Lateral Vehicle Control Based on Active Flight Control Technology

Young Bong Seo, Jae Weon Choi*
School of Mechanical Engineering and Research Institute of Mechanical Technology, Pusan National University, Pusan 609-735, Korea
Guang Ren Duan
Center for Control Theory and Guidance Technology, Harbin Institute of Technology, Harbin 150001, China

In this paper, a lateral vehicle control using the concept of control configured vehicle (CCV) is presented. The control objectives for the lateral dynamics of a vehicle include the ability to follow a chosen variable without significant motion change in other specified variables. The analysis techniques for decoupling of the aircraft motions are utilized to develop vehicle lateral control with advanced mode. Vehicle lateral dynamic is determined to have the steering input and control torque input. The additional vehicle modes are also defined to using CCV concept. We use right eigenstructure assignment techniques and command generator tracker to design a control law for an lateral vehicle dynamics. The desired eigenvectors are chosen to achieve the desired decoupling (i.e., lateral direction speed and yaw rate). The command generator tracker is used to ensure steady-state tracking of the driver's command. Finally, the developed design is utilized by using the lateral vehicle dynamic with four wheel.

Key Words: Active Control Technology (ACT), Control Configured Vehicle (CCV), Flight Control System, Lateral Vehicle Control, 4WS, Eigenstructure Assignment, Command Generator Tracker, Motion Decoupling, Multi-variable Control

1. Introduction

Advanced aircraft such as control configured vehicles (CCV) provides the capability to accomplish the desired flight movement. In order to develop advanced aircraft, active control technology is imported in the early state of flight design. Active control technology make a remarkable improvement in both flight capability and maneuverability by adding new loop to traditional SISO system, that is an extended MIMO system.

The elevator, rudder, flapron and canard of advance aircraft assign the aircraft to a new degree of freedom (longitudinal mode and lateral mode) which is related to position maintenance and change under altitude and direction, and make a flight movement to the specific direction (Siouris et al., 1995 ; Sobel and Shapiro, 1985a ; 1985b). Though these movement characteristics are above pilot's ability, it can be realized in combination with computer control. Control technology also ensure the stability problem such as fault management.

Meanwhile, vehicle has the basic movement characteristics such as acceleration, deceleration, moving, lane change, rotation and stop by the handle, actuator, and brake of vehicle. Vehicle control efforts have been focused on improving the vehicle maneuverability and the straight line stability (Kachroo and Tomizuka, 1995 ; Matsumoto et al.,

* Corresponding Author,
E-mail : choijw@pusan.ac.kr
TEL : +82-51-510-2470; **FAX** : +82-51-514-0685
School of Mechanical Engineering and Research Institute of Mechanical Technology, Pusan National University, Pusan 609-735, Korea. (Manuscript **Received** October 25, 2005; **Revised** April 24, 2006)

1991 ; Park and Kim, 1998 ; Wang and Tomizuka, 2001). But, the previous research has been conducted based on a basic movement of vehicle. In recent years, many researcher set a new trend toward driver-depended control in vehicle (Alleyne, 1997 ; Cho and Kim, 1996 ; Dugoff et al., 1970 ; Smith and Starkey, 1995 ; Will and Zak, 1997).

In this paper, the CCV concept for vehicle was expanded from the previous CCV concept for aircraft. The CCV vehicle is assumed that it can be developed from the early state of vehicle design as well as hardware addition or modification after vehicle design. We define that the CCV mode is the additional vehicle movement based upon the independent steering input of front/rear wheel. Vehicle lateral dynamic is determined to have the steering input and control torque input. The additional vehicle modes is also defined to using CCV concept. We use right eigenstructure assignment techniques and command generator tracker to design a control law for an lateral vehicle dynamics. The desired eigenvectors are chosen to achieve the desired decoupling (i.e., lateral direction speed and yaw rate). The command generator tracker is used to ensure steady-state tracking of the driver's command. The control objectives for the lateral dynamics achieve the ability to follow a chosen variable without significant motion change in other specified variables.

In section 2, the CCV modes for an advanced aircraft are described simply. The lateral vehicle dynamics with four wheel are described in Section 3. The mathematical formulation of the right eigenstructure assignment techniques and command generator tracker are described in section 4. The developed design is illustrated by using the lateral vehicle dynamic with four wheel in section 5. Finally, Section 6 summarizes the main results and conclusions.

2. CCV Mode for an Advanced Aircraft

As mentioned above, control configured vehicles (CCV) have been developed to provide the desired flight movement using active control technology. Active control technology make a re-

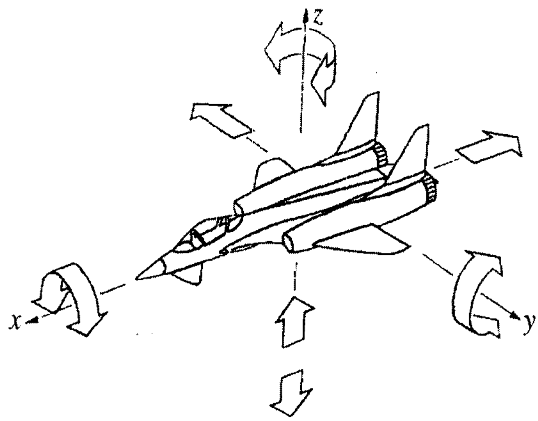


Fig. 1 The movement of 6-degree of freedom

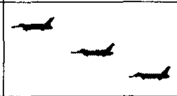
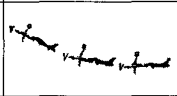
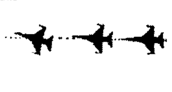
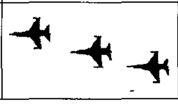
markable improvement in both flight capability and maneuverability by adding new loop to traditional SISO system, that is an extended MIMO system.

