

# Effects of Slot Combination and Skewed Slot on the Electromagnetic Vibration of a 4-pole Capacitor Motor under Load Condition

Isao Hirotsuka<sup>†</sup>, Yutaro Tsubouchi\* and Kazuo Tsuboi\*

**Abstract** - Recently, the reduction of electromagnetic vibration and noise of a capacitor motor (CRM) has become a very important subject from the standpoint of environmental improvement. Therefore, the authors have studied the characteristics of the dominant electromagnetic vibration of the CRM under load condition. In this paper, the effects of slot combination and skewed slot on the dominant electromagnetic vibration of a CRM under load condition are discussed both theoretically and experimentally. As a result, the characteristics of the dominant electromagnetic vibration for the slot combination and the reduction effect of the skewed slot on the electromagnetic vibration are clarified for a 4-pole CRM.

**Keywords:** capacitor motor, electromagnetic vibration, electromagnetic force wave, slot combination, skewed slot, backward magnetic field

## 1. Introduction

The capacitor motor (CRM) is widely used to drive industrial equipment and household electric appliances. Recently, the reduction in vibration and noise of the CRM has become a very important subject from the standpoint of environmental improvement. The CRM is structurally similar to the three-phase squirrel cage induction motor (IM). However, the electromagnetic vibration of the CRM is caused by both the forward and backward electromagnetic fields. Therefore, the causes and characteristics of the vibration of the CRM are more complicated than those of the IM. Moreover, the electromagnetic vibration of the CRM under load condition has not yet been analyzed theoretically. The authors have studied the characteristics of the dominant electromagnetic vibration of the CRM under a load condition [1]. As a result, the cause and tendency of the vibration for the load condition were clarified.

In this paper, the effects of the slot combination and skewed slot on the dominant electromagnetic vibration of the CRM under load condition are discussed both theoretically and experimentally. The theory, which has been reported previously, takes into consideration both the forward and backward electromagnetic fields as well as electromagnetic saturation. As a result, the characteristics of the dominant electromagnetic vibration for the slot combination and the reduction effect of the skewed slot on the electromagnetic vibration are clarified for a 4-pole CRM.

## 2. Dominant Electromagnetic Force Wave under Load Condition

The general equation of the electromagnetic force wave of the CRM under load condition has already been reported [2]. In this section, the occurrence frequency  $f_M$  and mode  $M$  of the dominant electromagnetic force wave and the effect of the skewed slot on the force are summarized.

Table 1 lists  $f_M$  and  $M$  of the dominant electromagnetic force wave. In this table,  $n_s$  is the number of stator slots,  $n$  is the number of rotor slots,  $p'$  is the number of pole pairs,  $s$

**Table 1** Dominant Electromagnetic Force

Classification of forces	Occurrence frequency: $f_M$ [Hz]	Mode: $M$
1	$\left  \frac{Kn}{p'}(1-s) - 2 \right  f$	[I]: $ (+p') + (+p' + Ln_s - Kn) $
		[II]: $ (+p' + Ln_s - Kn) + (+p') $
		[III A]: $ (\pm p') + (\mp p' + Ln_s - Kn) $
		[III B]: $ (\pm p' + Ln_s - Kn) + (\mp p') $
2	$\left  \frac{Kn}{p'}(1-s) \right  f$	[I]: $ (+p') - (+p' \pm Ln_s \mp Kn) $
		[II]: $ (+p' \pm Ln_s \mp Kn) - (+p') $
		[III A]: $ (\pm p') - (\mp p' \pm Ln_s \mp Kn) $
		[III B]: $ (\pm p' \pm Ln_s \mp Kn) - (\mp p') $
3	$\left  \frac{Kn}{p'}(1-s) + 2 \right  f$	[I]: $ (+p') + (+p' - Ln_s + Kn) $
		[II]: $ (+p' - Ln_s + Kn) + (+p') $
		[III A]: $ (\pm p') + (\mp p' - Ln_s + Kn) $
		[III B]: $ (\pm p' - Ln_s + Kn) + (\mp p') $

For simplicity, the forces are classified according to their respective expressions for  $f_M$  and  $M$ .

