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Bayesian Method on Sequential Preventive Maintenance Problem

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Abstract

This paper develops a Bayesian method to derive the optimal sequential preventive maintenance(PM) policy by determining the PM schedules which minimize the mean cost rate. Such PM schedules are derived based on a general sequential imperfect PM model proposed by Lin, Zuo and Yam(2000) and may have unequal length of PM intervals. To apply the Bayesian approach in this problem, we assume that the failure times follow a Weibull distribution and consider some appropriate prior distributions for the scale and shape parameters of the Weibull model. The solution is proved to be finite and unique under some mild conditions. Numerical examples for the proposed optimal sequential PM policy are presented for illustrative purposes.

Keywords : Sequential PM; Bayesian approach; Age reduction; Optimal PM schedule; Hybrid PM model; Prior distribution.

1. Introduction

A preventive maintenance(PM) policy specifies how the PM activities should be scheduled. Each PM action is taken to keep the repairable system at the desired level of successful operation and it may include minimal repair, perfect repair or replacement of the system as well as its components.

Many authors have proposed several PM models, either periodic or sequential, and obtained the best PM policies by optimizing several criteria regarding the operating cost.

Nakagawa (1986) considers several periodic and sequential PM policies for the system with minimal repair at each failure. For these models, the PM action is conducted at periodic times $kx, k = 1, 2, \cdots, N$, for periodic PM and at sequential

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times $x_1, x_1+x_2, \cdots, x_1+\cdots+x_N$, where $x_1, x_2, \cdots, x_N$ are not necessarily equal. When a failure occurs between the PMs, a minimal repair is done and the system remains in the same state as it was just prior to the failure. Canfield (1986) considers a periodic PM policy for which the PM slows the degradation process of the system, while the hazard rate keeps monotone increase with the age reduction at each PM. Park, Jung and Yum (2000) derive the optimal periodic PM schedules by incorporating various cost structures into Canfield's model. Lin, Zuo and Yam (2000) [LZY (2000)] propose a general sequential imperfect PM model under which each PM not only reduces the effective age of the system but also adjusts the hazard rate function. Such a model is referred to as a hybrid PM model in the sense that they combine two aspects of PM effects called the age reduction PM model and the hazard rate adjustment PM model.

The objective of this paper is to determine the optimal PM schedules based on LZY (2000) by adopting the Bayesian approach on certain unknown parameters of the PM model. As a criterion for the optimality of the PM model, we use the mean cost rate during the life cycle of the system until the system is replaced by a new one. The Bayesian approach could be quite flexible when the failure distribution of the system is either unknown or contains uncertain parameters, which is common in most of the practical situations. Mazzuchi and Soyer (1996) adopt a Bayesian approach on the random parameters of a Weibull model to solve the optimal replacement problem for both the block replacement protocols with minimal repair and the age replacement protocols by minimizing the expected long-run average cost. Their models have been extended by Juang and Anderson (2004), in which the minimal repair cost is assumed to be random as well.

In Section 2, the necessary notations and assumptions are listed and we present the hybrid PM model proposed by LZY's (2000). Section 3 discusses the Bayesian method to derive the optimal adaptive sequential PM schedules minimizing the mean cost rate by assigning appropriate prior distributions on both shape and scale parameters of the Weibull model. The mathematical expressions to formulate the mean cost rate during the life cycle of the system is derived as well. Section 4 provides numerical examples to illustrate the proposed Bayesian method and analyzes the effects of PM cost and replacement cost on the optimal adaptive sequential PM policy. Concluding remarks is given in Section 5.

2. Sequential imperfect PM model

The hybrid PM model due to LZY (2000) takes the advantages of the age reduction PM model and the hazard rate PM model by combining them. Under such a hybrid PM model, each PM not only reduces the effective age of the system to a certain value but also adjusts the slope of the hazard rate so that the degradation process of the system slows down due to the PM effects, although the slope of hazard rate keeps increasing with the number of PMs. Each PM is supposed to be performed at a sequence of intervals which may have unequal lengths.