The existing aircraft has been performed the rotation movement on x, y, z-axis and the movement of 4-degree of freedom on the airframe based on elevator, rudder and flapron. Also, aircraft has been performed the similar translation movement according to indirect control accompanied by body rotation.

On the other hand, advanced aircraft using ACT have been added y, z-direction movement in Fig. 1 which can move the direction of 6-degree of freedom independently. In order to control the additional degree of freedom, the additional control input devices must be developed in advance.

For the longitudinal dynamics of a control configured vehicle, the flapron and elevator form a set of redundant control surfaces capable of decoupling normal control forces and pitching moments. The decoupled motions include pitch pointing, vertical translation, and direct lift control. Pitch pointing is characterized by pitch attitude command without a change in flight angle, that is $\Delta\theta=\Delta\alpha$, $\Delta\gamma=0$ where $\gamma=\theta-\alpha$, $\Delta\theta$ is the variation of pitch angle, $\Delta\alpha$ is the variation of angle of attack and $\Delta\gamma$ is the variation of flight path angle. Vertical translation is characterized by flight path command without a change in pitch attitude, that is $\Delta\gamma=\Delta\alpha$, $\Delta\theta=0$. Direct lift con-

Table 1 The longitudinal/lateral dynamics of CCV

Pointing Mode	Translation Mode	Direct Force Mode
Pith Pointing	Vertical Translation	Direct Lift
		
Yaw Pointing	Lateral Translation	Direct Sideforce or Flat Turn
		

trol is characterized by normal acceleration command without a change in the angle of attack, that is $\Delta\theta=\Delta\gamma$, $\Delta\alpha=0$.

For the lateral dynamics of control configured vehicle, the vertical canard and rudder form a set of redundant surfaces that is capable producing lateral forces and yawing moments independently. The decoupled motions include yaw pointing, lateral translation, and direct sideforce. Yaw pointing is characterized by heading command without a change in lateral directional flight path angle, that is $\Delta\Psi=\Delta\beta$, $\Delta\Psi_g=0$ where $\Psi=\Psi_g-\beta$, $\Delta\Psi$ is the variation of yaw angle, $\Delta\beta$ is variation of the slip angle and $\Delta\Psi_g$ is the variation of flight path angle on the ground. Lateral translation is characterized by lateral directional flight path command without a change in leading, that is $\Delta\Psi=\Delta\beta$, $\Delta\Psi=0$. Direct sideforce is characterized by lateral acceleration command without a change in sideslip angle, that is $\Delta\Psi=\Delta\Psi_g$, $\Delta\beta=0$. All three lateral modes also require that there are be no change in bank angle.

Table 1 describes the longitudinal/lateral dynamics of a control configured vehicle.

3. Lateral Vehicle Dynamics with Four Wheel

3.1 Vehicle model via the dugoff tire model

The linear Dugoff tire model (Dugoff et al., 1970) is one of the simplest tire models used in simulation. It states that the force exerted by a tire is proportional to slip angle of the tire. The slip angle in Fig. 2 is the angle that the tire is making relative to the moving direction. To solve for the

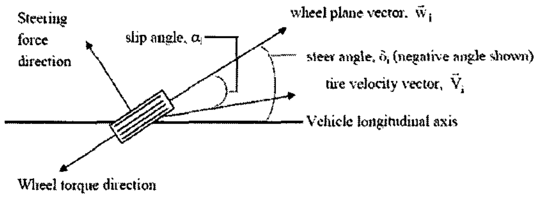


Fig. 2 A definition of the tire slip angle

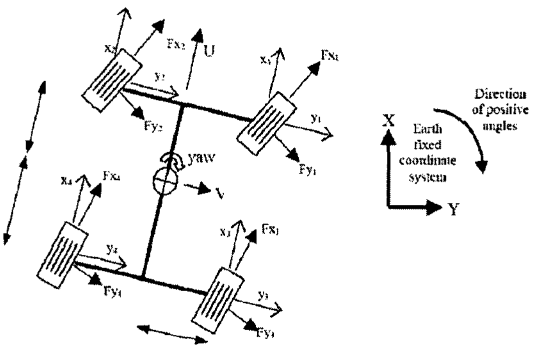


Fig. 3 A diagram showing the definition of the dimensions used the obtain vehicle lateral model

wheel forces, it is assumed that the torque inputs to each wheel act directly in the wheel plane, and that the steering forces act perpendicular to the wheel plane. Note that the torque input could either be a brake or acceleration input. The steering forces are assumed to be proportional to the wheel slip angle.

There is some contention over the best coordinate system to use to vehicle dynamics as depicted in Fig. 3, and the answer to this question depends too strongly on the application to allow an answer. For the "in-plane" dynamics under study, moving coordinate systems are most often utilized that are fixed to the vehicle's center-of-gravity. This allows the equations of motion to be rewritten so that the dynamics are studied in relation to the car (which in fact is the location that they are most measured from). It is important to note that the coordinate system is not fixed to the body, then the angular velocity of the vehicle is not the angular velocity of the coordinate system. To fix this problem, the preferred methods to orient the coordinate system along the vehicle's longitudinal axis so that moments of inertia of the body are constant. If a fixed (stationary) coordinate system were used, the moments of inertia would vary as the body changes orientation to the fixed axes. The first difficulty is in defining the acceleration of the moving coordinate system. It is often necessary to transform a vector from one coordinate system to another.