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is the slip,  $f$  is the power supply frequency,  $L$  and  $K$  are the orders of the stator and rotor slot harmonics with integer values, respectively,  $+p'$  denotes the fundamental component in the forward electromagnetic field, and  $-p'$  denotes the fundamental component in the backward electromagnetic field. The electromagnetic force wave [I] classified in Table 1 is generated by the interaction between the stator forward fundamental flux and rotor harmonic flux. Therefore, it is considered that the force wave [I] causes both under load condition and at no-load. The electromagnetic force wave [II] is generated by the interaction of the stator harmonic flux and rotor forward fundamental flux. Therefore, it is considered that the force wave [II] causes under load condition. The electromagnetic force waves [III<sub>A</sub>] and [III<sub>B</sub>] are generated by the interaction between the stator backward fundamental flux and rotor harmonic flux, or by the interaction between the stator harmonic flux and rotor backward fundamental flux. It is considered that the force waves [III<sub>A</sub>] and [III<sub>B</sub>] decrease with increasing load due to the effect of a backward magnetic field. These electromagnetic force waves [III<sub>A</sub>] and [III<sub>B</sub>] are affected by load condition and the value of the running capacitor.

When the rotor slots are skewed, the  $v'$ -th harmonic voltage including the fundamental voltage induced in the rotor bars decreases in proportion to the absolute value of the skew factor,  $|K_{sv'}|$ , shown in equation (1).

$$|K_{sv'}| = \left| \frac{n \sin(v' \sigma_r \pi / n)}{v' \sigma_r \pi} \right| \quad (1)$$

where  $\sigma_r$  is the ratio of the skewed slot pitch to the rotor slot pitch.

Therefore, since the  $v'$ -th harmonic current of the rotor bars decreases in proportion to  $|K_{sv'}|$ , the electromagnetic force wave is approximately proportional to  $|K_{sv'}|$ .

The order of the space harmonic of the magnetic stator flux density that produces the rotor magnetic flux density is defined as  $v_p' = (\pm p') \pm Ln_s$ , and the order of the space harmonic of the rotor magnetic flux density that produces the stator magnetic flux density is defined as  $v_q' = (\pm p') \pm Kn$ . As shown in reference [2], the electromagnetic force waves [I] and [III<sub>A</sub>] decrease in approximate proportion to  $|K_{svp'}|$ . Similarly, the electromagnetic force waves [II] and [III<sub>B</sub>] decrease in approximate proportion to  $|K_{svq'}|$ .

Since these three types of electromagnetic force waves are generated at the same frequency under load condition, the electromagnetic force wave at the frequency is the resultant electromagnetic force wave consisting of these force waves. Therefore, the resultant electromagnetic force wave does not necessarily decrease in proportion to the absolute value of the skew factor of a specific harmonic order.

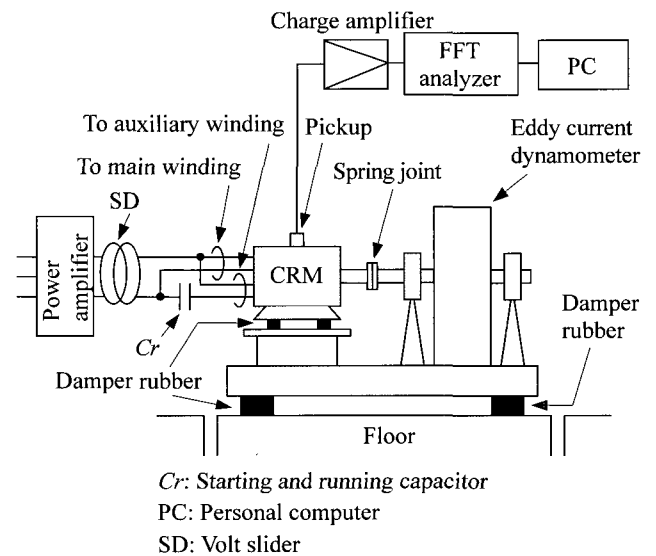
### 3. Test Motors And Experimental Method

