Journal of the Korean
Data & Information Science Society
2006, Vol. 17, No. 3, pp. 861 ~869

Optimal Burn-In Procedures for a System Performing Given Mission[1)]

Ji Hwan Cha[2)]

Absract

Burn-in is a widely used method to improve the quality of products or systems after they have been produced. In this paper, the problem of determining optimal burn-in time for a system which performs given mission is considered. It is assumed that the given mission time is not a fixed constant but a random variable which follows an exponential distribution. Assuming that the underlying lifetime distribution of a system has an eventually increasing failure rate function, an upper bound for the optimal burn-in time which maximizes the probability of performing given mission is derived. The obtained result is also applied to an illustrative example.

Keywords : Bathtub shaped failure rate function, Change points, Eventually increasing failure rate function, Mission time, Optimal burn-in, Wear-out points

1. Introduction

Burn-in is a technique applied with the intention of eliminating early failures of a system or product. Due to high failure rate in the early stages of system life, burn-in procedures have been recognized as a useful method for detecting early failures of components or systems before customer delivery. Without burn-in, a number of defective components could be delivered to customers. By applying burn-in, the manufacturer delivers fewer defective components, and consequently the lower failure rates reduce field-repair costs. An introduction to this important area of reliability can be found in Jensen and Petersen (1982) and Kuo and Kuo

1) This work was supported by the Pukyong National University Research Fund in 2003.
2) Assistant Professor, Division of Mathematical Sciences, Pukyong National University, Busan, 608-737, Korea.
E-mail: jhcha@pknu.ac.kr

(1983). Since too excessive or insufficient burn-in is either harmful to the performance of system or costly, one of the major problems is to decide how long the procedure should be. The best time to stop the burn-in procedure for a given criterion is called the optimal burn-in time. In the literature, certain cost structures have been studied. See, for example, Nguyen and Murthy (1982), Clarotti and Spizzichino (1991), Mi (1994a) (1996) (1997) and Cha (2000) (2001) (2003). Some other performance-based criteria, for example, the mean residual life, the reliability of a given mission time, or the mean number of failures, have also been considered for determining the optimal burn-in time(See also Mi (1991) (1994b), Block, Mi and Savits (1994)). An excellent survey of recent research on burn-in can be found in Block and Savits (1997).

Many practical problems require a system to accomplish a task in field operation with a given mission time. In this paper, we consider the problem of determining optimal burn-in time for a system which performs given mission. It is assumed that the given mission time is not a fixed constant but a random variable which follows an exponential distribution. In the literature, many objective cost or system performance functions related with burn-in have been discussed under the assumption of the bathtub shaped failure rate function. However, recently there have been many researches on the shape of the failure rate functions of mixture distributions which is not of the traditional bathtub shaped failure rate function. Under a general failure rate function assumption which includes the traditional bathtub shaped failure rate function as a special case, an upper bound for the optimal burn-in time will be derived.

This paper is organized as follows. In Section 2, a general assumption on the shape of failure rate function is introduced. It can be seen that this general assumption includes the traditional bathtub-shaped failure rate function as a special case. In Section 3, the probability of performing given mission for a burned-in system with burn-in time b will be obtained and an upper bound for the optimal burn-in time which maximizes the probability will be derived. In Section 4, an illustrative example will be presented. Finally in Section 5, some concluding remarks are discussed.

2. General Failure Rate Model

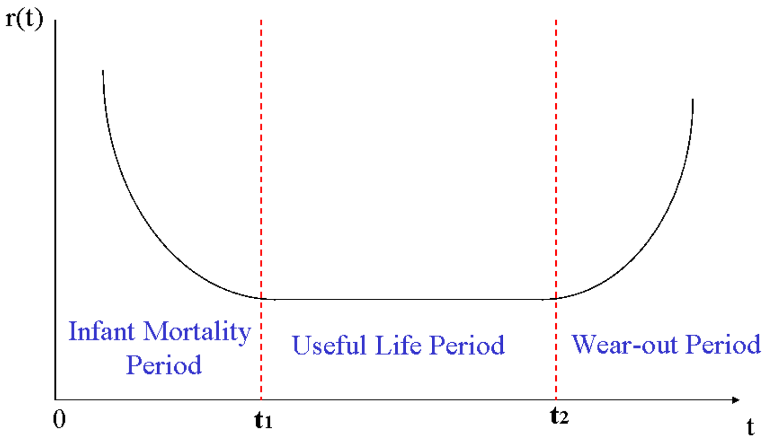
It is widely believed that many products, particularly electronic products or devices such as silicon integrated circuits, exhibit bathtub shaped failure rate functions. This belief is supported by much experience and extensive data collected by practitioners and researchers in many industries. Hence most of researches on burn-in has been done under the assumption of bathtub shaped failure rate function. The definition of bathtub shaped failure rate function is as follows.