The following notations and assumptions are used throughout this paper.

Notations

T	time to failure
$h(t)$	hazard rate without PM
$H(t)$	cumulative hazard rate without PM, $H(t)=\int_0^t h(x)dx$
$h_k(t)$	hazard rate between the $(k-1)$ st and the k th PMs
$H_k(t)$	cumulative hazard rate between the $(k-1)$ st and the k th PMs
x_k	interval length between the $(k-1)$ st and the k th PMs
z_k	the k th PM time, $z_k=\sum_{i=1}^{k} x_i$
y_k	effective age of the system just before the k th PM
a_k	adjustment factor in hazard rate due to the k th PM, $1=a_0 \leqq a_1 \leqq \cdots \leqq a_{N-1}$
A_k	$\prod_{i=0}^{k-1} a_i, \quad k=1,2,\cdots,N$
b_k	improvement factor in effective age due to the k th PM, $0=b_0 \leqq b_1 \leqq \cdots \leqq b_{N-1} < 1$
N	number of scheduled PMs before replacement
C_{pm}	unit cost of PM
C_{re}	cost of replacement
C_{mr}u	unit cost of minimal repair

Assumptions

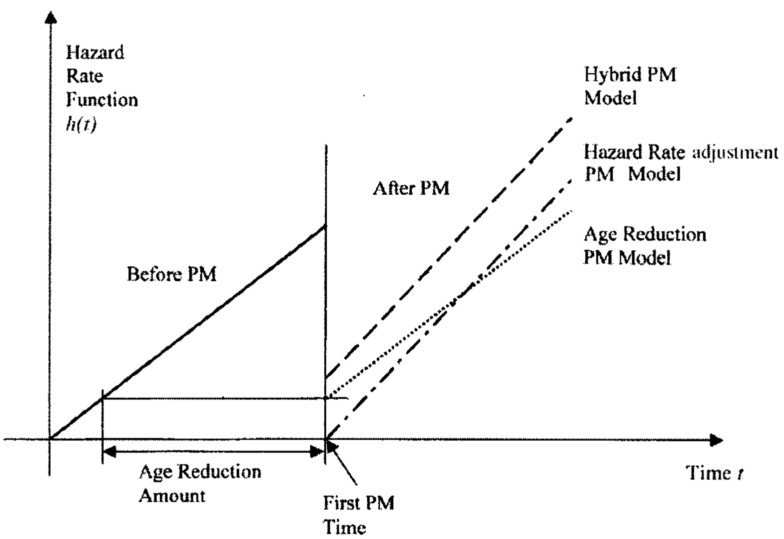
1) The system operates on the interval $(0,\infty)$.

2) When there are no PM interventions, the hazard rate of the system is

continuous and strictly increasing function.

3) It takes negligible time for PM, minimal repair and replacement.
4) PM is performed at $z_1, z_2, \cdots, z_{N-1}$, and the system is replaced at z_N by a new one.
5) The hazard rate function becomes $h_{k+1}(z_k) = A_{k+1}h(b_k y_k)$ immediately after the k th PM when it was $h_k(z_k) = A_k h(y_k)$ just before the PM. After the k th PM is performed, the hazard rate function is expressed as $A_{k+1}h(b_k y_k + x)$ for $x > 0$.

Let $h(t)$ be the hazard rate function of the system when no PM actions are taken and let y_k be the effective age of the system just before the k th PM under the hybrid model suggested by LZY (2000). Then, the system has the hazard rate $A_k h(t)$ between the $(k-1)$ st and the k th PMs, where t is measured as the effective age of the system. The effective age of the system becomes $b_{k-1}y_{k-1}$ right after the $(k-1)$ st PM and then becomes $y_k = x_k + b_{k-1}x_{k-1} + \cdots + b_{k-1}b_{k-2} \cdots b_2 b_1 x_1$ just before the k th PM. It is also clear that $y_k = x_k + b_{k-1}y_{k-1}$. If t is measured as the actual age of the system, then the hazard rate is equal to $h_k(z_{k-1})$ right after the $(k-1)$ st PM and then becomes $h_k(z_k)$ just before the k th PM. The hazard rate functions for the hybrid PM model, the hazard rate adjustment PM model and the age reduction PM model before and after the first PM is performed are shown in <Figure 2.1>.