Table 2 lists the specifications of the test motors. The rotor is selected from among 76 rotors; the number of rotor slots is from 27 to 45 with no skewed slot ( $\sigma_r = 0.0$ ) and three types of skewed slots ( $\sigma_r = 0.3, 0.7, \text{ and } 1.0$ ) for each rotor slot. The natural frequency of the test motor is measured at room temperature using an impact hammer. It is confirmed that the natural frequencies of the test motor are approximately constant even when either of the rotors is combined. However, the temperature rises when the electromagnetic vibration of the CRM is measured, and these natural frequencies appear to change slightly.

Fig. 1 shows the arrangement of the experimental apparatus. The vibrations in the radial direction are detected by the acceleration pickup, and they are analyzed by a fast Fourier transform (FFT) analyzer. An eddy current dynamometer is used as the load because of its small mechanical losses. The test motor is set on a resilient rubber base to prevent the effects of external vibration on the measurement. In order to avoid abnormal starting phenomena [3]

**Table 2** Specifications of the Test Motors

Number of poles, $2p'$	4
Rated output, $P_o$	0.75 kW
Rated voltage, $V$	200 V
Rated frequency, $f$	60 Hz
Rated capacitor, $Cr$	30 $\mu$ F
Number of stator slots, $n_s$	32
Number of rotor slots, $n$	27-45
Ratio of the skewed slot pitch to the rotor slot pitch, $\sigma_r$	0.0, 0.3, 0.7, 1.0
Natural frequencies	625, 1700, 3300, 4000 Hz



**Fig. 1** Arrangement of the experimental apparatus

and adjust the supply frequency  $f$ , a variable frequency power supply is used.

The applied voltage is sinusoidal, the rated voltage  $V$  is 200 V and the rated frequency  $f$  is 60 Hz. The load condition of the test motor is expressed as the ratio  $Lp$  [%] of the rated torque;  $Lp = 0\%$  at no load and  $Lp = 100\%$  at the rated load. In case of  $Lp = 0\%$ , because the CRM is connected to the load set, load condition of the CRM is not precisely no-load. However, from the results of many preparatory experiments, the vibration at  $Lp = 0\%$  has been verified to agree with the data measured on the CRM alone. Since  $f_v$  varies with  $Lp$ ,  $f_v$  is represented by a multiple of the source frequency  $f$  at  $s = 0$ .

### 4. Experimental Results

The experimentally measured dominant electromagnetic vibrations when  $n = 31$  are shown in Fig. 2. Many dominant vibrations occur when  $Lp = 0\%$  and  $\sigma_r = 0.0$ . When  $Lp = 100\%$ , the acceleration level  $f_a$  of 930-Hz and 2790-Hz components increases, however, many vibration components decrease. The reduction of  $f_a$  with increasing  $Lp$  means that the vibration is caused by the backward magnetic field [1]. When  $\sigma_r = 0.7$ , all the vibration components significantly decrease.

Fig. 3 shows the characteristics of the dominant electromagnetic vibration component for  $Lp$  and  $n$ . The symbol “o” in this figure indicates the occurrence of the dominant vibration. “o” is represented in three different sizes, depending on  $f_a$ . The oblique lines represent the  $f_M$  values in Table 1.