Definition 1. A failure rate function $r(t)$ is said to have a bathtub shape if there exist $0 \le t_1 \le t_2 \le \infty$ such that

$$r(t) = \begin{cases} \text{strictly decreases, if } 0 \le t \le t_1; \\ \text{is a constant, say } \lambda_{0,} \text{if } t_1 \le t \le t_2; \\ \text{strictly increases, i } t \ge t_2, \end{cases}$$

where t_1 and t_2 are called the first and second change points of $r(t)$.

The time interval $[0, t_1]$ is called the infant mortality period; the interval $[t_1, t_2]$, where $r(t)$ is flat and attains its minimum value, is called the normal operating life or the useful life; the interval $[t_2, \infty)$ is called the wear-out period. An example of bathtub shaped failure rate function is presented in the following Figure 1.



<Figure 1> Bathtub Shaped Failure Rate Function

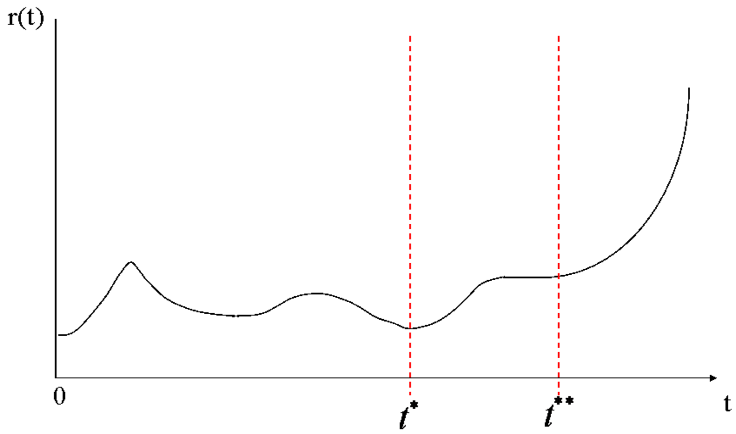
Although the bathtub shaped failure rate function is assumed in most of researches on burn-in, it does not model many other situations as pointed out in Wong (1988) (1989) (1991) and Klutke et. al. (2003). In regard to this point, we will consider burn-in problems under a more general assumption introduced in Mi (2003) which includes the usual bathtub shaped failure rate function as a special case.

Definition 2. A failure rate function $r(x)$ is eventually increasing if there exists $0 \le x_0 < \infty$ such that $r(x)$ strictly increases in $x \in [x_{0,} \infty)$. For an eventually increasing failure rate function $r(x)$ the first and second wear-out points t^* and t^{**} are defined by

$$t^{*} = inf\{t \geq 0 : r(x) \text{ is nondecreasing in } x \geq t\}$$
$$t^{**} = inf\{t \geq 0 : r(x) \text{ strictly increases in } x \geq t\}$$

An example of eventually increasing failure rate function is presented in Figure 2. Figure 2 is neither of the traditional bathtub shape nor of the modified bathtub-shape. Note also that if $r(x)$ has a bathtub shape with change points $t_1 \leq t_2 < \infty$, then it is eventually increasing with $t^{*} = t_1$ and $t^{**} = t_2$. Therefore, the eventually increasing failure rate function includes the traditional bathtub shaped failure rate function as a special case.

