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The Selection of Strategies for Variance Estimation under πPS Sampling Schemes

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Abstract

When using the well-known variance estimator of Sen (1953) and Yates and Grundy (1953) in inclusion probability proportional to size sampling, we often encounter the problems due to the calculation of the joint probabilities. Särndal (1996) and Knottnerus (2003) proposed alternative strategies for variance estimation to avoid those problems in the traditional method. We discuss some of practical issues that arise when they are used. Also, we describe the traditional strategy using a sampling procedure available in a statistical software. It would be one of the attractive choices for design-based variance estimation.

Keywords : Sen-Yates-Grundy variance estimator; Joint probabilities; Poisson sampling; systematic PPS sampling; Sampford's sampling method.

1. Introduction

In statistical agencies, there is considerable interest in the estimation of precision of survey estimates. But design-based variance estimation for most inclusion probability proportional to size (πPS) sampling schemes is often computationally difficult because it requires determining the joint probabilities. These quantities play the key role in the variance estimation using the Horvitz and Thompson (H-T) (1952) estimator or the generalized regression (GREG) estimator of a population total, and hence the problem of estimating variance is virtually identical with the problem of calculating the joint probabilities.

Särndal (1996) proposed a variance estimation strategy using Poisson sampling to replace the traditional fixed sample size πPS scenario using the H-T estimator. His approach uses the GREG estimator and entails the calculation of a weighted squared residual sum instead of the joint probabilities. He recommended this method for the Generalized Estimation System (GES) developed in Statistics Canada in 1994.

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Knottnerus (2003) in Statistics Netherlands also proposed a methodology based on a general framework in which the intraclass correlation coefficient instead of the joint probabilities plays a crucial role. He pays attention to systematic PPS sampling among the πPS sampling schemes and uses the H-T estimator, not the GREG estimator.

In this paper, we first review the three strategies for variance estimation mentioned above: (a) the traditional approach using fixed-size πPS sampling; (b) the approach using random-size πPS sampling; (c) the approximate approach using fixed-size πPS sampling. Second, for the two alternative approaches (b) and (c), we discuss some of practical issues that arise when they are used. It seems that they do not realize the essential advantages of πPS sampling or operational simplicity. Third, we give special emphasis to the traditional strategy using a sampling procedure called Sampford's method, which is available in a statistical software. Finally, we give an illustrative example to show the efficiency of the method.

2. Traditional Approach using Fixed-Size πPS Sampling

Considering a finite population $U = \{U_1, \cdots, U_N\}$ of N identifiable units, let S be a probability sample of the size n drawn from U according to a given sampling design $p(\cdot)$ with the inclusion probabilities $\pi_j = p(U_j \in S)$ and the joint probabilities $\pi_{ij} = p(U_i \in S \wedge U_j \in S)$.

Let Y_j denote the value of the character of interest, y , for the j th unit. Assuming $\pi_j > 0$ for all j , the unbiased H-T estimator $\hat{Y}_{HT}$ of the population total $Y = \sum_{j=1}^{N} Y_j$ is defined by

$$\hat{Y}_{HT} = \sum_S a_j Y_j \ , \tag{2.1}$$

where $a_j = 1/\pi_j$.

The variance of the H-T estimator, say $V(\hat{Y}_{HT})$, is given by

$$V(\hat{Y}_{HT}) = \sum\sum_U (a_i a_j / a_{ij} - 1)\, Y_i Y_j \ , \tag{2.2}$$

where $a_{ij} = 1/\pi_{ij}$ and $a_{jj} = a_j = 1/\pi_j$.

The well-known unbiased estimator of $V(\hat{Y}_{HT})$ proposed by Sen (1953) and Yates and Grundy (1953) is

$$\hat{V}(\hat{Y}_{HT}) = -(1/2)\sum\sum_S (1 - a_{ij} / a_i a_j)(a_i Y_i - a_j Y_j)^2 \ . \tag{2.3}$$

Let m_j be a known positive size measure, believed to be strongly correlated with Y_j . If the inclusion probabilities π_j are proportional to the m_j , then the method is called a πPS sampling scheme.