From Fig. 3, it appears that the occurrence frequency  $f_v$  of many dominant vibration components is in accord with  $f_M$ . However,  $f_v$  of some vibration components does not accord with  $f_M$ . These vibration components are generated by the effect of electromagnetic saturation. These components occur when  $K = 1$  and 2. They are affected by the mechanical system because their frequencies are near the natural frequency of the test motors. When  $Lp$  is increased,  $f_a$  of a large number of vibration components decreases. For example, the 1050-Hz component when  $n = 31$  shown in Fig. 2 occurs due to the electromagnetic force waves [III<sub>A</sub>] and [III<sub>B</sub>]. Since the backward magnetic fields decrease with an increase in  $Lp$  and the electromagnetic force waves [III<sub>A</sub>] and [III<sub>B</sub>] decrease,  $f_a$  of the 1050-Hz component decreases. In addition, as  $Lp$  increases,  $f_v$  of the 1050-Hz component shifts from the resonance frequency of the mechanical system. When rotor slots are skewed,  $f_a$  of the dominant electromagnetic vibration decreases. But, the reduction ratio of  $f_a$  for skew factor is different from each vibration component.

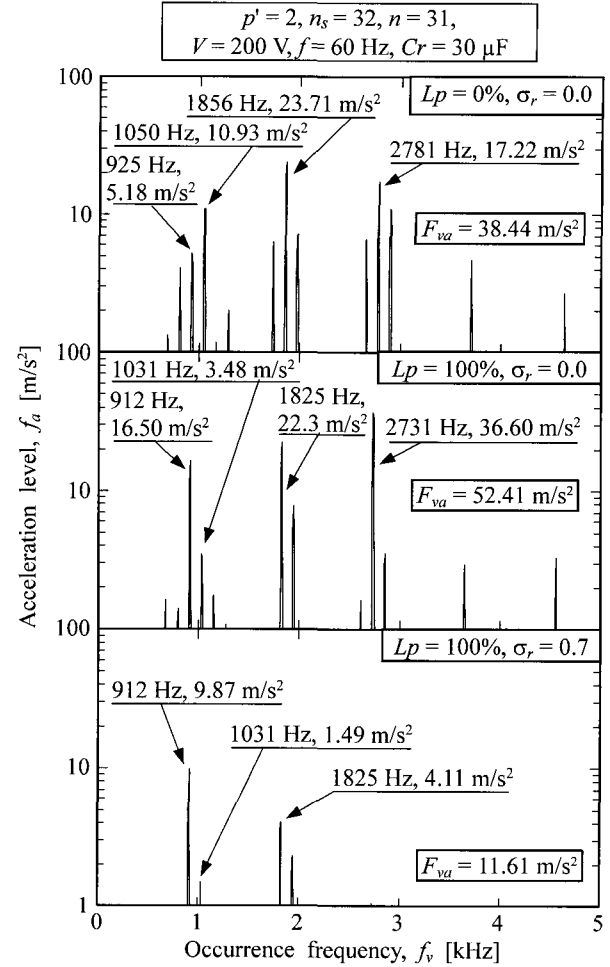


Fig. 2 Measured electromagnetic vibrations

Table 3 shows the measured vibrations and skew factors of the harmonic order at  $Lp = 100\%$ . For example, the main cause of the 1020-Hz component when  $n = 34$  is the electromagnetic force waves [I] and [II] of  $M = 2$ . The 1020-Hz component is approximately proportional to the absolute values of the skew factor of the following harmonic orders:  $v_p' = \pm 30$  and  $\pm 34$  and  $v_q' = \pm 32$  and  $\pm 36$ . Thus, it is clarified that the dominant electromagnetic vibration under load condition is approximately proportional to the absolute values of the skew factor for the harmonic order of the main causes.

Fig. 4 shows the overall value  $F_{va}$  of the vibratory acceleration up to 5 kHz for  $n$  and  $\sigma_r$ .  $F_{va}$  decreases with an increase in  $\sigma_r$ . When  $n_s + 3 \geq n \geq n_s - 3$ , that is,  $n$  almost equals  $n_s$  except  $n = n_s$ ,  $F_{va}$  is dominant because a large number of electromagnetic force waves with small modes are generated. However, when  $n = 28, 32, 36, 40, 44$  (that is,  $n$  is a multiple of  $2p'$ ),  $F_{va}$  is relatively small because no dominant electromagnetic force wave with a small mode is generated.