Mi (2003) considered optimal burn-in under the assumption of eventually increasing failure rate function. In this paper we will derive the upper bound for optimal burn-in time under the assumption of eventually increasing failure rate function. For more detailed discussions about general assumptions for the shape of failure rate function in burn-in model, see also Cha and Mi (2005).



<Figure 2> Eventually Increasing Failure Rate Function

3. Optimal Burn-In

Let a system have random life X which has distribution function $F(x)$, density $f(x)$, and failure rate function $r(x) = f(x)/\overline{F}(x)$, where $\overline{F}(x) = 1 - F(x)$ is the survival function of X. Without loss of generality, throughout this paper, we assume that $r(x)$ is continuous. Furthermore, let the lifetime of a system which has survived burn-in time b and its CDF be X_b and $F_b(x)$. Then

$$F_b(x) = P(X_b \le x) = P(X \le b + x | X > b) = \frac{\bar{F}(b) - \bar{F}(b+x)}{\bar{F}(b)},\ b, x \ge 0,$$

and

$$\overline{F_b}(x) \equiv 1 - F_b(x) = \frac{\bar{F}(b+x)}{\bar{F}(b)} = exp(-\int_b^{b+x} r(u)du),\ \forall b, x \ge 0.$$

Also, we denote the failure rate function of X_b as $r_b(x)$, which clearly is given by $r_b(x) = r(b+x),\ b, x \ge 0$. Let τ be a given mission time. It is assumed that τ is a random variable which follows an exponential distribution with its mean μ and its pdf $g(t) = (1/\mu)exp(-(1/\mu)t),\ t \ge 0$. Furthermore, we assume that X_b and τ are independent.

Many practical problems require a system to accomplish a task in field operation with a given mission time. The probability of performing given mission, $\eta(b)$, is given by

$$\begin{aligned}\eta(b) \equiv P(X_b > \tau) &= \int_0^\infty P(X_b > \tau | \tau = u) g(u) du \\ &= \int_0^\infty exp(-[\Lambda(b+u) - \Lambda(b)]) \cdot \frac{1}{\mu} exp(-\frac{1}{\mu}u) du \\ &= \frac{1}{\mu} \cdot exp(\Lambda(b) + \frac{1}{\mu}b) \cdot \int_b^\infty exp(-\Lambda(t) - \frac{1}{\mu}t) dt, \qquad (1)\end{aligned}$$

where $\Lambda(t) \equiv \int_0^t r(u)du$. In this paper, we consider optimal burn-in time which maximizes $\eta(b)$ in (1).

Let b^* be optimal burn-in time which satisfies

$$\eta(b^*) = max_{b \ge 0} \eta(b).$$

In most burn-in models, the optimal burn-in time cannot be given by an explicit form and must be obtained numerically. In this case, some bounds for the optimal burn-in time may be useful since the numerical search for the optimal burn-in time will be greatly reduced. The following result gives an upper bound for optimal burn-in time b^* when the failure rate function is eventually increasing.

Theorem 1. Suppose that the lifetime distribution function $F(t)$ has an eventually increasing failure rate function $r(t)$ with the first wear-out point t^*. Then t^* is an upper bound for optimal burn-in time. That is, b^* satisfies $0 \le b^* \le t^*$.

proof.

It suffices to show that $\eta'(b) < 0$, for all $b \ge t^*$, since this will imply that $\eta(b)$ is strictly decreasing in $b \in [t^*, \infty)$. By the assumption of the eventually increasing failure rate function assumption, $r(t) \ge r(b)$, for all $b \ge t^*$, and $r(t) > r(b)$, for all $b \ge t^{**}$. Hence it holds that

$$\begin{aligned}\eta'(b) &= \frac{1}{\mu}(r(b)+\frac{1}{\mu}) \cdot exp(\Lambda(b)+\frac{1}{\mu}b) \cdot \int_b^\infty exp(-\Lambda(b)+\frac{1}{\mu}t)dt - \frac{1}{\mu} \\ &< \frac{1}{\mu} \cdot exp(\Lambda(b)+\frac{1}{\mu}b) \cdot \int_b^\infty (r(t)+\frac{1}{\mu}) \cdot exp(-\Lambda(t)-\frac{1}{\mu}t)dt - \frac{1}{\mu} \\ &= \frac{1}{\mu} \cdot \left[-exp(-\Lambda(t)-\frac{1}{\mu}t)\right]_b^\infty \cdot exp(\Lambda(b)+\frac{1}{\mu}b) - \frac{1}{\mu} \\ &= 0,\end{aligned}$$

for all $b > t^*$. This completes the proof.