The classical πPS scenario for variance estimation is as follows:

Construct the πPS sampling satisfying

a. fixed sample size design

b. exact design unbiasedness for the estimator of population total and its variance

c. some desirable properties such as $a_i a_j - a_{ij} < 0$ for all $i \neq j$

Then use the H-T estimator to estimate the population total and compute the joint probabilities π_{ij} which are generally all unequal. Finally use them to compute the variance estimator $\hat{V}(\hat{Y}_{HT})$.

Note that the variance (2.2) and variance estimator (2.3) of the H-T estimator involve the joint probabilities π_{ij} . For most of πPS sampling schemes, especially with $n > 2$, considerable difficulties can arise in the determination of these quantities. Recently alternative methods for variance estimation that do not depend on the π_{ij} have been developed, as shown in the following sections.

3. Approach using Random-Size πPS Sampling

The desirable features a, b and c quoted in the classical πPS scenario for variance estimation conflict with the goal: correct computation of the joint probabilities in a double-sum calculation with $n(n-1)/2$ terms using the Sen-Yates-Grundy formula (2.3). Särndal (1996) considered abandoning some features, that is, a and b, and proposed the following alternative approach using the generalized regression (GREG) estimator:

1. Compute the inclusion probabilities $\pi_j = nm_j / \sum_U m_j$, $j = 1, \cdots, N$. Carry out the Poisson sampling with these π_j .
2. To estimate the population total Y , use the GREG estimator $\hat{Y}_{GREG}$ below generated by $\boldsymbol{x}_j = \boldsymbol{x}_{0j}$. Here $\boldsymbol{x}_j$ is the value for the unit j of a auxiliary vector $\boldsymbol{x}$ whose total, $\boldsymbol{X} = \sum_U \boldsymbol{x}_j$, is assumed to be known from a reliable source and $\boldsymbol{x}_{0j}$ is the observed value of $\boldsymbol{x}_j$ where $\boldsymbol{X}_0 = \sum_U \boldsymbol{x}_{0j}$.

$$\hat{Y}_{GREG} = \hat{Y}_{HT} + \hat{B}'(\boldsymbol{X}_0 - \hat{\boldsymbol{X}}_{0HT}) \quad , \tag{3.1}$$

where $\hat{X}_{0HT} = \sum_S a_j x_{0j}$ and $\hat{B}$ is the vector of regression coefficient estimators.

3. Variance estimation is obtained via the weighted squared residual form of

$$\hat{V} = \sum_S a_j \phi_j g_j^2 e_j^2 \quad , \tag{3.2}$$

where $\phi_j = a_j - 1$ and g_j and e_j are the calibration factor and the regression residual for the unit j , respectively (See Särndal (1996), pages 1290-1291).

As shown in (3.2), the variance estimator does not depend on the joint probabilities π_{ij} and is always non-negative.

4. Approximate Approach using Fixed-Size πPS Sampling

Unlike the classical variance estimator, the one suggested by Knottnerus (2003) is not based on the joint probabilities. His approach depends on the generalized sampling autocorrelation coefficient ρ , that is, the correlation coefficient between two randomly chosen observations from a random sample S that is selected from a population according to an unequal probability sampling design. In the literature this ρ is often used for indicating the loss of efficiency in comparison between systematic sampling (or cluster sampling) and simple random sampling. Then ρ is used as a measure of homogeneity and usually called the intracluster or the intraclass correlation coefficient(See Cochran (1977), Kish (1995) and Särndal et al. (1992)). Knottnerus' approach is summarized as follows.