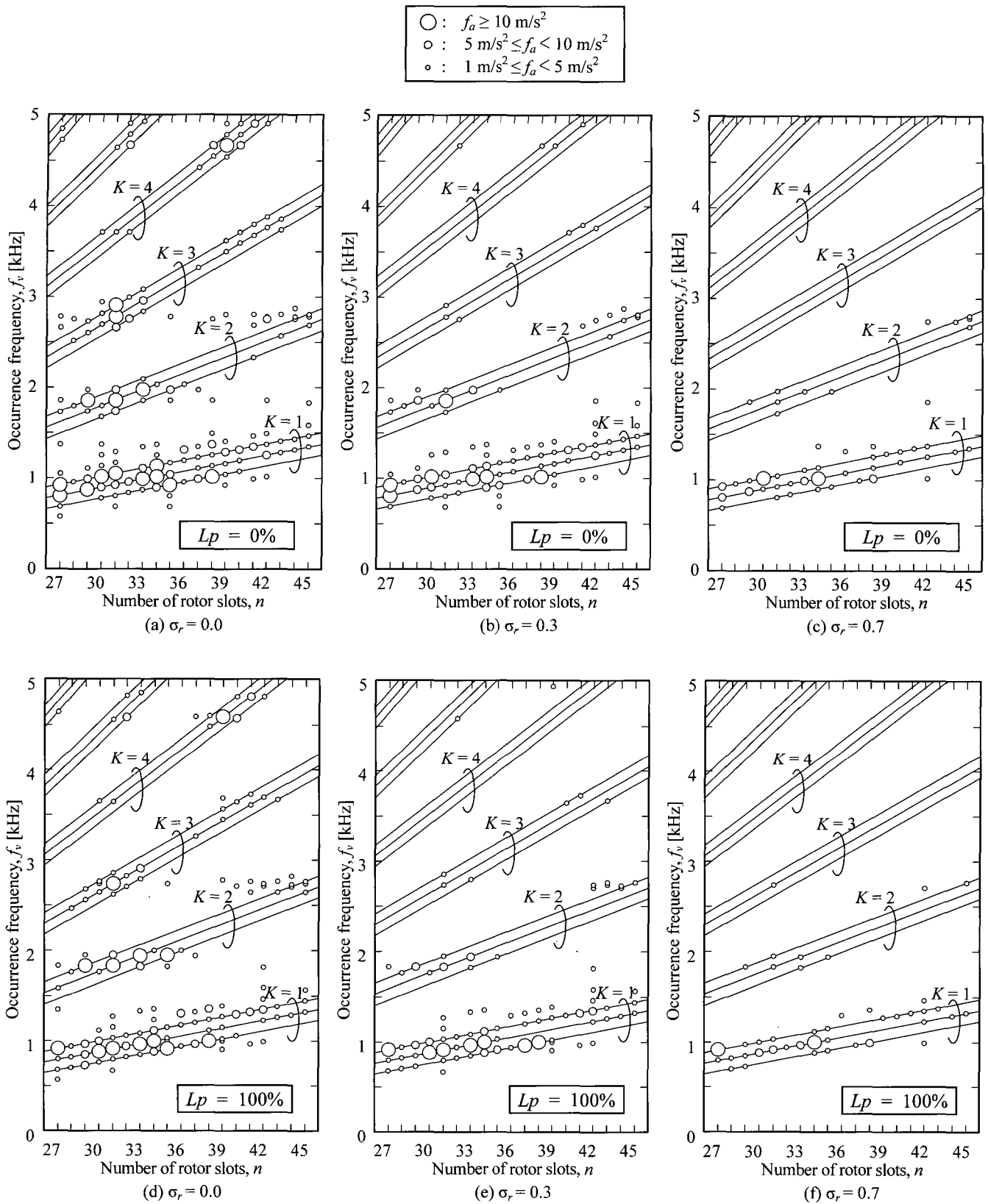
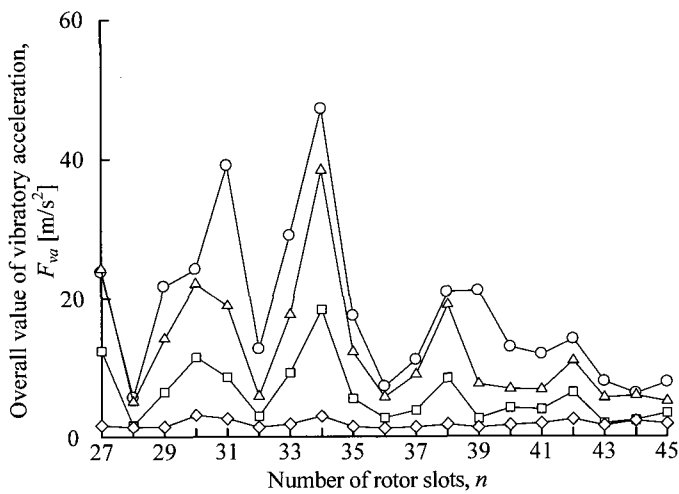
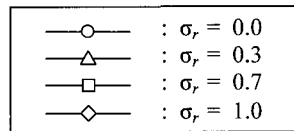


