λ is clipped to 0, as shown in Figure 5-b). Figure 8 shows the equalizer outputs open eyes at the point where λ is about to fall back from 1. When the channel has been abruptly changed, CV increased and correspondingly λ raised, and the adaptation mode changed to blind mode. As blended blind adaptation successfully equalizes the received signals to open eye, the λ decreases as CV decreases and switched to DD-LMS mode.

Ⅴ. Conclusion

In this paper, we have proposed a configurable blind adaptation algorithm for equalization combining existing blind equalization algorithms and DD-LMS in order to have both rapid (re)acquisition ability and optimal MSE performance. The proposed "Run-and-Go" algorithm

apply existing blind algorithms such as CMA or Sato Algorithm for the unreliable signal region and apply DD-LMS for the reliable signal region, and the reliable region is configured by the estimated signal quality. Simulation results for static multipath channels and dynamic multipath channels confirmed the performance of proposed algorithm.

Reference

[1] Johnson, Jr. et. al. ″Blind Equalization using Constant Modulus Criterion: A Review,″ *Proceedings IEEE,* vol. 86, October 1998.

[2] J. Mazo, ``Analysis of decision-directed equalizer convergence,'' *Bell Syst. Tech. J.,* pp. 1857--1876, Dec. 1980.

[3] A. Benveniste and M. Goursat, ″Blind equalizers,″ *IEEE Transactions on Communications*, vol COM-32, no. 8, pp-871-883, August 1984.

[4] G. Picchi and G. Parti, ″Blind equalziation and carrier recovery using a ″stop-and-go″ decision-directed algorithm,″ *IEEE Transactions on Communications,* vol COM-35, no. 9, September 1987,

[5] D. Hatzinakos, ``Blind equalization using stop-and-go adaptation rules,'' *Optical Engineering*, pp. 1181--1188, June 1992.

[6] J. R. Treichler, C. R. Johnson, Jr., and M.~G. Larimore, *Theory and Design of Adaptive Filters*, Prentice Hall, 2001.

[7] D. N. Godard, ″Self-recovering equalization and carrier tracking in two dimensional data communication systems,″ *IEEE Transactions on Communications.* vol 28, no, 7, July 1997.

[8] Y. Sato ″A method of self-recovering equalization for multilevel amplitude modulation systems,″ *IEEE Transactions on Communications,* June 1975.

[9] A. Benveniste, M. Goursat, and G. Ruget, ″Robust identification of a nonmimum phase system: Blind adjustment of a linear equalizer in data communications″ IEEE Transactions on Automatic Control, vol. 25, no. 3, June 1980.

[10] S. S. Wilks, *Mathematical Statistics*. Wiley, New York, 1962.

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