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On Estimating Burr Type XII Parameter Based on General Type II Progressive Censoring

Chansoo Kim[1]

Abstract

This article deals with the problem of estimating parameters of Burr Type XII distribution, on the basis of a general progressive Type II censored sample using Bayesian viewpoints. The maximum likelihood estimator does not admit closed form but explicit sharp lower and upper bounds are provided. Assuming squared error loss and linex loss functions, Bayes estimators of the parameter k, the reliability function, and the failure rate function are obtained in closed form. Finally, a simulation study is also included.

Keywords : Burr Type XII distribution; General progressive Type II censoring; HPD credible interval; Linex loss.

1. Introduction

The two parameter Burr Type XII distribution(which is denoted by BurrXII(c, k) was first introduced in the literature in Burr (1942). Its capacity to assume various shapes often permits a good fit when used to describe biological, clinical or other experimental data. It has also been applied in areas of quality control, reliability, duration and failure time modelling.

The probability density function(p.d.f), cumulative distribution function(c.d.f), reliability function and failure rate of the BurrXII(c, k) distribution are given respectively by

$$\begin{aligned} f(x) &= ckx^{c-1}(1+x^c)^{-(k+1)},\ x>0,\ k>0,\ c>0, \\ F(x) &= 1-(1+x^c)^{-k}, \\ R(t) &= (1+t^c)^{-k}, \end{aligned} \tag{1}$$

and

$$h(t) = \frac{kct^{c-1}}{1+t^c}.$$

In many life test studies, it is common that the lifetimes of test units may not be able to record exactly. For example, in the Type II censoring, the test terminates after a predetermined number of failure occurred in order to save time

1) Assistant Professor, Department of Applied Mathematics, Kongju National University, Gongju 314-705, Korea. E-mail : chanskim@kongju.ac.kr

or cost. Furthermore, some test units may have to be removed at different stages in the study for various reasons. This would lead to progressive censoring. Progressive Type II censored sampling is an important method of obtaining data in lifetime studies. Live units removed early on can be readily used in other tests, thereby saving cost to the experimenter, and a compromise can be achieved between time consumption and the observation of some extreme values. Although censoring occurs frequently in many applications, there are relatively few work on it. Some early work can be found in Cohen (1963), Mann (1971), Thomas and Wilson (1972), Viveros and Balakrishnan (1994), Balakrishnan and Sandhu (1996), Balakrishnan and Aggarwala (2000), and Fernandez (2004).

Let us consider the following progressive Type II censoring scheme which was generalized by Balakrishnan and Sandhu (1996). Suppose n randomly selected units were placed on a life test; the first r failure times, $X_1, \cdots, X_r$ are not observed; at time X_{r+1}, R_{r+1} units are removed from the test randomly; at time X_{r+2}, R_{r+2} units are removed from the test randomly, so on. Finally, at the time of the mth failure, X_m, the experiment is terminated and the remaining R_m units are removed from the test. Therefore, $X_{r+1} \leq \cdots \leq X_m$ are the lifetimes of the completely observed units to fail and $R_{r+1}, \cdots, R_m$ are the number of units withdrawn from the test at these failure times. At $(i+1)$th failure, there are n_i units on test where

$$n_i = n - i - \sum_{j=r+1}^{i} R_j, \quad i = r+1, \cdots, m-1 \ .$$

The R_i's, m and r are prespecified integers which must satisfy the following conditions:

$$0 \leq r < m \leq n, \ 0 \leq R_i \leq n_{i-1} - 1 \ \text{for} \ i = r+1, \cdots, m-1 \ \text{with} \ n_r = n - r$$

and $R_m = n_{m-1} - 1$.

In the next section, we deal with the problem of estimating the parameter k, the reliability function and the failure rate function of BurrXII(c, k) under a squared error loss(SEL) and linex loss(LL) functions. The prior distribution for the parameter of the model has been taken as a natural conjugate prior.