After selecting the sample S using a given sampling design without replacement, the elements of the sample are ordered according to a random permutation, resulting in a randomly ordered sample $s = \{u_1, \cdots, u_n\}$, where n is fixed. When tracing both the random sample selection mechanism and the random permutation of the sample elements, it can be shown that the so-called first- and second-order drawing probabilities, p_j and $p_{j_1 j_2}$ are given by

$$p_j = P(u_i = U_j) = P(u_i = U_j \ \wedge \ U_j \in S)$$
$$= P(u_i = U_j \mid U_j \in S) P(U_j \in S) = \frac{1}{n}\pi_j \tag{4.1}$$

$$p_{j_1 j_2} = P(u_{i_1} = U_{j_1} \wedge u_{i_2} = U_{j_2}) = \frac{1}{(n-1)} \frac{1}{n} \pi_{j_1 j_2} \quad . \tag{4.2}$$

Next define $Z_j = Y_j / p_j$, $j = 1, \cdots, N$ and the corresponding z values of the randomly ordered elements in s by $z_1, z_2, \cdots, z_n$ with $z_i = y_i / p_i$. Then the expectation and the variance of z_i can be derived as well as the covariance between z_i and z_j :

$$\mu_z = E(z_i) = \sum_{j=1}^{N} p_j Z_j = Y \tag{4.3}$$

$$\sigma_z^2 = Var(z_i) = \sum_{j=1}^{N} p_j (Z_j - \mu_z)^2 = \sum_{j=1}^{N} \frac{Y_j^2}{p_j} - Y^2 \tag{4.4}$$

$$Cov(z_i, z_j) = E(z_i - \mu_z)(z_j - \mu_z) = \sum_{i=1}^{N}\sum_{j=1}^{N} p_{ij} \frac{Y_i Y_j}{p_i p_j} - Y^2 \quad . \tag{4.5}$$

Now denoting an unbiased estimator of Y or μ_z in (4.3) by $\hat{Y}_z$ or $\hat{\mu}_z$, we have

$$\hat{Y}_z = \hat{\mu}_z = \frac{1}{n}\sum_{i=1}^{n} z_i = \frac{1}{n}\sum_{i=1}^{n} \frac{y_i}{p_i} \quad . \tag{4.6}$$

Note that since $\pi_i = np_i$ according to (4.1), it turns out that $\hat{Y}_z$ is identical to $\hat{Y}_{HT}$ from (2.1). Then for samples of size n

$$Var(\hat{Y}_z) = [1 + (n-1)\rho_z] \frac{\sigma_z^2}{n} \quad , \tag{4.7}$$

where ρ_z is called the generalized sampling autocorrelation coefficient and expressed as

$$\rho_z = \frac{Cov(z_i, z_j)}{\sigma_z^2} \quad , \quad 1 \leq i \neq j \leq n \quad . \tag{4.8}$$

Note that since $\hat{Y}_{HT} = \hat{Y}_z$ above, the variance of $\hat{Y}_{HT}$ from (2.3) equals

$$Var(\hat{Y}_{HT}) = [1 + (n-1)\rho_z] \frac{\sigma_z^2}{n} \quad . \tag{4.9}$$

An unbiased estimator of $Var(\hat{Y}_z)$ is denoted by

$$\widehat{Var}(\hat{Y}_z) = \left[1 + n\frac{\rho_z}{1-\rho_z}\right] \frac{s_z^2}{n} \quad , \tag{4.10}$$

where $s_z^2 = \sum_{i=1}^{n}(z_i - \sum_{i=1}^{n} z_i/n)^2/(n-1)$.

When ρ_z is known, the variance estimator (4.10) is always non-negative and does not require any joint probabilities. But when ρ_z is unknown, it must be estimated and the joint probabilities are absolutely needed. The idea of the estimation procedure for ρ_z is that the fixed value z_j can be decomposed numerically into two components: (i) a part Φ_j that is linear in the powers of p_j and (ii) an uncorrelated remainder Ω_j . By using this decomposition, ρ_z can be estimated by

$$\hat{\rho}_z = \frac{\sigma_\varphi^2 \hat{\rho}_\varphi + (s_z^2 - \sigma_\varphi^2)\hat{\rho}_\omega}{s_z^2 + (\hat{\rho}_\varphi - \hat{\rho}_\omega)\sigma_\varphi^2} , \tag{4.11}$$

where $\sigma_\varphi^2 = \sum_{j=1}^{N} p_j \hat{\Phi}_j^2$ with the estimate of the Φ_j , and $\hat{\rho}_\varphi$ and $\hat{\rho}_\omega$ are the estimated sampling autocorrelation coefficients depending on the parts Φ_j and Ω_j , respectively.