<Figure 2.1> Hazard rate function for the hybrid model

Under the hybrid PM model, the mean cost rate is expressed as

$$C_{LZY} = \frac{C_{mr}\sum_{k=1}^{N} A_k\left[H(y_k) - H(b_{k-1}y_{k-1})\right] + (N-1)C_{pm} + C_{re}}{\sum_{k=1}^{N-1}(1-b_k)y_k + y_N}. \tag{2.1}$$

where the numerator of the expression (2.1) means the total cost needed until the system is replaced at N th PM by a new one and the denominator denotes the total time until then.

Based on the mean cost rate function given in equation (2.1), LZY (2000) develop non-Bayesian optimal sequential PM policies by calculating the PM times and the number of scheduled PMs for which the mean cost rate is minimized.

3. Bayesian method on sequential PM model

For non-Bayesian method to find the optimal sequential PM policy, the parameters are assumed to be fixed. However, in most of the practical situations, the failure distribution is either unknown or contains several unknown parameters and thus they must be estimated from the failure data. In such cases, the Bayesian approach could be quite effective to estimate these unknown parameters by assuming the prior distributions for them.

In this section, we discuss an optimal sequential PM policy based on the LZY's (2000) hybrid PM model in the context of Bayesian concepts by considering the random parameters. To derive the Bayesian optimal sequential PM policy, we consider the case when the failure times follow a Weibull distribution with the following hazard rate function.

$$h(t) = \alpha\beta t^{\beta-1}, \quad t \geqq 0, \alpha > 0, \beta > 1, \tag{3.1}$$

where α and β are the scale and shape parameters, respectively. Under the hybrid PM model discussed in Section 2, the hazard rate function can be obtained recursively as

$$h_k(t) = \begin{cases} \alpha\beta t^{\beta-1} & \text{for} \quad 0 \leqq t \leqq z_1 \\ A_k\alpha\beta\left(t - \sum_{i=1}^{k-1}(1-b_i)y_i\right)^{\beta-1} & \text{for} \quad z_{k-1} < t \leqq z_k, \end{cases} \tag{3.2}$$

for $k = 1, 2, \cdots, N$, $h_1(t) = h(t)$ and $x_0 = y_0 = z_0 = 0$. By applying the results given in Nakagawa (1986), we obtain the explicit formula for the mean cost rate as follows.

$$C(y_1,y_2,\cdots,y_N,N)=\frac{C_{mr}\sum_{k=1}^{N}\{H_k(z_k)-H_k(z_{k-1})\}+(N-1)C_{pm}+C_{re}}{\sum_{k=1}^{N-1}(1-b_k)y_k+y_N}$$

$$=\frac{C_{mr}\sum_{k=1}^{N}A_k\left\{\alpha y_k^{\beta}-\alpha(b_{k-1}y_{k-1})^{\beta}\right\}+(N-1)C_{pm}+C_{re}}{\sum_{k=1}^{N-1}(1-b_k)y_k+y_N}. \qquad (3.3)$$

The formula (3.3) can also be derived by using the expression (2.1) as well. To determine the optimal sequential PM schedules for the Weibull model by adopting the Bayesian approach, we utilize the adaptive estimation schemes to update uncertainty about α and β based on the failure data of the system observed during the current life cycle and thus, reevaluate the optimal PM schedules for the next life cycle.

To obtain the no-data estimation for the optimal sequential PM schedules in the initial life cycle, we assign a gamma distribution and a discretized beta distribution as prior distributions for scale parameter α and shape parameter β, respectively. Such priors have been widely used, for instance in Mazzuchi and Soyer (1996) and Juang and Anderson (2004) due to its great flexibility in presenting the prior uncertainty. In particular, although the shape parameter is continuous in nature, it is more suitable in this model to discretize the value of β by a finite number of intervals. By adjusting the size of each interval, the prior may have more flexibility to describe the pattern of β .