With this transformation, the control inputs are completely decoupled and the tire forces are now linear in the inputs and states (Alleyne, 1997; Smith and Starkey, 1995). Finally, we can now conclude the model development by representing the system in state-space format as:

$$F=A_F\bar{x}+B_F\bar{u} \tag{1}$$

where

$$F=[x_1\ x_2\ x_3\ x_4\ y_f\ y_r]^T$$

$$\bar{x}=[V\ \dot{\Theta}]^T$$

$$\bar{u}=[\delta_f\ \delta_r\ \Delta T_1\ \Delta T_2\ \Delta T_3\ \Delta T_4]$$

$$A_F=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\dfrac{2C_{\alpha f}}{U} & -\dfrac{2aC_{\alpha f}}{U} \\ -\dfrac{2C_{\alpha r}}{U} & \dfrac{2bC_{\alpha r}}{U} \end{bmatrix}$$

$$B_F=\begin{bmatrix} 0 & 0 & \frac{1}{r} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{r} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{r} \\ \frac{2T_{nom,f}}{r}\delta_f+2C_{rf} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2T_{nom,r}}{r}\delta_f+2C_{\alpha r} & 0 & 0 & 0 & 0 \end{bmatrix}$$

The linear vehicle dynamics can now be derived by substituting the above expression into system equation, $\dot{\bar{x}}=A_X\bar{x}+B_XF$

$$\begin{aligned}\dot{\bar{x}}&=A_X\bar{x}+B_X(A_F\bar{x}+B_F\bar{u})\\&=(A_X+B_XA_F)\bar{x}+B_XB_F\bar{u}\\&=A\bar{x}+B\bar{u}\end{aligned} \tag{2}$$

where,

$$A=\begin{bmatrix} -2\dfrac{C_{\alpha f}+C_{\alpha r}}{mU} & -U-2\dfrac{aC_{\alpha f}-bc_{\alpha r}}{mU} \\ -2\dfrac{aC_{\alpha f}-bC_{\alpha r}}{I_zU} & -2\dfrac{a^2C_{\alpha f}+b^2C_{\alpha r}}{I_zU} \end{bmatrix}$$

$$B=\begin{bmatrix} \frac{2}{m}\left(\frac{T_{nom,f}}{r}+C_{\alpha f}\right) & \frac{2}{m}\left(\frac{T_{nom,r}}{r}+C_{\alpha r}\right) & 0 & 0 & 0 & 0 \\ \frac{2a}{I_z}\left(\frac{T_{nom,f}}{r}+C_{\alpha f}\right) & -\frac{2b}{I_z}\left(\frac{T_{nom,r}}{r}+C_{\alpha r}\right) & -\frac{d}{I_zr} & \frac{d}{I_zr} & -\frac{d}{I_zr} & \frac{d}{I_zr} \end{bmatrix}$$

and:

m =the vehicle mass

V =the vehicles velocity (assumed to be primarily in longitudinal direction)

δ_i =the steer input into the i^{th} vehicle

a, b =the longitudinal distance from the C.G. to the front and rear axle

Ψ =the yaw angle of the vehicle, measured w.r.t. the ground

I_z =the moment of inertia about the z-axis

$C_{\alpha f}, C_{\alpha r}$=front and rear wheel cornering stiffness

$T_{norm,f,r}$=the nominal torque produced by the front and rear tires

ΔT_i=the controlled torque input into the i^{th} tire

3.2 CCV mode of the vehicle

The operate mode for vehicle are defined as 'negative, neutral, positive' mode as depicted in Fig. 4.

For the negative mode, the front wheel steer against the rear wheel in low vehicle speed, that is $\delta_f=-\delta_r=\alpha$, where δ_f is the variation of front wheel steering, δ_r is the variation of rear wheel steering and α is the variation of steering angle. In neutral mode, both direction of the front wheel and the rear wheel maintain the same neutral direction, i.e., $\delta_f=\delta_r=0$. Finally, in the positive mode, the front wheel and rear wheel steer with same direction in high vehicle speed, that is $\delta_f=\delta_r=\alpha$. This allows the new vehicle motion to be

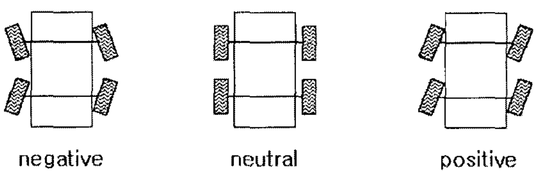


Fig. 4 The operate modes of vehicle

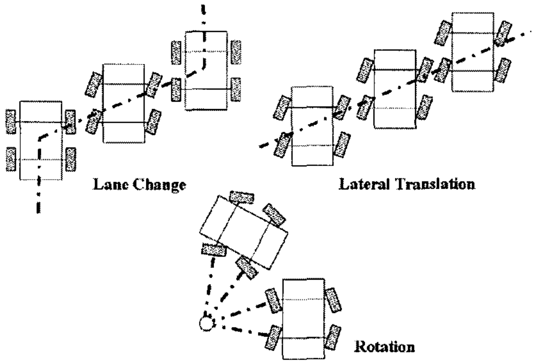


Fig. 5 CCV modes of the vehicle

re-defined so that the CCV run mode are studied in relation to the car. Figure 5 shows the possible mode on car, four wheel steering is changed according to speed.

(1) Lane Change Mode

Maintaining level moving, change yaw angle temporarily, it can be change moving path

$$\delta_f=\delta_r=0 \rightarrow \delta_f=\delta_r=a \rightarrow \delta_f=\delta_r=0$$

(2) Lateral Translation Mode

With only change yaw angle, it can be go straight on ahead

$$\delta_f=\delta_r=a$$

(3) Rotation Mode

Front wheel steer against the rear wheel, it can be moving circular path

$$\delta_f=-\delta_r=a$$

4. Formulations of CCV Mode Control

Consider the linear, time-invariant continuous-time controllable and observable system model described by the matrix differential equation (also known as the matrix triple (F, G, C)

$$\dot{x}(t)=Fx(t)+Gu(t) \quad (3)$$

$$y(t)=Cx(t) \quad (4)$$

where $x(t)=(x_1, x_2, \cdots, x_n)^T$ is an n-vector, called the system state vector, $u(t)=(u_1, u_2, \cdots, u_m)^T$ is an m-vector, called the control function $(m \leq n)$ and $y(t)$ is the r vector, called the output vector. F and G are constant, $n \times n$ and $n \times m$ matrices, respectively. C is also a constant, $r \times n$ matrix. We assume that $rank\, G=m \neq 0$ and $rank\, G=r \neq 0$. The system model for the present investigation is illustrated in Fig. 6. In Fig. 6, the control input takes the form