■

From Theorem 1, we can see that it is sufficient to consider only $b \in [0, t^*]$ to find the optimal burn-in time b^* when the failure rate function is eventually increasing.

The following result, which is readily obtained from Theorem 1, gives an upper bound for optimal burn-in time b^* when the failure rate function is bathtub shaped.

Corollary 1. Suppose that the lifetime distribution function $F(t)$ has a bathtub shaped failure rate function $r(t)$ with the change points $t_1 \le t_2 < \infty$. Then t_1 is an upper bound for optimal burn-in time. That is, b^* satisfies $0 \le b^* \le t_1$.

proof.

If $r(t)$ is a bathtub shaped failure rate function with change points $t_1 \le t_2 < \infty$ then it is an eventually increasing with $t^* = t_1$. Hence the desired results follow from Theorem 1.

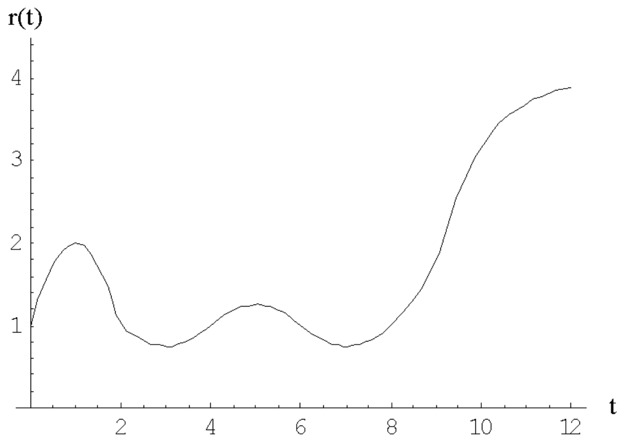
■

4. Numerical Example

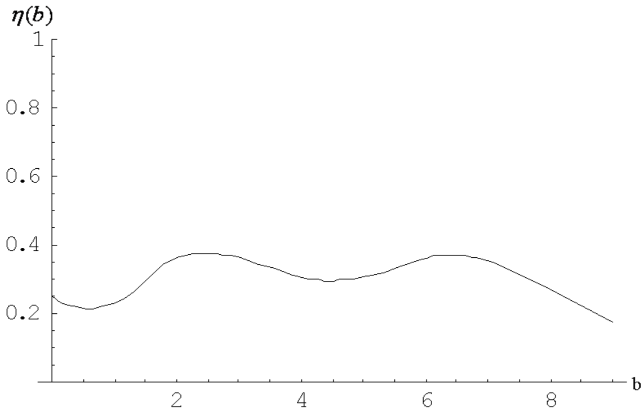
In this section, an illustrative example will be given. Suppose that the failure rate function of the system is given by

$$r(t) = \begin{cases} -(t-1)^2 + 2, \ if \ 0 \le t \le 2, \\ (1/4)(t-3)^2 + 3/4, \ if \ 2 \le t \le 4, \\ -(1/4)(t-5)^2 + 5/4, \ if \ 4 \le t \le 6, \\ (1/4)(t-7)^2 + 3/4, \ if \ 6 \le t \le 9, \\ 4 - (9/4) exp(-(t-9)), \ if \ 9 \le t. \end{cases}$$

The graph for the failure rate function is presented in Figure 3.



<Figure 3> Failure Rate Function



<Figure 4> The Graph for $\eta(b)$

Then the failure rate function is eventually increasing with two wear-out points $t^* = t^{**} = 7.0$. Assume that the mean of the mission time μ is given by $\mu = 2.0$. By Theorem 1 in the previous section, an upper bound for the optimal burn-in time is given by $t^* = 7.0$. Hence it is sufficient to consider only $b \in [0, 7.0]$ to find the optimal burn-in time b^*. The graph of the probability of performing given mission $\eta(b)$ is presented in Figure 4. By numerical search, the optimal burn-in time is given by $b^* = 2.44$ and the maximum probability is $\eta(b^*) = 0.37666412$.