The prior distributions for α and β are given by

$$f(\alpha)=\frac{b^a}{\Gamma(a)}\alpha^{a-1}e^{-b\alpha}, \quad a>0, b>0 \qquad (3.4)$$

and

$$\Pr(\beta=\beta_l)=\int_{\beta_l-\delta/2}^{\beta_l+\delta/2}g(\beta)d\beta\equiv P_l, \qquad (3.5)$$

respectively, where $\beta_l=\beta_L+\delta(2l-1)/2$ and $\delta=(\beta_U-\beta_L)/m$ for $l=1,2,\cdots,m$. Here, $\sum_{l=1}^{m}P_l=1$ and $g(\beta)$ is a beta density to be defined as

$$g(\beta)=\frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)}\frac{(\beta-\beta_L)^{c-1}(\beta_U-\beta)^{d-1}}{(\beta_U-\beta_L)^{c+d-1}}, \quad 0\leq\beta_L\leq\beta\leq\beta_U, c>0, d>0. \qquad (3.6)$$

Initially, α and β are independent priors and thus, the joint prior distribution of α and β can be obtained as the product of two marginals given in (3.4) and (3.5). That is,

$$p(\alpha,\beta) = f(\alpha)\Pr(\beta = \beta_l) = \frac{b^a}{\Gamma(a)}\alpha^{a-1}e^{-b\alpha}\cdot P_l.$$

To determine the optimal sequential PM schedules based on the prior distributions of α and β, we need to formulate the mean cost rate by taking the expectation on (3.3) with respect to α and β. Given the priors (3.4) and (3.5), the mean cost rate can be expressed as

$$C_B(y_1,y_2,\cdots,y_N,N) = E_{\alpha,\beta}C(y_1,y_2,\cdots,y_N,N)$$

$$= E_{\alpha,\beta}\left[\frac{C_{mr}\sum_{k=1}^{N}A_k\{\alpha y_k^{\beta}-\alpha(b_{k-1}y_{k-1})^{\beta}\}+(N-1)C_{pm}+C_{re}}{\sum_{k=1}^{N-1}(1-b_k)y_k+y_N}\right]$$

$$= \frac{\sum_{l=1}^{m}\left[C_{mr}\frac{a}{b}\sum_{k=1}^{N}A_k\{y_k^{\beta_l}-(b_{k-1}y_{k-1})^{\beta_l}\}+(N-1)C_{pm}+C_{re}\right]P_l}{\sum_{k=1}^{N-1}(1-b_k)y_k+y_N}. \tag{3.7}$$

3.1. When N is known

If we differentiate the mean cost rate given in (3.7) with respect to each y_k, $k=1,2,\cdots,N$, and set them equal to 0, then we have

$$C_{mr}\frac{a}{b}\sum_{l=1}^{m}A_N\beta_l y_N^{\beta_l-1}P_l = C_B(y_1,y_2,\cdots,y_N,N) \tag{3.8}$$

and

$$\sum_{l=1}^{m}\beta_l\{A_k y_k^{\beta_l-1} - A_{k+1}b_k(b_k y_k)^{\beta_l-1}\}P_l = \sum_{l=1}^{m}\beta_l A_N(1-b_k)y_N^{\beta_l-1}P_l, \tag{3.9}$$

for $k=1,2,\cdots,N-1$.

Theorem 3.1. For a fixed $y_N(0<y_N<\infty)$, the solution of equation (3.9) with respect to $y_k(y_k>0)$ exists and is unique if $1-a_k b_k>0, k=1,2,\cdots,N-1$, and $\beta_l>1$, $l=1,2,\cdots,m$.