The SEL and linex loss functions are the following forms:

$$L(\theta, \hat{\theta}) = (\theta - \hat{\theta})^2,$$

$$L(\theta, \hat{\theta}) = b(e^{a(\hat{\theta} - \theta)} - a(\hat{\theta} - \theta)) - 1), \ a \neq 0, \ b > 0, \tag{2}$$

where a and b are shape and scale parameters of the loss function, respectively.

One of the most popular asymmetric loss function is the linex loss function which is introduced by Vraian (1975) and further properties of this loss function have been investigated by Zellner (1986). For a small values of a (near to zero), the linex loss function is same as SEL and for choice of negative or positive values of a, it gives more weight to overestimation or underestimation.(For details see Zellner (1986))

Section 2 is considered a general progressive Type II censoring scheme and the problem of MLE and a Bayes estimation of the Burr Type XII distribution. Subsection 2.1 concerned with ML estimation. A Bayesian approach to estimation under SEL and linex loss function is presented in subsection 2.2. Finally, a simulation study is given in Section 3.

2. Estimation based on general progressive samples

Suppose that n randomly selected units from BurrXII(c, k) population, where c is known and k is unknown, are put on test under a general progressive Type II censoring scheme. Let $\boldsymbol{x}=(x_{r+1},\cdots,x_m)$ be the observed sample.

The likelihood function of k is then

$$L(k|\boldsymbol{x}) = \frac{n!}{r!(n-r)!}\left(\prod_{j=r}^{m-1} n_j\right)[F(x_{r+1})]^r \prod_{i=r+1}^{m} f(x_i|k)[R(x_i)]^{R_i}. \tag{3}$$

In accordance with (1) and (3), the likelihood becomes proportional to

$$L(k|\boldsymbol{x}) \propto k^{m-r}(1-\exp(-kq))^r \exp(-kw), \tag{4}$$

where q is the observed value of $Q=\log(1+x_{r+1}^c)$ and w is the observed value of the statistic $W(X)=\sum_{i=r+1}^{m}(R_i+1)\log(1+x_i^c)$.

2.1. Maximum likelihood estimation

The maximum likelihood estimator(MLE) of k, denoted by $\hat{k}$, can be derived by solving the equation

$$L'(k|\boldsymbol{x}) = \frac{\partial \log L(k|\boldsymbol{x})}{\partial k} = \frac{rq}{\exp(kq)-1} + \frac{m-r}{k} - w = 0\,. \tag{5}$$

Obtaining a closed form expression for $\hat{k}$ is not possible. The solution could be obtained by using Newton's method. If $r=0$, then

$$\hat{k} = \frac{m}{\sum_{i=1}^{m}(R_{i+1})\log(1+x_i^c)}\,.$$

The following theorem provides sharp bounds on the value of $\hat{k}$. Using the inequality

$$\max\left(0, \frac{6-2x}{6+x}\right) < \frac{x}{\exp(x)-1} < \frac{1}{1+\frac{x}{2}+\frac{x^2}{6}} ,$$

and Cardan's formula, the following theorem can be constructed.

Theorem 1. The MLE of k, $\hat{k}$ satisfies inequality that $\hat{k}_L \le \hat{k} \le \hat{k}_U$, where

$$\hat{k}_{U=}\left(u+\sqrt{u^2+v^3}\right)^{1/3} - \left(-u+\sqrt{u^2+v^3}\right)^{1/3} - \frac{a}{3}$$

and

$$\hat{k}_L = \max\left(\frac{m-r}{w}, \frac{\rho+\sqrt{\rho^2+24qwm)}}{2qw}\right),$$

where

$\rho = (m-3r)q-6w$, $u = ab/6-c/2-(a/3)^2$,$v = b/3-(a/3)^2$, $a = (3w-(m-r)q)/wq$, $b = (3(m-r)q-6w)/wq2$ and $c = -6m/wq^2$.