Fig. 3 Features of the electromagnetic vibration under load condition

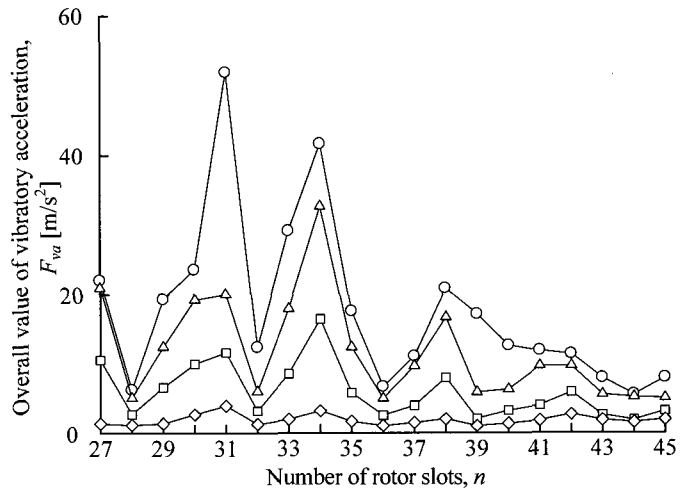
**Table 3** Main Causes and Reduction Ratio ( $L_p = 100\%$ )