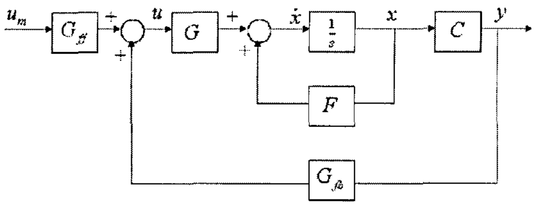


Fig. 6 Feedback/feedforward control diagram

$$u=G_{ff}u_m+G_{fb}y \quad (5)$$

where G_{ff} is the feedforward gain matrix and G_{fb} is the optimal feedback gain matrix. The techniques of eigenstructure assignment and command generator tracking have been employed to compute G_{fb} and G_{ff}, which will insure that the pilot's command is followed by properly distributing it to the necessary control surfaces. Essential to the command generator tracker is the computation of the required feedforward gain matrix G_{ff}, whereas the feedback gain matrix G_{fb} is determined by eigenstructure assignment technique as illustrated in Figure 6. The transformation $G_{ff}u_m$ of the pilot command u_m allows the dynamic state equation to be written by

$$\dot{x}(t)=(F+GG_{fb}C)x(t)+GG_{ff}u_m \quad (6)$$

4.1 Output eigenstructure assignment

Eigenstructure assignment is well-suited for incorporating the classical specifications on damping, settling time, and mode or disturbance decoupling into a modern multivariable control framework (Sobel and Shapiro, 1985a), and has been shown to be a useful tool for flight control design (Sobel and Shapiro, 1985a; Andry et al., 1983). The eigenstructure assignment technique is used to design flight control laws for aircraft with many control efforts, and the technique together with suitable feedforward design can be achieve static decoupling with internal stability, which is an important requirement in many flight control system (Sobel and Shapiro, 1985a; 1985b).

The eigenstructure assignment method can be divided into two groups (Choi et al., 1994), that is, the right eigenstructure (eigenvalue/right eigenvectors) assignment and the left eigenstructure (eigenvalue/left eigenvectors) assignment, and their roles in designing a control system are distinctly different. The right eigenstructure assignment is widely used to solve the mode decoupling problem (Siouris et al., 1995 ; Andry et al., 1983 ; White, 1991), and a design a controller for the vibration suppression of flexible structures (Liebst and Garrard, 1986), and can be applied to disturbance decoupling problems (Wonham, 1989). On the other hand, the left eigenstructure is used to define the controllability measure (Choi et al., 1995a ; Hamdan and Nayfeh, 1989) and also can be used to design an effective and dis turbance suppressible controller (Choi et al., 1995 ; Zhang et al., 1990).

Given a self-conjugate set of desired eigenvalues $\{\lambda_i\}$, $i=1, 2, \cdots, r$ and a corresponding self-conjugate set of desired eigenvectors $\{\phi_i^d\}$, $i=1, 2, \cdots, r$, find a real $m\times r$ matrix G_{fb} such that r of the eigenvalues of the closed-loop system $F+GG_{fb}C$ are precisely those of the self-conjugate set $\{\lambda_i\}$ and the corresponding eigenvectors of the closed-loop system $F+GG_{fb}C$ are close to the respective components of the self-conjugate set $\{\phi_i^d\}$.

The following theorem describes the general eigenstructure assignment problem for the system with repeated eigenvalues in detail. In the following, matrices Φ_0, Ψ_0, W_0 and Z_0 are defined, respectively

$$\begin{aligned}\Phi_0&=[\Phi_1, \Phi_2, \cdots, \Phi_p],\ \Psi_0=[\Psi_1, \Psi_2, \cdots, \Psi_s]\\ W_0&=[W_1, W_2, \cdots, W_p],\ Z_0=[Z_1, Z_2, \cdots, Z_s]\end{aligned} \tag{7}$$

where ϕ_i is $n\times d_i$ submatrix of the form

$$\phi_i=[\phi_{i1}, \phi_{12}, \cdots, \phi_{id_i}] \tag{8}$$

and similarly, matrices Ψ_i, W_i and Z_i have the same forms.

Theorem 1. (Kwon and Youn, 1987)

Let $\Lambda=\Lambda_1\cup\Lambda_2$ such that $\Lambda_1=\{\lambda_1, \cdots, \lambda_p\}$ and $\Lambda_2=\{\lambda_{p+1}, \cdots, \lambda_s\}$ are symmetric with $\sum_{i=1}^{p}d_i=r$ and $\sum_{i=p+1}^{s}d=n-r$. If $\Lambda=\Lambda_1\cup\Lambda_2$ exists such that there exist vectors ϕ_{ij}, for $i=1, \cdots, p, j=1, \cdots, d_i$ and ψ_{ij}, for $i=p+1, \cdots, s, j=1, \cdots, d_i$ satisfying 1) $\Psi_0^T\phi_0=0$, and 2) $C\Phi_0$ is of full rank and $\lambda_i=\lambda_k^*$ implies $\psi_{ij}=\psi_{kj}^*$ where

$$\begin{aligned}&[F-\lambda_iI\,|\,G]\left[\frac{\phi_{ij}}{w_{ij}}\right]=\phi_{ij-1}\\ &[F^T-\lambda_iI\,|\,C^T]\left[\frac{\psi_{ij}}{z_{ij}}\right]=\psi_{ij+1}\end{aligned} \tag{9}$$

where, $\phi_{ij}=\psi_{id_i+1}=0$, then there exists a real matrix G_{fb} such that a set of eigenvalues of the closed-loop system $F+GG_{fb}C$ is Λ, and $\phi_{ij}(i=1, \cdots, p, j=1, \cdots, d_i)$ and $\psi_{ij}(1=p+1, \cdots, s, j=1, \cdots, d_i)$ constitute corresponding generalized right eigenvectors and left eigenvectors, respectively.