5. Concluding Remark

In the literature, many objective cost or system performance functions related with burn-in have been discussed under the assumption of the bathtub shaped failure rate function and it has been obtained that the optimal burn-in time must be before the first change point t_1 if the underlying lifetime distribution has a bathtub shaped failure rate function. However, recently there have been many researches on the shape of the failure rate functions of mixture distributions which is not of the traditional bathtub shaped failure arte function.. In this paper, we considered more general failure rate function models for burn-in procedures, eventually increasing failure rate function. It has been shown that the general failure rate function model considered in this paper include the traditional bathtub shaped failure rate function as a special case. Under the general failure rate function model, we have obtained an upper bound for optimal burn-in time. By an illustrative example, we have shown that the optimal burn-in time can be found with ease using the bound that we have obtained.

Acknowledgement

The author thanks the referee and the Editor for helpful comments and careful readings of this paper.

References

1. Block, H. W. and Savits, T. H. (1997). Burn-In, *Statistical Science 12,* 1-19.
2. Block, H. W., Mi, J. and Savits, T. H. (1994). Some Results on Burn-In, *Statistica Sinica 4*, 525-533.
3. Cha, J. H. (2000). On a Better Burn-In Procedure, *Journal of Applied Probability 37*, 1099-1103.
4. Cha, J. H. (2001). Burn-In Procedures for a Generalized Model, *Journal of Applied Probability 38*, 542-553.
5. Cha, J. H. (2003). A Further Extension of the Generalized Burn-In

Model, *Journal of Applied Probability 40*, 264-270.
6. Cha, J. H. and Mi, J. (2005). Optimal Burn-in Procedures in a Generalized Environment, International Journal of Reliability, *Quality and Safety Engineering 12*, 189-202.
7. Clarotti, C. A. and Spizzichino, F (1991). Bayes Burn-In Decision Procedures, *Probability in the Engineering and Informational Sciences 4*, 437-445.
8. Jensen, F. and Petersen, N. E. (1982). *Burn-In*. John Wiley, New York.
9. Kuo, W. and Kuo, Y. (1983). Facing the Headaches of Early Failures: A State-of-the-Art Review of Burn-In Decisions. *Proc. IEEE. 71*, 1257-1266.
10. Klutke, G., Kiessler, P. C. and Wortman, M. A. (2003). A Critical Look at the Bathtub Curve, *IEEE Transactions on Reliability 52*, 125-129.
11. Mi, J. (1991). *Optimal Burn-In*. Doctoral Thesis. Dept. Statistics, Univ. Pittsburgh.
12. Mi, J. (1994a). Burn-In and Maintenance Policies. *Advances in Applied Probability. 26*, 207-221.
13. Mi, J. (1994b). Maximization of a Survival Probability and Its Application, *Journal of Applied Probability 31,* 1026-1033.
14. Mi, J. (1996). Minimizing Some Cost Functions Related to Both Burn-In and Field Use. *Operations Research.* 44, 497-500.
15. Mi, J. (1997). Warranty Policies and Burn-In. *Naval Research Logistics. 44*, 199-209.
16. Mi, J. (2003). Optimal Burn-in Time and Eventually IFR, *Journal of the Chinese Institute of Industrial Engineers 20*, 533-542.
17. Nguyen, D. G. and Murthy, D. N. P. (1982). Optimal Burn-In Time to Minimize Cost for Products Sold under Warranty, *IIE Transactions 14*, 167-174.
18. Wong, L. K. (1988). The Bathtub Does Not Hold Water Any More, *Quality and Reliability Engineering International 4*, 279-286.
19. Wong, L. K. (1989). The Roller-coaster Curve Is In, *Quality and Reliability Engineering International 5*, 29-36.
20. Wong, L. K. (1991). The Physical Basis for the Roller-coast Hazard Rate Curve for Electronics, *Quality and Reliability Engineering International 7*, 489-495.

[received date : May. 2006, accepted date : Jun. 2006]