According to Theorem 1, $\hat{k} \in (\hat{k}_L, \hat{k}_U)$, it is convenient to employ the rule of false position to determine the MLE. In that case, the first iteration provides:

$$\hat{k}_L + \frac{(\hat{k}_U - \hat{k}_L)L'(\hat{k}_L|x)}{L'(\hat{k}_L|x) - L'(\hat{k}_U|x)} = \hat{k} .$$

It is also clear that the MLEs of $h(t)$ and $R(t)$ are given by $\hat{k}(t) = \hat{k}ct^{c-1}/(1+t^c)$ and $\hat{R}(t) = (1+x^c)^{-\hat{k}}$.

2.2. Bayesian estimation

Suppose that the parameter c is assumed to be known and k has a gamma prior density of the form

$$\pi(k) = \frac{\beta^\alpha}{\Gamma(\alpha)} k^{\alpha-1} e^{-k\beta}, \quad k > 0, \quad \alpha,\beta > 0 . \tag{6}$$

It follows from (3) and (6), that the posterior density of k is given by

$$\pi(k|x) = \frac{(w+\beta)^{m+\alpha-r}\, k^{m+\alpha-r-1}}{\Gamma(m+\alpha-r)\, C_q[w+\beta, m+\alpha-r]} (1-\exp(-kq))^r \exp(-k(w+\beta)) , \tag{7}$$

where

$$C_q[a,b] = a^b \sum_{l=0}^{r} \frac{(-1)^l \frac{r!}{l!(r-l)!}}{(a+lq)^b}, \quad a,b > 0, \text{ with } C_0 = 1.$$

From (7), the posterior distribution of k given x is found to be

$$\Pi(k|x) = \int_0^k \pi(t|x)dt = \frac{D_q[w+\beta, m+\alpha-r; k]}{C_q[w+\beta, m+\alpha-r]} , \tag{8}$$

where

$$D_q[a,b;k]=a^b\sum_{l=0}^{r}\frac{(-1)^l\frac{r!}{l!(r-l)!}}{(a+lq)^b}I[b;(a+lq)k]$$

and $I[\,\cdot\,;\,\cdot\,]$ is the incomplete gamma ratio function defined by

$$I[b\,;\theta]=\frac{1}{\Gamma(b)}\int_0^{\theta}z^{b-1}\exp(-z)dz,\ b,\theta>0.$$

2.2.1. Bayes estimators under SEL

Assuming the commonly used squared error loss(SEL) function, $L(k,\hat{k}_S)=(k-\hat{k}_S)^2$, the Bayes estimator of k(i.e., the value $\hat{k}_S$ that minimizes the posterior expected loss) is the mean of posterior density (7), which is given by

$$\hat{k}_S=E(k|\boldsymbol{X})=\frac{C_q[w+\beta,m+\alpha-r+1]}{C_q[w+\beta,m+\alpha-r]}\frac{m+\alpha-r}{w+\beta}. \tag{9}$$

Other problems of interest are those of estimating the failure rate $h(t)$ and the reliability function $R(t)$, with fixed $t>0$. For SEL, the Bayes estimators of $h(t)$ and $R(t)$ are found to be

$$\hat{h}(t)_S=E\left(\frac{kct^{c-1}}{1+t^c}\right)=\frac{ct^{c-1}}{1+t^c}\frac{m+\alpha-r}{w+\beta}\frac{C_q[w+\beta,m+\alpha-r+1]}{C_q[w+\beta,m+\alpha-r]} \tag{10}$$

and

$$\hat{R}(t)_S=E((1+t^c)^{-k})=\frac{C_q[w+\beta+q_t,m+\alpha-r]}{C_q[w+\beta,m+\alpha-r]}\left(\frac{w+\beta}{w+\beta+q_t}\right)^{m+\alpha-r}, \tag{11}$$

where $q_t=\log(1+t^c)$.