Proof. Let $g(y_k)$ denote the left hand side of equation (3.9). Then $g(y_k)$ is 0 when $y_k=0$. If $1-a_k b_k>0$ and $\beta_l>1$, the derivative of $g(y_k)$ with respect to y_k is shown to be positive as follows.

$$\sum_{l=1}^{m}\beta_l(1-\beta_l)y_k^{\beta_l-2}A_k(1-a_kb_k^{\beta_l})P_l \geqq \sum_{l=1}^{m}\beta_l(1-\beta_l)y_k^{\beta_l-2}A_k(1-a_kb_k)P_l > 0.$$

Thus, $g(y_k)$ is a strictly increasing function of y_k and so the solution of equation (3.9) with respective to y_k is finite and unique.

It was pointed out in LZY (2000) that the condition of $1-a_kb_k>0$ means that the hazard rate adjustment factor a_k should be smaller than the inverse of the improvement factor b_k, which in turn, implies that the hazard rate right after the k th PM, $a_kh_k(b_ky_k)$, should be smaller than the hazard rate just before the PM, $h(y_k)$. Such a condition can be satisfied if each PM tends to improve the system, which is quite reasonable.

Substituting each solution, $y_k(k=1,2,\cdots,N-1)$, of equation (3.9) into equation (3.8), we obtain

$$\frac{a}{b}\sum_{l=1}^{m}\left[A_N\beta_l y_N^{\beta_l-1}\left\{\sum_{k=1}^{N-1}(1-b_k)y_k+y_N\right\}-\sum_{k=1}^{N}A_k\left\{y_k^{\beta_l}-(b_{k-1}y_{k-1})^{\beta_l}\right\}\right]P_l = \frac{(N-1)C_{pm}+C_{re}}{C_{mr}}, \tag{3.10}$$

where each y_k, $(k=1,2,\cdots,N-1)$ is a function of y_N. Thus, the left hand side of equation (3.10) becomes a function of y_N alone.

Theorem 3.2. If $1-a_kb_k>0$, $k=1,2,\cdots,N-1$ and $\beta_l>1$, $l=1,2,\cdots,m$, then the solution of equation (3.10) with respect to $y_N(y_N>0)$ exists and is unique.

Proof. Since

$$\sum_{l=1}^{m}\beta_l y_k^{\beta_l}-1A_k(1-a_kb_k^{\beta_l})P_l \geqq \sum_{l=1}^{m}\beta_l y_k^{\beta_l}-1A_k(1-a_kb_k)P_l > 0,$$

for $k=1,2,\cdots,N-1$, the solution of equation (3.9) with respect to y_k is zero when $y_N=0$. Thus, if $y_N=0$, the left hand side of equation (3.10) equals zero and thus is smaller than the right hand side of (3.10). In addition, the derivative of equation (3.10) with respective to y_N is shown to be positive as follows.

$$\frac{a}{b}\sum_{l=1}^{m}\left[A_N\beta_l(\beta_l-1)y_N^{\beta_l-2}\left\{\sum_{k=1}^{N-1}(1-b_k)y_k+y_N\right\}\right]P_l > 0.$$

Thus, the left hand side of equation (3.10) is a strictly increasing function of $y_N(y_N>0)$ and equals zero at $y_N=0$. Consequently, the solution of equation (3.10) with respect to $y_N(y_N>0)$ exists and is unique.

Once the values of $y_k, k=1,2,\cdots,N$, are obtained, then x_k can be calculated from the relation $x_k=y_k-b_{k-1}y_{k-1}, k=1,2,\cdots,N$ when N is known.

3.2. When N is unknown

As for the case when neither N nor $x_1, x_2, \cdots, x_N$ is known, we may proceed similarly as when N is known. Firstly, we assume that N is fixed and determine the values of $x_1^*, x_2^*, \cdots, x_N^*$ as a function of N alone by applying the method discussed in Section 3.1. Then, the value of N^* is determined as

$$N^* = \min_{N \geq 1} C_B(x_1^*(N), x_2^*(N), \cdots, x_N^*(N), N).$$

Once the value of N^* is determined, then $x_1^*, x_2^*, \cdots, x_{N^*}^*$ can be obtained by replacing N by N^* in its expressions.