A design for output eigenstructure assignment for finding a desired gain matrix satisfying the desired eigenstructure is given below.

Firstly, find maximal rank matrices N_i, S_i, $\bar{N}_k$ and $\bar{S}_k$ such that

$$\begin{aligned}&N_i=\left[\frac{N_{1i}}{N_{2i}}\right],\ S_i=\left[\frac{S_{1i}}{S_{2i}}\right]\\ &\bar{N}_k=\left[\frac{\bar{N}_{1k}}{\bar{N}_{2k}}\right],\ \bar{S}_k=\left[\frac{\bar{S}_{1k}}{\bar{S}_{2k}}\right]\end{aligned} \tag{10}$$

for $i=1, \cdots, p$; $k=p+1, \cdots, s$ satisfying the following relation

$$[F-\lambda_iI\,|\,G]\left[\frac{S_i}{N_i}\right]=\left[\frac{I}{0}\right] \tag{11}$$

$$[F^T-\lambda_kI\,|\,G]\left[\frac{\bar{S}_k}{\bar{N}_k}\right]=\left[\frac{I}{0}\right] \tag{12}$$

where $N_i\in C^{(n+m)\times m}$, $S_i\in C^{(n+m)\times n}$, $\bar{N}_k\in C^{(n+r)\times r}$, $\bar{S}_k\in C^{(n+r)\times n}$.

Second, form the generalized right and left eigenvectors for $i=1, \cdots, p$; $k=p+1, \cdots, s$ as follows :

$$\phi_{ij}=S_{1i}\phi_{ij-1}+N_{1i}P_{ij},\ i=1, \cdots, d_i \tag{13}$$

$$\psi_{kj}=\bar{S}_{1k}\psi_{kj+1}+\bar{N}_{1k}p_{kj},\ i=1, \cdots, d_k \tag{14}$$

where $\phi_{i0}=\psi_{kd_k+1}$ and vectors $p_{ij}(i=1, \cdots, s;\ j=1, \cdots, d_i)$ are selected to satisfy conditions of generalized eigenstructure assignment theorem and the desired eigenvectors.

Third, calculate vector chains and construct matrix W_0 as follows:

$$w_{ij}=S_{2i}\phi_{ij-1}+N_{2i}p_{ij},\ i=1,\cdots,p;\ j=1,\cdots,d_i \quad (15)$$

$$W_0=[w_{11}|\cdots|w_{ij}|\cdots|w_{pd_i}] \quad (16)$$

Finally, calculate the output feedback gain matrix

$$G_{fb}=W_0(C\Phi_0)^{-1} \quad (17)$$

REMARKS. For the mode decoupling problem, the right eigenvectors are considered only. Thus, if we assume that the desired right eigenvector has a structure given by

$$\phi_{ij}^d=[\phi_{ij_1},\ \text{x, x, x, x},\ \phi_{ij_k},\ \text{x, x, x, x},\ \phi_{ij_n}]^T$$

where ϕ_{ij_k}'s are the designer specified components while x's are unspecified components. Then, vectors p_{ij} are selected to satisfy the specified components of the desired eigenvector, at the worst, in the least square sense according to the number of independent control actuators (i.e., m).

Note that the number of assignable eigenvalues, (right) eigenvectors, and entries in each eigenvector described in Theorem 1 are $\max(m, r)$ and $\min(m, r)$, respectively, which considers with the results of Srinathkumar (Srinathkumar, 1978).

4.2 Command generator tracker

The feedforward gain matrix G_{ff} for achieving required motion control is obtained by command generator tracker which is one of the explicit model following methods. Suppose that an explicit model representing the desired behavior of an aircraft is described by

$$\dot{x}_m=F_m x+G_m u_m \quad (18)$$

$$y_m=C_m x+D_m u_m \quad (19)$$

Further, define the controlled (or tracked) variables of the aircraft by

$$y_t=Tx \quad (20)$$

The objective is to determine a flight control law such that controlled aircraft variables closely approximate the outputs of the explicit model. We now briefly summarize the derivation of the command generator tracker. Suppose that $y_t=y_m$ at some $t=t_0$. Then, let u^* be the input to the aircraft which guarantees that $y_t=y_m$ for all $t=t_0$. We also let x^* and u^* be the corresponding aircraft state and the ideal aircraft input, respectively. These quantities satisfy the same dynamics as the aircraft such that

$$\dot{x}^*=Fx^*+Gu^* \quad (21)$$

$$y^*=Cx^* \quad (22)$$

$$y_t^*=Tx^* \quad (23)$$

The controlled variable of the ideal aircraft is equal to the model output. Thus,

$$y_t=y_m \quad (24)$$

Assume that x^* and u^* are given by

$$x^*=V_{11}x_m+V_{12}u_m+\mathrm{HOD}(u_m) \quad (25)$$

$$u^*=V_{21}x_m+V_{22}u_m+\mathrm{HOD}(u_m) \quad (26)$$

Both x^* and u^* are the linear in the state and input of the explicit model. The matrices V_{ij} are assumed to be constant and the expression HOD $(\cdot)$ represents higher-order derivatives.

If we restrict out discussion to model step inputs, then u_m is a step function, and consequently $\mathrm{HOD}(u_m)=0$ for all $t>t_0$. For the aircraft control problem, u_m is simply the pilot's command.