If $r=0$, then the Bayes estimators of k, $h(t)$ and $R(t)$ have the following forms:

$$\hat{k}_S=\frac{m+\alpha}{w+\beta},$$

$$\hat{h}(t)_S=\frac{ct^{c-1}}{1+t^c}\frac{m+\alpha}{w+\beta},$$

and

$$\hat{R}(t)_S=\left(1+\frac{\log(1+t^c)}{w+\beta}\right)^{-(m+\alpha)}$$

where $w=\sum_{i=1}^{m}(R_i+1)\log(1+x_i^c)$ and these estimators are equivalent to the estimators which is based on progressive Type II censoring.

2.2.2. Bayes estimators under Linex Loss

The use of SEL has been justified in statistical literature on various grounds because it is the most commonly used loss function. But SEL is symmetric loss function, which gives equal importance to underestimation and overestimation with equal magnitude. It is generally agreed upon that in life testing and reliability context overestimation and underestimation may not be of equal importance. Thus, the use of asymmetric loss function seems to be more justified. Among these, one of the most popular asymmetric loss function is the linex loss(LL) function. In this subsection, we deal with the problems of estimation of parameter k, reliability function $R(t)$ and failure rate $h(t)$ under linex loss function.

From (7), the Bayes estimator of k is the value $\hat{k}_{LL}$ that minimizes the posterior expected loss. The Bayes estimators of k, $h(t)$ and $R(t)$ are followings, respectively:

$$\begin{aligned}\hat{k}_{LL} &= -\frac{1}{a}log\left(E(e^{-ak}|\boldsymbol{X})\right) \\ &= \frac{1}{a}\left(\log\left(\frac{C_q[w+\beta, m+\alpha-r]}{C_q[w+\beta+a, m+\alpha-r]}\right)+(m+\alpha-r)\log\left(1+\frac{a}{w+\beta}\right)\right),\end{aligned} \tag{12}$$

$$\begin{aligned}\hat{h}(t)_{LL} &= -\frac{1}{a}log\left(E\left(\exp\left(-\frac{akct^{c-1}}{1+t^c}\right)|\boldsymbol{X}\right)\right) \\ &= \frac{1}{a}\left(\log\left(\frac{C_q[w+\beta, m+\alpha-r]}{C_q[w+\beta+q_{a,t}, m+\alpha-r]}\right)+(m+\alpha-r)\log\left(1+\frac{q_{a,t}}{w+\beta}\right)\right),\end{aligned} \tag{13}$$

and

$$\hat{R}(t)_{LL} = -\frac{1}{a}log\left\{E\left(e^{-a(1+t^c)^{-k}}|\boldsymbol{X}\right)\right\}, \tag{14}$$

where $q_{a,t} = act^{c-1}/(1+t^c)$.

If $r=0$, then $\hat{k}_{LL} = (m+\alpha)/a\log(1+a/(w+\beta))$ and $\hat{h}(t)_{LL} = (m+\alpha)/a\log(1+q_{a,t}/(w+\beta))$.

The posterior expectation given by (14) can not, generally, be obtained in a closed form. Therefore, in such situations, we use numerical integration technique, which can be computationally intensive in high dimensional parameter spaces. One can also use approximate methods such as the approximate form of Lindley (1980) or that of Tierney and Kadane(T-K) (1986). We adopt here the T-K approximation since its error is of order $O(n^{-2})$, while the error in using Lindley's approximate form is of order $O(n^{-1})$.

2.2.3. The approximation of Tierney and Kadane

This procedure was developed by Tierney and Kadane (1986) to evaluate an approximation of the posterior moments and marginal densities. Let $l(k;x)$ be the likelihood function of k based on the n observations and $\pi(k|x)$ denote the posterior distribution of k. Then the posterior mean of a function $\phi(k)$ can be written as

$$E(\phi(k)|\boldsymbol{X}) = \int \phi(k)\pi(k|\boldsymbol{x})dk = \frac{\int e^{nL^*}dk}{\int e^{nL}dk} \tag{15}$$

where $L(k) = 1/n\log\pi(k|\boldsymbol{x})$ and $L^*(k) = L(k) + 1/n\log\phi(k)$.