3.3. Adaptive sequential Bayesian method

In this section, we discuss the concept of an adaptive sequential PM strategy which is based on the posterior distributions of α and β . When the failure data is recorded at the end of each life cycle, the priors for α and β are adaptively updated and hence become the prior distributions for the next life cycle. Let N_k and $\underline{T}_k = (T_{k1}, T_{k2}, \cdots, T_{kn_k})$ denote the number of failures and the failure times between the $(k-1)$ st PM and the k th PM for $k = 1, 2, \cdots, N$. Then, the joint probability density of $(\underline{T}_k, N_k)$ can be written as

$$f(t_{k1}, t_{k2}, \cdots, t_{kn_k}) = \left\{ \prod_{j=1}^{n_k} h_k(t_{kj}) \right\} \exp\{-H_k(z_k)\},$$

where $H_k(t) = \int_{z_{k-1}}^{t} h_k(s)ds$. To simplify the notations, we let $\underline{t} = \{\underline{t}_1, \underline{t}_2, \cdots, \underline{t}_N\}$ denote the vector of observed failure times throughout the life cycle of the system. Given $\underline{t}$, the posterior distributions of α and β are derived and then it becomes the prior distributions for the next life cycle of the system. The likelihood function of α and β can be written as

$$\begin{aligned} L(\alpha, \beta \mid \underline{t}) &= \prod_{k=1}^{N} f(t_{k1}, t_{k2}, \cdots, t_{kn_k}) \qquad (3.11) \\ &= \left\{ \prod_{k=1}^{N} \prod_{j=1}^{n_k} A_k \alpha \beta \left(t_{kj} - \sum_{i=1}^{k-1} (1-b_i) y_i \right)^{\beta - 1} \right\} \\ &\cdot \exp\left[-\sum_{k=1}^{N} A_k \alpha \left\{ \left(z_k - \sum_{i=1}^{k-1} (1-b_i) y_i \right)^{\beta} - \left(z_{k-1} - \sum_{i=1}^{k-1} (1-b_i) y_i \right)^{\beta} \right\} \right]. \end{aligned}$$

Using Bayes' theorem, the joint posterior distribution of α and β can be expressed as $f(\alpha,\beta_l \mid \underline{t}) = \dfrac{\alpha^{a+\sum_{k=1}^{N} n_k - 1} \beta_l^{\sum_{k=1}^{N} n_k} g_1(\beta_l)\exp\{-\alpha g_2(\beta_l)\} P_l}{\sum_{h=1}^{m} \left[P_h \beta_h^{\sum_{k=1}^{N} n_k} g_1(\beta_h) \Gamma\left(a+\sum_{k=1}^{N} n_k\right) \Big/ (g_2(\beta_h))^{a+\sum_{k=1}^{N} n_k} \right]}$,

where $g_1(\cdot)$ and $g_2(\cdot)$ are defined as

$$g_1(\gamma) = \prod_{k=1}^{N} \prod_{j=1}^{n_k} A_k \left(t_{kj} - \sum_{i=1}^{k-1} (1-b_i) y_i \right)^{\gamma-1}$$

and

$$g_2(\gamma) = b + \sum_{k=1}^{N} A_k \left\{ \left(z_k - \sum_{i=1}^{k-1} (1-b_i) y_i \right)^{\gamma} - \left(z_{k-1} - \sum_{i=1}^{k-1} (1-b_i) y_i \right)^{\gamma} \right\},$$

respectively. Since, $f(\alpha|\beta_l, \underline{t}|) = f(\alpha,\beta_l,\underline{t}) / \Pr(\beta=\beta_l,\underline{t})$, the conditional posterior distribution becomes a gamma distribution with parameters of a^* and b^* which has the following probability density function.

