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A Note on Statistically Monotonic and Bounded Sequences of Fuzzy Numbers

Dug Hun Hong[1)] · Kyung Tae Kim[2)]

Abstract

This note gives a counterexample showing that a main theorem of Aytal and Pehliven's (Information Sciences 176 (2006) 734-744) result on statically monotonic and bounded sequences of fuzzy numbers is not valid.

Keywords : Fuzzy numbers, Statistically boundedness, Statistically convergent sequence of fuzzy numbers, Statistically monotonicity

1. Introduction

Recently, Aytal and Pehliven(2006) introduced the statically monotonicity and boundedness of a sequence of fuzzy numbers and they derived an analogue of monotone convergence theorem. The object of this note is to provide a counterexample to Theorem 2.11 in the paper of Aytal and Pehliven(2006). We essentially use the same notations as in Aytal and Pehliven(2006). We recall some definitions.

A fuzzy number X is a fuzzy set in R with a normal, fuzzy convex and upper semi-continuous membership function of bounded support. These properties imply that for any $\alpha \in (0,1]$, the $\alpha-$level set $X^{\alpha}:=\{x \in R : X(x) \geq \alpha\} = [\underline{X}^{\alpha}, \overline{X}^{\alpha}]$ is a non-empty convex subset of R, as is the support X^0. By $L(R)$, we denote the set of all fuzzy numbers. Every fuzzy point a_1 $(a \in R)$ defined by $a_1(x) = 1$ for $x = a$ and 0, otherwise

1) Department of Mathematics, Myongji University, Kyunggido 449-728, South Korea
E-mail : dhhong@mju.ac.kr
2) Department of Electronics and Electrical Information Engineering Kyungwon University, Sungnam Kyunggido, South Korea

is a fuzzy number. Define a map $\overline{d} : L(R) \times L(R) \rightarrow R$ by

$$\overline{d}(X, Y) := \sup_{\alpha \in (0,1]} \max\{|\underline{X}^{\alpha} - \underline{Y}^{\alpha}|, |\overline{X}^{\alpha} - \overline{Y}^{\alpha}|\}.$$

Here, $\overline{d}$ is a metric on $L(R)$. We follow the order relation on $L(R)$ defined by

$$X \leq Y \text{ iff } \underline{X}^{\alpha} \leq \underline{Y}^{\alpha} \text{ and } \overline{X}^{\alpha} \leq \overline{Y}^{\alpha} \text{ for any } \alpha \in [0,1].$$

A subset E of $L(R)$ is said to be bounded above if there exist a fuzzy number μ, called an upper bound of E, such that $X \leq \mu$ for every $X \in E$. Lower bound is defined similarly.

E is said to be bounded if it is both bounded below and below.

The notion of natural density is given by $\delta(K) := \lim_{n \to \infty} \frac{|\{k \in K : k \leq n\}|}{n}$ where $K \subset N$, and $|\{k \in K : k \leq n\}|$ denotes the number of elements in $\{k \in K : k \leq n\}$.

A sequence of fuzzy numbers $X = \{X_k\}$ is statistically convergent to a fuzzy number X_0 provided that for every $\varepsilon > 0$ the set $\{k \in N : \overline{d}(X_k, X_0) \geq \varepsilon\}$ has natural density zero.

A sequence $X = \{X_k\}$ is said to be statistically monotone increasing if there exists a subset

$$K = \{k_1 < k_2 < \cdots\} \subset N$$

such that $\delta(K) = 1$ and $X_{k_n} \leq X_{k_{n+1}}$ for every $n \in N$. A statistically monotone decreasing sequence can be defined similarly. Every statistically monotone increasing or statistically monotone decreasing sequence can be called as statistically monotonic.

A sequence $X = \{X_k\}$ is called statistically bounded if there exists a real number M such that the set $\{k \in N : \overline{d}(X_k, 0_1) \geq M\}$ has natural density zero.

2. Counterexample

The following result is a main theorem of Aytal and Pehliven(2006).

Theorem 2.1 (Aytal and Pehliven(2006)) A statistically monotone sequence is statistically convergent if and only if it is statistically bounded.

The necessity is obvious but the sufficiency is not correct. For this, we consider the following elementary counterexample.

Counterexample. Let

$$X_n(x) := \begin{cases} x+1 & \text{if} \quad -1 \le x \le 0 \\ (1-x)^{\frac{1}{n}} & \text{if} \quad 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Then clearly the sequence X_n is statistically monotone increasing and bounded and we have that

$$\underline{X}_{n(\alpha)} := \alpha - 1 \text{ and } \overline{X}_{n(\alpha)} := 1 - \alpha^n, \text{ for } n = 1, 2, \cdots.$$

Then it is elementary to check that there does not exist a fuzzy number X_0 such that $st-\lim_k X_k := X_0$, which means that X_n does not converge statistically.

Reference

1. S. Aytal and S. Pehliven(2006). Statically monotonic and statically bounded sequences of fuzzy numbers, Information Sciences 176, 734-744.

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