Note that when a sample is drawn according to Brewer's (1963) method for $n = 2$ only, $\hat{\rho}_\varphi$ is given by

$$\hat{\rho}_\varphi = \sum_{i=1}^{N} \sum_{j \neq i}^{N} \frac{p_i p_j (1 - p_i - p_j)\hat{\Phi}_i \hat{\Phi}_j}{\gamma(1 - 2p_i)(1 - 2p_j)\sigma_\phi^2} , \tag{4.12}$$

where γ is defined by $\gamma = \left\{ 1 + \sum_{j=1}^{N} p_j / (1 - 2p_j) \right\} / 2$ consisting of the joint probabilities in Brewer's method, denoted by

$$\pi_{ijB} = \frac{2p_i p_j (1 - p_i - p_j)}{\gamma(1 - 2p_i)(1 - 2p_j)} . \tag{4.13}$$

He showed how the π_{ijB} can be employed for finding $\hat{\rho}_\varphi$ approximately for systematic PPS sampling for $n > 2$.

5. Some Issues for Alternative Approaches

Because of the complexity of the variance estimation due to the joint probabilities, especially in cases of $n > 2$, Särndal (1996) and Knottnerus (2003) developed the alternative approaches mentioned above.

Now we discuss some issues when they are used in practice. First, Särndal's approach does not depend on the joint probabilities at all and hence the variance estimator is simpler to handle mathematically. Poisson sampling, which is one of the πPS sampling schemes, is essential to implement the approach. The sampling method that is a generalization of Bernoulli sampling is easy to execute and has the property of $a_{ij} = a_i a_j$ for any $i \neq j$, which results in the simple variance estimator (3.2) expressed by the weighted squared residual form.

But the sample size in Poisson sampling is random. Since random sample size may substantially increase the variance, survey samplers often prefer fixed sample size designs. Also, many surveys in governmental statistical agencies and major survey institutes currently adopt deep stratification that there are many strata and extremely small samples from each stratum are selected. The counties for the Current Population Surveys of the U.S. Census Bureau and for the Survey Research Center in the University of Michigan may be the typical examples of national samples that are selected under the deep stratification. When using these samples, there is some concern that although the GREG estimator uses the available auxiliary information, it may be appreciably biased. Additionally, as noted by Hidiroglou, Estevao and Arcaro (2000), the GES in Statistics Canada still computes variance estimates for stratified designs under simple random sampling and probability proportional size (PPS) with replacement sampling. Considering this fact, it would be desirable that some efficient πPS sampling schemes having fixed sample size are examined for variance estimation.

Second, Knottnerus' approach uses the generalized sampling autocorrelation coefficient. If the coefficient is known, the variance estimation process is quite simple because the variance estimator (4.10), which does not depend on the joint probabilities, is available. But if it is not known, then the process become complicated. First of all, the numerical decomposition of the fixed value z_j using a kind of regression model is not simple. Also, his approach uses systematic PPS sampling that is one of πPS sampling schemes. This sampling method is easy to implement, while the exact evaluation of the joint probabilities for that method is cumbersome as the sample size increases. Thus the following approximation using the joint probabilities in Brewer's (1963) sampling method is unavoidable.

$$\pi_{ij} \approx \left(\binom{n}{2}\right)\pi^{*}_{ijBR} = \frac{(n-1)\pi_i \pi_j (n-\pi_i-\pi_j)}{\gamma^{*}(n-2\pi_i)(n-2\pi_j)} , \tag{5.1}$$

where $\gamma^{*} = \left\{1+\sum_{j=1}^{N}\pi_j/(n-2\pi_j)\right\}/2$.