Subject to the restriction on u_m, the ideal aircraft state and input become

$$\begin{bmatrix} x^* \\ u^* \end{bmatrix}=\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}\begin{bmatrix} x_m \\ u_m \end{bmatrix} \quad (27)$$

Under very mild restrictions, the solution for the ideal aircraft input u_m^* is given by

$$u^*=V_{21}x_m+V_{22}u_m \quad (28)$$

where

$$V_{11}=\Omega_{11}V_{11}F_m+\Omega_{12}C_m \quad (29)$$

$$V_{12}=\Omega_{11}V_{11}G_m+\Omega_{12}D_m \quad (30)$$

$$V_{21}=\Omega_{21}V_{11}F_m+\Omega_{22}C_m \quad (31)$$

$$V_{22}=\Omega_{21}V_{11}G_m+\Omega_{22}D_m \quad (32)$$

and the Ω_{ij} are given by

$$\Omega=\left[\begin{array}{c|c}\Omega_{11} & \Omega_{12} \\ \hline \Omega_{21} & \Omega_{22}\end{array}\right]=\left[\begin{array}{c|c}F & G \\ \hline H & 0\end{array}\right]^T$$

4.3 Vehicle lateral control

For our vehicle lateral control problem we desire the controlled variables of the vehicle to track the driver's commands. This may be achieved by choosing an identity model. The output of this model is equal to its input. That is,

$$y_m=u_m \tag{33}$$

Hence, the output y_m of this model is also driver's command u_m which the controlled variables y_t of the vehicle track.

The identity model is described by

$$F_m=0,\ G_m=0,\ C_m=0,\ D_m=I \tag{34}$$

Using (34), (29)-(32) become

$$V_{11}=0 \tag{35}$$

$$V_{12}=\Omega_{12} \tag{36}$$

$$V_{21}=0 \tag{37}$$

$$V_{22}=\Omega_{22} \tag{38}$$

and the ideal vehicle state and input become

$$x^*=\Omega_{12}u_m \tag{39}$$

$$u^*=\Omega_{22}u_m \tag{40}$$

It should be noted that the ideal vehide state x^* and the ideal vehicle input u^* depend only on the driver's command u_m and the feedforward gains Ω_{12} and Ω_{22} depend only on the vehicle matrices F, G and the tracking matrix T.

To incorporate output feedback into the design, we let

$$\bar{x}=x-x^* \tag{41}$$

$$\bar{u}=u-u^* \tag{42}$$

$$\bar{y}=y-y^* \tag{43}$$

Then,

$$\dot{\bar{x}}=F\bar{x}+G\bar{u} \tag{44}$$

$$\bar{y}=C\bar{x} \tag{45}$$

The feedback control law for (44) and (45) is given by

$$\bar{u}=G_{fb}\bar{y}=G_{fb}(y-y^*) \tag{46}$$

or

$$\begin{aligned} u&=u^*+\bar{u} \\ &=u^*+G_{fb}(y-y^*) \\ &=u^*+G_{fb}C(x-x^*) \end{aligned} \tag{47}$$

Upon substituting (39) and (40) into (47) we obtain

$$u=\underbrace{[\Omega_{22}-G_{fb}C\Omega_{12}]\,u_m}_{\text{feedforward}}+\underbrace{G_{fb}y}_{\text{feedback}} \tag{48}$$

Note that the feedforward gains depend upon the feedback gains which were previously computed by using eigenstructure assignment. The resulting closed-loop system can be represented by

$$\begin{aligned} \dot{x}(t)&=[F+GG_{fb}C]x(t) \\ &\quad+G[\Omega_{22}-G_{fb}C\Omega_{12}]\,u_m \\ &\triangleq F_c x(t)+GG_{ff}u_m \end{aligned} \tag{49}$$

5. Simulations and Results

If we consider the lateral movement to the exclusion of brake and drive, the most effective control method is the steer angle control in the relation of tire slip angle and lateral force are linear. In these system, controller can be designed in order to change rear wheel toe angle only or both front and rear toe angle.

For the vehicle lateral dynamics under study, system (2) are most often utilized in order to decouple lateral velocity and yaw rate. The control objectives for the lateral dynamics of a vehicle include the ability to follow a chosen variable without significant motion change in other specified variables. Motion decoupling vehicle modes are also effective in case of lane change because desired movement of vehicle easily can be achieved using the proposed control method.

Considered a lateral vehicle dynamics with the parameter value as depicted in Table 2, then the numerical model of given system are as follows :

$$F=\begin{bmatrix} 4.8627 & -29.1797 \\ -0.9382 & 7.1743 \end{bmatrix}$$

$$G=\begin{bmatrix} -73.7465 & -62.3693 & 0 & 0 & 0 & 0 \\ -58.0596 & 84.3173 & -0.0012 & 0.0012 & -0.0012 & 0.0012 \end{bmatrix}$$

$$C=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Table 2 The parameter values of a vehicle (Alleyne, 1997)

Parameter	Value	Unit
m	1670	Kg
I_z	2100	Kg·m^2
a	0.99	m
b	1.7	m
C_{α_f}	−61595	N/rad
C_{α_r}	−52095	N/rad
$T_{norm,f}$	5	Kg·m^2/sec^2
$T_{norm,r}$	5	Kg·m^2/sec^2
r	0.3	m
d	0.76	m

Let the desired eigenvalues of the closed-loop system so that the natural frequency of the remaining eigenvalue can be three times as large as the one of a dominant eigenvalue as follows:

$$\Lambda^d=(-1,\ -3)$$

As earlier description, for the mode decoupling problem, the right eigenvectors are considered only. Thus, we choose the desired right eigenvectors to decouple lateral velocity and yaw rate as follows

$$\Phi^d=\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where, 'x' elements in the desired eigenvectors represent element that are not specified because they are not directly related to the decoupling objective. But, 'x' elements is concerned in system robustness, we choose the value to minimize the eigenvector sensitivity.