$$f(\alpha \mid \beta_l, \underline{t}) = \frac{(g_2(\beta_l))^{a+\sum_{k=1}^{N} n_k - 1}}{\Gamma\left(a+\sum_{k=1}^{N} n_k\right)} \alpha^{a+\sum_{k=1}^{N} n_k - 1} \cdot \exp\{-\alpha(g_2(\beta_l))\}.$$

where $a^* = a + \sum_{k=1}^{N} n_k$ and $b^* = g_2(\beta_l)$ are the updated shape and scale parameters. In addition, since $\Pr(\beta=\beta_l \mid \underline{t}) = f(\alpha,\beta_l \mid \underline{t}) / f(\alpha \mid \beta_l, \underline{t})$, the posterior distribution of β can be written as

$$\Pr(\beta=\beta_l \mid \underline{t}) = P_l^* = P_l \times \frac{\beta_l^{\sum_{k=1}^{N} n_k} g_1(\beta_l) / (g_2(\beta_l))^{a+\sum_{k=1}^{N} n_k}}{\sum_{h=1}^{m} \left[P_h \beta_h^{\sum_{k=1}^{N} n_k} g_1(\beta_h) / (g_2(\beta_h))^{a+\sum_{k=1}^{N} n_k} \right]}.$$

Note that the posterior distributions of α and β are no longer independent. The optimal sequential PM schedules based on the posterior distributions of α and β can be calculated in a straightforward manner by replacing a, b and P_l in equation (3.7) by a^*, b^* and P_l^*, respectively, and by finding N^* and the optimal PM intervals $x_1^*, x_2^*, \cdots, x_{N^*}^*$.

4. Numerical example

To illustrate the Bayesian method proposed in Section 3, we consider the case when the failure times follow a Weibull distribution and derive the Bayesian optimal sequential PM schedules based on both prior and posterior distributions for two parameters α and β. The values of parameters are taken as $C_{pm}=1.0$, $a_k=(6k+1)/(5k+1)$, $b_k=k/(2k+1)$, $k=0,1,2,\cdots$. As for the values of C_{re} and C_{mr}, we consider the various ratios of C_{mr}/C_{pm} and C_{re}/C_{pm} so that we can analyze the impact of each costs on the results. Regarding the prior parameters, we take $a=2.0$, $b=3.0$, $c=d=2.0$, $\beta_L=2.0$, $\beta_U=4.0$, $m=20$ and thus δ is set equal to $(4.0-2.0)/20=0.1$.

<Table 4.1> Optimal Bayesian PM schedules based on adaptive schemes when $C_{mr}/C_{pm}=2.0$

C_{re}/C_{pm}	Cycle	Failure Times	Optimal PM Number	Optimal PM Interval	Mean Cost Rate
2	0		1	$x_1^*=0.9111$	3.3029
	1	0.8745	1	$x_1^*=0.8555$	3.4728
	2	0.5315	1	$x_1^*=0.8238$	3.6032
	3	0.7108	1	$x_1^*=0.7947$	3.6888
5	0		5	$x_1^*=0.9319$ $x_2^*=0.5247$ $x_3^*=0.4246$ $x_4^*=0.3664$ $x_5^*=0.4935$	4.9505
	1	0.7665 2.2774 2.3235 2.4238	5	$x_1^*=0.8508$ $x_2^*=0.4787$ $x_3^*=0.3872$ $x_4^*=0.3340$ $x_5^*=0.4503$	5.4296
	2	0.7665 2.2774 2.3235 2.4238	5	$x_1^*=0.8033$ $x_2^*=0.4547$ $x_3^*=0.3694$ $x_4^*=0.3200$ $x_5^*=0.4285$	5.6242
	3	0.8611 1.5074 2.2936	5	$x_1^*=0.7914$ $x_2^*=0.4499$ $x_3^*=0.3667$ $x_4^*=0.3187$ $x_5^*=0.4242$	5.6204

<Table 4.1> illustrates the adaptive nature of the Bayesian approach by considering three life cycles of the system. At the end of each cycle, the current failure data is used to update the parameters a, b and P_l and thus, the optimal sequential PM schedules during the next life cycle is renewed based on the updated parameter values. In this example, we start with the simulated data using the hazard function given in (3.1) with $\alpha=1$ and $\beta=3$ and the numerical results are listed in <Table 4.1>. To analyze the effects of the PM cost and the replacement cost on the optimal PM schedules, we consider three cases when the ratio of C_{re}/C_{pm} is equal to $2.0, 5.0$, 10.0 and for fixed ratio of $C_{mr}/C_{pm}=2.0$.