Note that this expression is quite different from one to $O(N^{-4})$ derived by Hartley and Rao under the same sampling scheme (See page 369, Hartley and Rao (1962)).

In summary, the alternative approaches use certain πPS sampling schemes that survey samplers would not prefer in several aspects.

6. Traditional Approach using Efficient Sampling

Bayless and Rao (1970) empirically investigated the efficiencies of many unequal probability sampling methods for $n=3,4$. They showed that a strictly πPS sampling scheme, called Sampford's (1967) method, which is theoretically interesting, is one of the methods that perform well with respect to the variance as well as variance estimator. This method to cover $n>2$ is well-known as an extension of the procedures suggested by Brewer (1963) mentioned before and Rao (1965) for $n=2$ only.

But the difficulties in calculating the joint probabilities are not exceptional even for this method. Sampford's method is a rejective procedure, which selects a sample with replacement and only accept the sample that contain n different units. The computations may become tedious for the large samples. Although they can be programmed for computer evaluation, the uncritical use in computer programs may result in substantial round-off errors. The difficulty in their calculation stems from the large number of decimals which must be stored if they are to be calculated with any acceptable degree of accuracy. Because of these problems, Asok and Sukhatme (1976) derived an approximation of π_{ij} correct to $O(N^{-4})$ under Sampford method.

In recent version of the SAS/STAT (2004) and the SPSS (2004), some equal or unequal sampling methods including Sampford's method are available for sample selection and optionally, joint probabilities of selection are also available for certain πPS sampling methods. The SURVEYSELECT procedure for analysis of sample survey data in the SAS system provides the exact joint probabilities for Sampford's method for large-size sample, although it has not been noted by survey samplers until recently.

7. Numerical Illustration

As mentioned before, Knottnerus' approach based on systematic PPS sampling uses the approximate expression for π_{ij} given by (5.1), which is different from one derived by Hartley and Rao (1962) under the same sampling method. For Sampford's method, the approximation of π_{ij} derived by Asok and Sukhatme (1976) may be one of the choices in calculating the joint probabilities. Also, we can calculate the exact joint probabilities for Sampford's method by using the SAS. Thus, the following comparisons may be possible: (i) the direct comparison of the values of π_{ij} 's; (ii) the comparison of variances, obtained by (2.2).

<Table 7.1> is the data of 35 Scottish farms, which is given in Sampford (1962). We consider selecting the samples of $n=3$ under Sampford's method and systematic PPS sampling. In this case there exist 6545 possible samples.

<Table 7.2> presents a comparison of π_{ij} 's and variances between four approaches: (a) the exact method under Sampford's method; (b) the approach by Asok and Sukhatme under Sampford's method; (c) Knottnerus' appproach under systematic PPS sampling; (d) Hartley and Rao's approach under systematic PPS sampling. The sets of joint probabilities in the table are a part of 595 all possible joint probabilities chosen to put some distances between units and the variances at the bottom of the table are calculated using all joint probabilities.

As given in the table, the joint probabilities obtained by the approach of Asok and

<Table 7.1> Recorded Acreage of Crops and Grass for 1947 and Acreage under Oats in 1957 for 35 farms in Orkney

Farm no.	Recorded Crops and grass m_j	Oats 1957 Y_j	Farm no.	Recorded Crops and grass m_j	Oats 1957 Y_j
1	50	17	19	140	43
2	50	17	20	140	48
3	52	10	21	156	44
4	58	16	22	156	45
5	60	6	23	190	60
6	60	15	24	198	63
7	62	20	25	209	70
8	65	18	26	240	28
9	65	14	27	274	62
10	68	20	28	300	59
11	71	24	29	303	66
12	74	18	30	311	58
13	78	23	31	324	128
14	90	0	32	330	38
15	91	27	33	356	69
16	92	34	34	410	72
17	96	25	35	430	103
18	110	24			

Sukhatme show a close approximation to those by the exact method. The probabilities for the approaches of Knottnerus and Hartley and Rao are also similar each other, but the probabilities for the approach of Knottnerus is closer to those by the exact method under Sampford's method. This may be due largely to the use of the joint probabilities in Brewer's (1963) sampling, as expressed in (5.1).