According to the design procedure of the proposed algorithm, feedback gain matrix can be obtained as follows:

$$G_{fb}=\begin{bmatrix} -0.0443 & 0.1856 \\ -0.0416 & 0.2484 \\ +0 & -0 \\ -0 & +0 \\ +0 & -0 \\ -0 & +0 \end{bmatrix}$$

where, '±0' elements in the matrix denote element that are very small value less than 10^{-7}.

The response of the close-loop system (2) due to control input is presented using the right and left modal matrices of the system by (Choi et al., 1995)

$$y(t)=C\Phi^a\int_0^t e^{\Lambda^a(t-\tau)}\{\Psi^{aT}GG_{ff}u(\tau)\}\,d\tau$$

where Λ^a is the diagonal matrix of achieved eigenvalues, C is the output matrix, GG_{ff} is the input matrix, and Φ^a and Ψ^a denote the right and left modal matrices of the closed-loop system, respectively. Substituting the obtained result into above equation, the lateral velocity and yaw rate of closed-loop system was decoupled by the proposed scheme as follows:

$$\begin{bmatrix} V \\ \dot{\Theta} \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\int_0^t e^{\begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}(t-\tau)}\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \delta_f(\tau) \\ \delta_r(\tau) \\ \Delta T_1(\tau) \\ \Delta T_2(\tau) \\ \Delta T_3(\tau) \\ \Delta T_4(\tau) \end{bmatrix}\right\}d\tau$$

We choose the identity model that is the controlled variables of the vehicle to track the driver's commands. According to the proposed algorithm, the resulting matrices T, Ω_{12}, Ω_{22} and the feedforward gain matrix G_{ff} can be obtained as follows:

$$T=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},\ \Omega_{12}=\begin{bmatrix} 1 & +0 & 0 & 0 & 0 & 0 \\ +0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Omega_{22}=\begin{bmatrix} 0.0357 & -0.2046 & 0 & 0 & 0 & 0 \\ 0.0357 & -0.2260 & 0 & 0 & 0 & 0 \\ -0 & +0 & 0 & 0 & 0 & 0 \\ +0 & -0 & 0 & 0 & 0 & 0 \\ -0 & +0 & 0 & 0 & 0 & 0 \\ +0 & -0 & 0 & 0 & 0 & 0 \end{bmatrix},\ G_{ff}=\begin{bmatrix} -0.0086 & -0.0190 & 0 & 0 & 0 & 0 \\ -0.0059 & 0.0225 & 0 & 0 & 0 & 0 \\ -0 & -0 & 0 & 0 & 0 & 0 \\ +0 & +0 & 0 & 0 & 0 & 0 \\ -0 & -0 & 0 & 0 & 0 & 0 \\ +0 & +0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The controllers for the three input types will be developed separately and simulations will be given in each case. The controllers for each input will all be based on the proposed design scheme. For a case for Four Wheel Steering, two steering input and four torque input will be applied to the wheels as follows:

$$\begin{bmatrix}\dot{V}\\ \ddot{\Theta}\end{bmatrix}=\begin{bmatrix}-2\dfrac{C_{\alpha_f}+C_{\alpha_r}}{mU} & -U-2\dfrac{aC_{\alpha_f}-bC_{\alpha_r}}{mU}\\ -2\dfrac{aC_{\alpha_f}-bC_{\alpha_r}}{I_zU} & -2\dfrac{a^2C_{\alpha_f}+b^2C_{\alpha_r}}{I_zU}\end{bmatrix}\begin{bmatrix}V\\ \dot{\Theta}\end{bmatrix}$$

$$+\begin{bmatrix}\dfrac{2}{m}\left(\dfrac{T_{nom,f}}{r}+C_{\alpha_f}\right) & \dfrac{2}{m}\left(\dfrac{T_{nom,r}}{r}+C_{\alpha_r}\right) & 0 & 0 & 0 & 0\\ \dfrac{2a}{I_z}\left(\dfrac{T_{nom,f}}{r}+C_{\alpha_f}\right) & -\dfrac{2b}{I_z}\left(\dfrac{T_{nom,r}}{r}+C_{\alpha_r}\right) & -\dfrac{d}{I_zr} & \dfrac{d}{I_zr} & -\dfrac{d}{I_zr} & \dfrac{d}{I_zr}\end{bmatrix}\begin{bmatrix}\delta_f\\ \delta_r\\ \Delta T_1\\ \Delta T_2\\ \Delta T_3\\ \Delta T_4\end{bmatrix}$$

Simulation scenarios for the lane change mode, lateral translation mode and rotation mode are taken as follows, respectively.

The lateral velocity and yaw rate of the lane change mode are changed to 1 m/sec and 1 rad/sec at 5 sec, and maintained for 5 sec. For the lateral translation mode, the lateral velocity and yaw rate are fixed to 1 m/sec and 1 rad/sec at all time. Finally the lateral velocity and yaw rate of the rotation mode are maintained to 1 m/sec and 1 rad/sec at all time, the front wheel is steered against the rear wheel.

For the lane change mode and lateral translation mode, both direction of the front wheel and the rear wheel maintain the same direction. Meanwhile, for the rotation mode, the front wheel is steered against the rear wheel, a sign of gain matrices could be obtained as the opposite sign, that are given by

$$G_{fb}=\begin{bmatrix}-0.0443 & 0.1856\\ 0.0416 & -0.2484\\ +0 & -0\\ -0 & +0\\ -0 & +0\\ +0 & -0\end{bmatrix},\quad G_{ff}=\begin{bmatrix}-0.0086 & -0.0190 & 0 & 0 & 0 & 0\\ 0.0059 & -0.0225 & 0 & 0 & 0 & 0\\ -0 & -0 & 0 & 0 & 0 & 0\\ +0 & +0 & 0 & 0 & 0 & 0\\ +0 & +0 & 0 & 0 & 0 & 0\\ -0 & -0 & 0 & 0 & 0 & 0\end{bmatrix}$$

For a lane change mode, the output responses, the error of output, and the inputs are represented as depicted in Figs. 7-9.