Following T-K approximation, the equation in (15) can be approximated as following forms;

$$E(\phi(k)|\boldsymbol{X}) = \left(\frac{|\sigma^*|}{|\sigma|}\right)^{\frac{1}{2}} \exp\left(n(L^*(\hat{k}^*) - L(\hat{k}))\right), \tag{16}$$
$$= \left(\frac{|\sigma^*|}{|\sigma|}\right)^{\frac{1}{2}} \frac{\phi(\hat{k}^*)\pi(\hat{k}^*|\boldsymbol{x})}{\pi(\hat{k}|\boldsymbol{x})},$$

where $\hat{k}^*$ and $\hat{k}$ maximizes $L^*(k)$ and $L(k)$, respectively and σ^* and σ are negative of the inverses of the second derivatives of $L^*(k)$ and $L(k)$ at $\hat{k}^*$ and $\hat{k}$ respectively.

We apply this approximation to obtain the Bayes estimators of the reliability function $R(t)$ given by (14). In this case, $\phi(k) = \exp(-a(1+t^c)^{-k})$ and the functions L and L^* are given by respectively,

$$L(k) = \frac{1}{n}\{(m+\alpha-r-1)\log k + r\log(1-e^{-kq}) - k(w+\beta)\} \tag{17}$$

and

$$L^*(k) = L(k) - \frac{a(1+t^c)^{-k}}{n}\ . \tag{18}$$

Let $L_1 = \partial L(k)/\partial k$ and $L_{11} = \partial^2 L(k)/\partial k^2$ be the first and second derivatives of $L(k)$. Then, the posterior mode $\hat{k}$ is obtained by setting L_1 to be zero and solving the resulting nonlinear equation in k.

Similar derivatives are needed to determine the mode $\hat{k}^*$ of $L^*(k)$. Let $L_1^* = \partial L^*(k)/\partial k$ and $L_{11}^* = \partial^2 L^*(k)/\partial k^2$ be the first and second derivatives of $L^*(k)$. Differentiate (18) with respect to k and set the result L_1^* to be zero. The mode $\hat{k}^*$ of k can be obtained iteratively by solving the two resulting equations.

Substituting from L_{11} and L_{11}^* in (16), the Bayes estimator $\hat{R}(t)_{LL}$ of a function $\phi(k) = \exp(-a(1+t^c)^{-k})$ under linex loss function takes of the form

$$\hat{R}(t)_{LL} = -\frac{1}{a}log\left\{\left(\frac{\sigma}{\sigma^*}\right)^{\frac{1}{2}}\left(\frac{\hat{k}^*}{\hat{k}}\right)^{m+\alpha-r-1}\left(\frac{1-e^{-\hat{k}^*q}}{1-e^{-\hat{k}q}}\right)\exp\left(-a(1+t^c)^{-\hat{k}^*} - (w+\beta)(\hat{k}^* - \hat{k})\right)\right\} \tag{19}$$

where

$$\sigma = (m+\alpha+r-1)/\hat{k}^2 + rqe^{\hat{k}q}/(e^{\hat{k}q}-1)^2,$$

and

$$\sigma^* = (m+\alpha+r-1)/\widehat{k^{*2}} + rqe^{\hat{k}^* q}/(e^{\hat{k}^* q}-1)^2 + a(\log(1+t^c))(1+t^c)^{-\hat{k}^*}.$$

3. A simulation study and Comparisons

Applying the algorithms of Aggarwala and Balakrishnan (1998), the following steps are used to generate a general progressive Type II censored sample the BurrXII$(c,\ k)$ distribution.

(1) Generate V_m from Beta distribution with parameters $n-r$ and $r+1$.

(2) Independently generate Z_{r+i} from $U(0,1)$ for $i=1,\cdots,m-r-1$.