For systematic PPS sampling, Knottnerus' joint probabilities provide a smaller variance than in Hartley and Rao. For Sampford's method, it has a smaller variance when using exact joint probabilities. Note that the exact joint probabilities under Sampford's method give the lower variance than in Knottnerus' approach under systematic PPS sampling.

Therefore we would say that the exact joint probabilities for Sampford method preserve the attractive low variance.

<Table 7.2> Comparison of the exact π_{ij} 's and approximate π_{ij} 's

(i,j)	Sampford's method		Systematic PPS sampling	
	Exact π_{ij}	Approximate π_{ij} (Asok and Sukhatme)	Approximate π_{ij} (Knottnerus)	Approximate π_{ij} (Hartley and Rao)
(1, 2)	0.000439	0.000439	0.000439	0.000441
(1, 5)	0.000527	0.000528	0.000528	0.000530
(3, 4)	0.000530	0.000531	0.000531	0.000532
(3,10)	0.000623	0.000623	0.000624	0.000625
(5, 6)	0.000634	0.000634	0.000635	0.000636
(5,15)	0.000967	0.000968	0.000968	0.000970
(7, 8)	0.000711	0.000711	0.000712	0.000713
(7,20)	0.001552	0.001553	0.001554	0.001556
(9,10)	0.000780	0.000781	0.000781	0.000783
(9,25)	0.002464	0.002465	0.002465	0.002467
(11,12)	0.000930	0.000930	0.000931	0.000933
(11,30)	0.004094	0.004094	0.004092	0.004090
(13,14)	0.001247	0.001248	0.001249	0.001251
(13,35)	0.006389	0.006382	0.006381	0.006365
(15,16)	0.001492	0.001493	0.001493	0.001496
(15, 5)	0.000967	0.000968	0.000968	0.000970
(17,18)	0.001890	0.001891	0.001891	0.001894
(17,10)	0.001159	0.001160	0.001160	0.001163
(19,20)	0.003556	0.003558	0.003558	0.003561
(19,15)	0.002291	0.002292	0.002292	0.002295
(21,22)	0.004442	0.004444	0.004443	0.004447
(21,20)	0.003974	0.003976	0.003976	0.003980
(23,24)	0.006965	0.006968	0.006966	0.006969
(23,25)	0.007367	0.007370	0.007368	0.007371
(25,26)	0.009395	0.009397	0.009395	0.009397
(25,30)	0.012345	0.012345	0.012344	0.012342
(27,28)	0.015767	0.015765	0.015767	0.015768
(27,35)	0.023196	0.023177	0.023202	0.023188
(29,30)	0.018213	0.018210	0.018215	0.018215
(29, 5)	0.003359	0.003359	0.003358	0.003356
(31,32)	0.020824	0.020818	0.020830	0.020829
(31,10)	0.004094	0.004094	0.004092	0.004088
(33,34)	0.029050	0.029030	0.029077	0.029074
(33,15)	0.006085	0.006082	0.006080	0.006073
Variance	68319.534	68342.535*	68326.105	68360.497*

Note. Exact π_{ij} are obtained in SURVEYSELECT procedure in the SAS system

*: The computed value of variance is correct to $O(N^0)$.

8. Conclusion

A lot of efficient unequal probability sampling schemes, especially some efficient πPS sampling methods have been developed. Many of them have not been used in practice due to the decision problems of the joint probabilities. But owing to the features incorporated in the statistical software, the calculation of those probabilities no longer has the restrictions for certain sampling schemes such as Sampford' method. The traditional fixed · sample size πPS scenario for variance estimation using the method may be one of the attractive choices.

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