In each figure, the result of mode decoupling eigenenstructure assignment and command generator tracker is depicted by the solid line, and the result of general eigenstructure assignment and command generator tracker is depicted by the dotted line. The outputs of the vehicle using our method tracks the reference inputs more properly in Fig. 7. In Fig. 8, the error of our method is smaller than the previously method, the error of output are zero after converge into a steady-state.

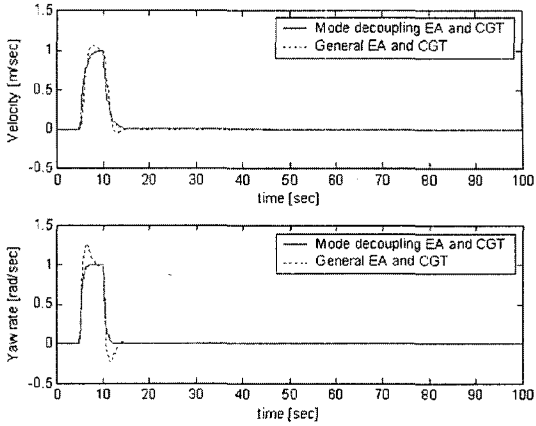


Fig. 7 The output responses of lane change mode

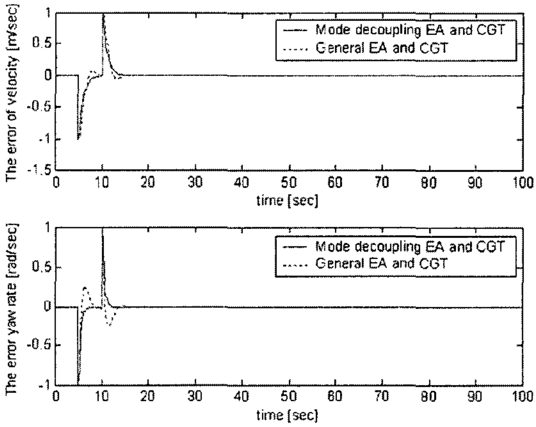


Fig. 8 The output errors of lane change mode

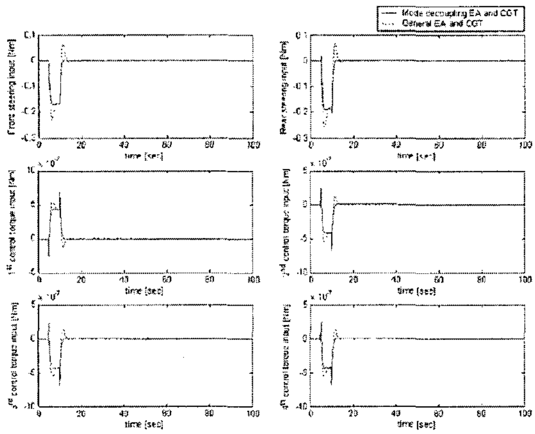


Fig. 9 The inputs of lane change mode

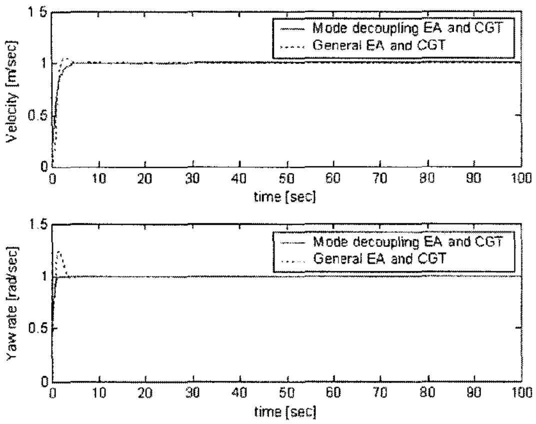


Fig. 10 The output responses of lateral translation mode

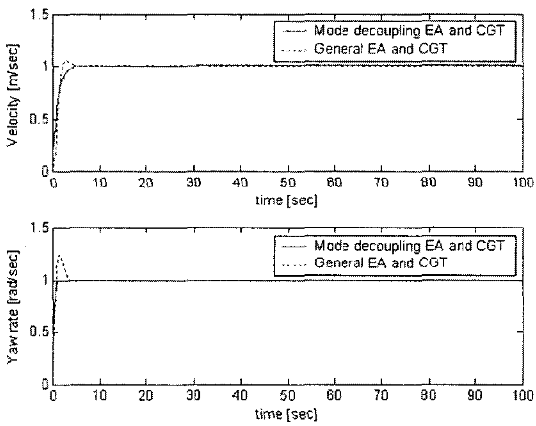


Fig. 11 The output responses of rotation mode

The front and rear steering input is primarily used while control torque input are very small as depicted in Fig. 9.

The result for a lateral translation mode and rotation mode are similar with a lane change mode. Figs. 10 and 11 represents the output responses of each mode, respectively. From compare with the general eigenstructure assignment, the proposed method tracks the reference inputs more properly in Figs. 10 and 11.

6. Concluding Remark

In this research, the analysis techniques for decoupling of the aircraft motions are utilized to develop vehicle lateral control with advanced mode and CCV vehicle modes was defined to achieve additional vehicle movement. We used right eigenstructure assignment techniques and command generator tracker to design a control law for an lateral vehicle dynamics. The desired eigenvectors are chosen to achieve the desired decoupling (i.e., lateral direction speed and yaw rate). The command generator tracker is used to ensure steady-state tracking of the driver's command. As the result of simulation, the output of lateral dynamic can be achieved the ability to follow a chosen variable without significant motion change in other specified variables.

Acknowledgments

This work was supported by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD). (KRF-2005-214-D00271).

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