(3) Set $V_{r+i} = Z_{r+i}^{\frac{1}{a_{r+i}}}$, $a_{r+i} = i + \sum_{j=m-i+1}^{m} R_j$, $i=1,\cdots,m-r-1$.

(4) Set $U_{r+i} = 1 - V_{m-i+1} V_{m-i+2} \cdots V_m$, $i=1,\cdots,m-r$.

(5) Set $X_i = F^{-1}(U_i)$, $i=r+1,\cdots,m$.

This is the desired general progressive Type II censored sample from the BurrXII$(c,\ k)$. The MLE and Bayes estimates with respect to SEL and linex loss functions of the parameter k, the reliability function $R(t)$ and the failure rate function $h(t)$ are compared via Monte Carlo simulation study according to the following steps.

(1) For given $c=2$, $k=3$ and $r=1$, generate a general progressive Type II censored sample of size $m=8$ with given $R_2=0, R_3=3, R_4=0, R_5=3,$ $R_6=0,$ $R_7=0,$ $R_{8=5}$ from the BurrXII$(c,\ k)$ whose p.d.f is given by (1) according to the above simulation algorithm.

(2) Following above step, the m ordered observed failure times with a general progressive Type II censoring scheme $(R_2, \cdots, R_m)$ are presented in <Table 1>.

<Table 1> Failure Time $\boldsymbol{X}$ and a censoring scheme $\boldsymbol{R}$

i	1	2	3	4	5	6	7	8
X_i	-	0.2636	0.2999	0.3050	0.3295	0.3341	0.5354	0.5466
R_i		0	3	0	3	0	0	5

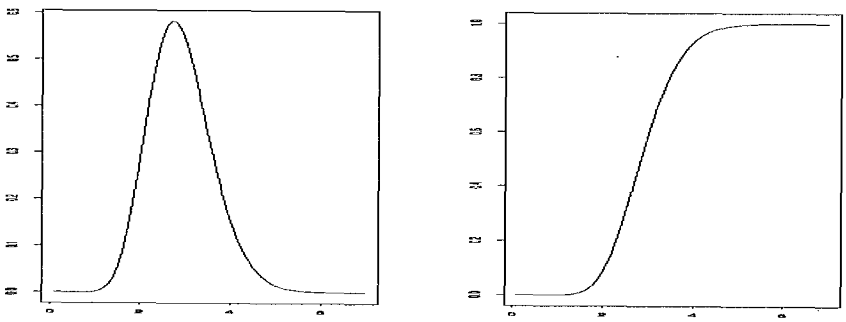
According to Theorem 1, it is found that $\hat{k}_L = 2.787469$ and $\hat{k}_U = 2.787582$. Since $\hat{k} \in (\hat{k}_L,\ \hat{k}_U)$,it is convenient to employ the rule of the false position to determine the MLE. In that case, the first iteration provides:

$$\hat{k}_L + \frac{(\hat{k}_U - \hat{k}_L) L'(\hat{k}_L|x)}{L'(\hat{k}_L|x) - L'(\hat{k}_U|x)} = 2.787499$$

It is also clear that the MLEs of $R(t)$ and $h(t)$ are given respectively by

$\hat{R}(t) =$ 0.5368 and $\hat{h}(t) = 2.230$ at $t = 0.5$.

The computational results for the Bayes estimates relative to SEL and linex loss functions are displayed in Tables (2-5) for the case of known parameter c and the prior parameters α=9 and β=3 which yield the value of k=3 (as the true value). The MLE, posterior mean, median and mode of k under SEL are displayed in <Table 2>. Since the posterior density $\pi(k|x)$ is unimodal and positively skewed, it is clear that $\hat{k}_S > \hat{k}_{median} > \hat{k}_{\text{mode}}$.