<Table 4.1> *Continued*

C_{re}/C_{pm}	Cycle	Failure Times	Optimal PM Number	Optimal PM Interval	Mean Cost Rate
10	0		8	$x_1^*=1.0584$ $x_2^*=0.5969$ $x_3^*=0.4836$ $x_4^*=0.4178$ $x_5^*=0.3694$ $x_6^*=0.3301$ $x_7^*=0.2967$ $x_8^*=0.4215$	6.4473
	1	0.4710 0.8975 0.9075 0.9212 0.9615 2.7486 3.9194	9	$x_1^*=0.9093$ $x_2^*=0.5208$ $x_3^*=0.4270$ $x_4^*=0.3732$ $x_5^*=0.3339$ $x_6^*=0.3021$ $x_7^*=0.2749$ $x_8^*=0.2510$ $x_9^*=0.3516$	6.9197
	2	0.4170 0.9404 1.0103 1.0503 1.3081 2.6728 3.6682 3.6919	9	$x_1^*=0.8868$ $x_2^*=0.5042$ $x_3^*=0.4111$ $x_4^*=0.3574$ $x_5^*=0.3180$ $x_6^*=0.2861$ $x_7^*=0.2589$ $x_8^*=0.2351$ $x_9^*=0.3332$	7.3717
	3	0.2591 0.3558 0.8103 1.5508 1.6968 2.1539 3.1454	9	$x_1^*=0.8368$ $x_2^*=0.4811$ $x_3^*=0.3956$ $x_4^*=0.3467$ $x_5^*=0.3110$ $x_6^*=0.2821$ $x_7^*=0.2575$ $x_8^*=0.2358$ $x_9^*=0.3281$	7.3751

In our adaptive scheme, we first derive the optimal PM schedules with no failure data and then, the next failure data are adaptively generated based on that schedules. Once the failure data is generated, then the prior parameters a, b and P_l are updated and the renewed sequential optimal PM schedules for the next life cycle is calculated.

<Table 4.1> shows that when all the costs are fixed, then the PM intervals get shorter as the number of PMs increases except the last one, which could be due to the fact that at the end of the last PM the system is replaced by a new one.

It is also observed that the more the parameter values are updated in the adaptive schemes, the shorter the PM intervals become although the difference is very small. It is quite interesting to note from <Table 4.1> that as the replacement cost becomes much higher than the PM cost, then not only does the number of PMs increase, but also the PM intervals become slightly greater to reduce the mean cost rate.

5. Concluding remarks

This paper considers the hybrid PM model suggested by LZY (2000) to derive the optimal sequential PM schedules by adopting the Bayesian approach. The hybrid model combines the age reduction PM model and the hazard rate adjustment PM model and thus, each PM not only reduces the effective age of the system, but also slows down the degradation process under this model. In this paper, we discuss the Bayesian method to determine the optimal sequential PM schedules which minimize the mean cost rate during the life cycle of the system. For Bayesian context, the parameters characterizing the PM model are considered to be random, instead of fixed constants.

To obtain the Bayesian optimal sequential PM schedules, we consider the case when the failure times follow a Weibull model with scale and shape parameters α and β and assume the gamma prior and the discretized beta prior for these two parameters. Applying the adaptive estimation schemes to update the values of α and β based on the failure data observed during the current life cycle, the optimal PM schedules are renewed for the next life cycle sequentially.

It is interesting to note that the structures of replacement cost, PM cost and minimal repair cost have the significant effects on the optimal sequential PM schedules. Such costs not only affects the number of scheduled PMs, but also adjusts the PM intervals. This is mainly due to the fact that the structure of various costs is essential to determine the mean cost rate and we use the mean cost rate as the criterion for optimality of the proposed sequential PM policy.

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