Number of rotor slots, $n$	Frequency, $f_v$ [Hz]	Classification of force	Mode, $M$		$K$	$L$	$v_p'$ $v_q'$	$f_a$ [ $m/s^2$ ]	$f_a$ [%]: ratio for $\sigma_r = 0.0$ , $ K_{svp} $ and $ K_{svq} $ [%] in parentheses			
			[I] [II]	[III <sub>A</sub> ] [III <sub>B</sub> ]					$\sigma_r = 0.0$	$\sigma_r = 0.3$	$\sigma_r = 0.7$	$\sigma_r = 1.0$
27	930	3	1	5	1	1	$\begin{pmatrix} -30^* \\ -34^* \\ 25 \\ 29 \end{pmatrix}$	21.0	95 $\begin{pmatrix} 83 \\ 78 \\ 88 \\ 84 \end{pmatrix}$	49 $\begin{pmatrix} 26 \\ 13 \\ 44 \\ 30 \end{pmatrix}$	5 $\begin{pmatrix} 10 \\ 18 \\ 8 \\ 7 \end{pmatrix}$	
29	1860	3	2	6	2	2	$\begin{pmatrix} -62^* \\ -66^* \\ 56 \\ 60 \end{pmatrix}$	13.8	47 $\begin{pmatrix} 45 \\ 39 \\ 53 \\ 48 \end{pmatrix}$	14 $\begin{pmatrix} 21 \\ 19 \\ 21 \\ 22 \end{pmatrix}$	2 $\begin{pmatrix} 6 \\ 11 \\ 4 \\ 3 \end{pmatrix}$	
31	1860	2	2	2	2	2	$\begin{pmatrix} \pm 60^* \\ \pm 62 \\ \pm 64^* \\ \pm 66 \end{pmatrix}$	22.3	42 $\begin{pmatrix} 53 \\ 50 \\ 48 \\ 45 \end{pmatrix}$	18 $\begin{pmatrix} 21 \\ 22 \\ 22 \\ 21 \end{pmatrix}$	3 $\begin{pmatrix} 3 \\ 0 \\ 3 \\ 6 \end{pmatrix}$	
	2790	2	3	1	3	3	$\begin{pmatrix} \pm 91^* \\ \pm 94 \\ \pm 95^* \\ \pm 98 \end{pmatrix}$	36.6	11 $\begin{pmatrix} 13 \\ 10 \\ 9 \\ 5 \end{pmatrix}$	4 $\begin{pmatrix} 3 \\ 6 \\ 7 \\ 9 \end{pmatrix}$	4 $\begin{pmatrix} 2 \\ 1 \\ 2 \\ 5 \end{pmatrix}$	
34	1020	2	2	6	1	1	$\begin{pmatrix} \pm 30 \\ \pm 32^* \\ \pm 34 \\ \pm 36^* \end{pmatrix}$	39.3	80 $\begin{pmatrix} 89 \\ 87 \\ 86 \\ 84 \end{pmatrix}$	41 $\begin{pmatrix} 48 \\ 42 \\ 37 \\ 31 \end{pmatrix}$	7 $\begin{pmatrix} 13 \\ 6 \\ 0 \\ 6 \end{pmatrix}$	
38	1020	1	2	6	1	1	$\begin{pmatrix} -36^* \\ -40^* \\ 30 \\ 34 \end{pmatrix}$	18.2	79 $\begin{pmatrix} 87 \\ 84 \\ 91 \\ 89 \end{pmatrix}$	33 $\begin{pmatrix} 42 \\ 32 \\ 57 \\ 47 \end{pmatrix}$	4 $\begin{pmatrix} 6 \\ 5 \\ 25 \\ 12 \end{pmatrix}$	

\*:  $v_q'$ -th space-harmonic



(a)  $L_p = 0\%$



(b)  $L_p = 100\%$

**Fig. 4** Overall value of vibratory acceleration for rotor slots

## 5. Conclusion

In this study, the effects of slot combination and skewed slot on the dominant electromagnetic vibration of 4-pole CRM under load condition were discussed theoretically and experimentally. The main conclusions are summarized as follows:

- (1) The characteristics of the dominant electromagnetic vibration of 4-pole CRM based on the slot combination were discussed experimentally and the following results were obtained.
  - (a) When  $n_s+3 \geq n \geq n_s-3$  except  $n = n_s$ , many dominant electromagnetic vibrations occur and  $F_{va}$  becomes dominant.
  - (b) The dominant electromagnetic vibration does not occur when the number of rotor slots is a multiple of the number of poles.
- (2) The relationship between the electromagnetic force wave and rotor skew was shown theoretically. The effect of the skewed slot on the electromagnetic vibrations was discussed experimentally. It was clarified that the dominant electromagnetic vibration decreases in approximate proportion to the absolute value of the skew factor of the harmonic order, which is the main cause of the vibration.

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