<Figure 1> Posterior density and Posterior distribution of k

<Table 2> MLE, posterior mean, median, mode 90% two-sided and HPD credible interval for k under SEL

Parameter	MLE	Posterior mean	Posterior median	Posterior mode
k	2.7875	2.8962	2.8395	2.7258
	90% two-sided credible interval	Interval width	90% HPD credible interval	Interval width
	(1.8453, 4.1402)	2.2949	(1.7590, 4.0123)	2.2533

The two-sided 90% credible interval for k is given by (1.845311, 4.140184) in <Table 2> Due to a little asymmetric of $\pi(k|x)$, this interval is 0.03157 units wider than the corresponding HPD credible interval, (1.748977, 4.012280).

<Table 3> Bayes estimate of k for various a

Parameter	a	$\hat{k}_{LL}$
k	1.0	2.674289
	0.01	2.89361
	-1.0	3.175091

The Bayes estimates of MLE, k, $R(t)$ and $h(t)$ with respect to SEL and the linex loss function are obtained for various a in <Table 3> and <Table 4>. In <Table 3>, the Bayes estimates of the parameter k relative to linex loss function are sensitive to the values of the a.

<Table 4> MLE and Bayes estimate of k, $R(t)$ and $h(t)$ under SEL and linex loss function at $t = 0.5$

Parameter	MLE	Bayes estimate(SEL)	Bayes estimate(Linex)
k	2.787499	2.896455	2.89360
$R(t)$	0.536863	0.530319	0.56498
$h(t)$	2.230	2.306924	2.31534

For small value a, the linex loss function is almost symmetric and not far from a SEL. Therefore, the Bayes estimates under SEL and linex loss functions are almost the same. If a goes to a negative value, then it tends to give more weight to overestimation. Otherwise, it gives more weight to underestimation.

References

[1] Aggarwala, R. and Balakrishnan, N. (1998). Some properties of progressive censored order statistics from arbitrary and uniform distributions with applications to inference and simulation. *Journal of Statistical Planning and Inference*, Vol. 70, 35-49.

[2] Balakrishnan, N. and Aggarwala, R. (2000). *Progressive censoring: Theory, Methods, and Applications*. Boston, MA: Birkhauser.

[3] Balakrishnan, N. and Sandhu, R.A. (1996). Best linear unbiased and maximum likelihood estimation for exponential distributions under general progressive Type-II censored samples. *Sankhya B*, Vol. 58, 1-9.

[4] Burr, IW. (1942). Cumulative frequency functions. *Annals of Mathematics*, Vol. 43, 1441-1449.

[5] Cohen, A.C. (1963). Progressively censored samples in life testing. *Technometrics*, Vol. 5, 327-339.

[6] Fernandez, A.J. (2004). On estimating exponential parameters with general type II progressive censoring. *Journal of Statistical Planning and Inference*, Vol. 121, 135-147.

[7] Lindley, D.V. (1980). Approximate Bayesian methods. *Trabajos de Stadistca*, Vol. 21, 223-237.

[8] Mann, N.R. (1971). Best linear invariant estimation for Weibull parameters under progressive censoring. *Technometrics*, Vol. 13, 521-533.

[9] Thomas, D.R. and Wilson, W.M. (1972). Linear order statistic estimation for the two parameter Weibull and extreme-value distribution under Type II progressively censored samples. *Technometrics*, Vol. 14, 679-691.

[10] Tierney, L. and Kadane, J. B. (1986). Accurate approximations for posterior moments and marginal densities. *Journal of the American Statistical Association*. Vol. 81,82-86.

[11] Viveros, R. and Balakrishnan, N. (1994). Interval estimation of life characteristics from progressively censored samples. *Technometrics*, Vol. 36, 84-91.

[12] Varian, H. (1975). *A Bayesian approach to real estate assessment*, in Fienber S.E., Zellner A.(Eds.), Studies in Bayesian Econometrics and Statistics in honour of Leonard J. Savage, North-Holland, Amsterdam, 195-208.

[13] Zellner, A. (1986) Bayesian estimation and prediction using asymmetric loss function. *Journal of the American Statistical Association*, Vol. 81, 446-451.

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