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Goodness-of-fit Tests for the Weibull Distribution Based on the Sample Entropy

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Abstract

For Type-II censored sample, we propose three modified entropy estimators based on the Vasieck's estimator, van Es' estimator, and Correa's estimator. We also propose the goodness-of-fit tests of the Weibull distribution based on the modified entropy estimators. We simulate the mean squared errors (MSE) of the proposed entropy estimators and the powers of the proposed tests. We also compare the proposed tests with the modified Kolmogorov-Smirnov and Cramer-von-Mises tests which were proposed by Kang et al. (2003).

Keywords : Cramer-von-Mises test, Entropy estimator, Kolmogorov-Smirnov test, Type- II censored sample, Weibull distribution

1. Introduction

The entropy of the random variable X may be regarded as a descriptive quantity and was introduced by Shannon (1948). The entropy is a measure of the extent to which the probability is concentrated on a few points or dispersed over many point.

Vasicek (1976) proposed the estimator of entropy and introduced a test of normality based on the property of the normal distribution. In the normal distribution, the entropy exceeds that of any other distribution with a density that has the same variance. Ebrahimi et al. (1992) introduced the test for exponentiality based on the Kullback-Leibler (KL) information. van Es (1992) proposed entropy

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estimator based on spacings and Correa (1995) proposed a new estimator of the entropy that is based on log linear regression. Kim and Lee (1998) studied a test of fit for the Weibull distribution and the extreme value distribution based on KL information.

The goodness-of-fit tests using entropy have been studied by many authors. Grzegorzewski and Wieczorkowski (1999) proposed a goodness-of-fit test for exponentiality based on the entropy. Park (1999) provided the sample entropy of order statistics and presented one application of the sample entropy of order statistics as a test of normality versus skewness. Esteban et al. (2001) compared the powers of four normality test using different entropy estimates by means of Monte Carlo simulations. They presented the critical values of the test statistics. Taufer (2002) considered transformations of the observations which turn the test of exponentiality into one of uniformity and use the corresponding test based on entropy. Şenğlu and Sürücü (2004) computed the power of modified test based on the sample entropy and compared with Shapiro-Wilks test, Tiku test based on sample spacings, and sample correlation test.

In this paper we propose three modified entropy estimators based on Type-II censored samples and the goodness-of-fit tests of the Weibull distribution based on the proposed entropy estimators.

We also simulate the MSEs of the proposed entropy estimators and the power of the proposed tests. We also compare the proposed tests with the modified Kolmogorov-Smirnov and the Cramer-von-Mises tests which were proposed by Kang et al. (2003).

2. Estimation of the entropy

For the continuous random variables, the entropy of the random variable X is defined as

$$H(X;\theta) = -E[\ln f(X;\theta)] = -\int_{-\infty}^{\infty} f(x;\theta)\ln f(x;\theta)\,dx, \tag{2.1}$$

where $F(x)$ and $f(x)$ are cdf and pdf of the random variable X.

It can be re-expressed as

$$H(X;\theta) = \int_0^1 \ln \frac{d}{dp} F^{-1}(p)\,dp.$$

The entropy of the Weibull distribution with pdf

$$f(x) = \frac{\delta}{\theta^{\delta}} x^{\delta-1} \exp\left\{-\left(\frac{x}{\theta}\right)^{\delta}\right\}, \quad x>0,\ \theta>0,\ \delta>0$$

is

$$\ln \frac{\theta}{\delta} + \gamma - \frac{\gamma}{\delta} + 1, \tag{2.2}$$

where γ is Euler's constant. That is, $\gamma = -\psi(1)$ and $\psi(x) = \frac{d}{dx} \ln \Gamma(x)$.

Let

$$X_{(r+1)} \le X_{(r+2)} \le \cdots \le X_{(n-s)} \tag{2.3}$$

be a doubly Type-II censored sample where the first r and the last s observations are censored.

First, we propose the modified Vasicek's entropy estimator based on Type-II censored sample (2.3) as follows;

$$H^{r,s}_{m,n} = \frac{1}{n-r-s} \sum_{i=r+1}^{n-s} \ln \left\{ \frac{n}{2m} (X_{(i+m)} - X_{(i-m)}). \right\} \tag{2.4}$$

where $X_{(i)} = X_{(r+1)}$ for $i < r+1$ and $X_{(i)} = X_{(n-s)}$ for $i > n-s$.

Second, we propose the modified van Es' entropy estimator based on Type-II censored sample (2.3) as follows;

$$V^{r,s}_{m,n} = \frac{1}{n-m-r-s} \sum_{i=r+1}^{n-m-s} \ln \left\{ \frac{n+1}{m} (X_{(i+m)} - X_{(i)}) \right\} + \sum_{k=m}^{n} \frac{1}{k} + \ln(m) - \ln(n+1). \tag{2.5}$$

Third, we propose the modified Correa's entropy estimator based on Type-II censored sample (2.3) as follows;

$$C^{r,s}_{m,n} = -\frac{1}{n-r-s} \sum_{i=r+1}^{n-s} \ln(b_i), \tag{2.6}$$

where

$$b_i = \frac{\sum_{j=i-m}^{i+m} (X_{(j)} - \overline{X}_{(i)})(j/n - i/n)}{\sum_{j=i-m}^{i+m} (X_{(j)} - \overline{X}_{(i)})^2}$$

and

$$\overline{X}_{(i)} = \sum_{j=i-m}^{i+m} \frac{X_{(j)}}{2m+1}.$$

3. Goodness-of-fit test based on entropy

It is important that how well a sample of data agrees with a given distribution as its population. We will use the proposed entropy estimators for the goodness-of-fit test of the Weibull distribution. Since the entropy has a special property for the exponential distribution, we change the Weibull distribution into the exponential distribution. The powers of the proposed tests based on the modified entropy estimators are compared with the powers of the modified Kolmogorov-Smirnov test (D) and Cramer-von-Mises test (W^2) which were proposed by Kang et al. (2003).

Let X be a random variable with pdf $f(x)$ and cdf $F(x)$. The hypothesis we are interested in is

$$H_0 : f(x) = f_0(x) \text{ against } H_1 : f(x) \neq f_0(x),$$

where $f_0(x)$ is pdf of the Weibull distribution.

If X is an exponential random variable with $P[X>0]=1$, and $E(X)=\lambda$ then the entropy of the random variable X is $H(X)=\log\lambda+1$, $\lambda>0$ and the entropy is a maximum (see Kotz and Johnson (1982)). Hence, we will use this property for the entropy of the exponential distribution.

It is easy to see that $V=X^{\delta}$ has its density function given by

$$f(v)=\frac{1}{\theta^{\delta}} \exp^{-\frac{v}{\theta^{\delta}}}, \quad v>0. \tag{2.7}$$

That is, V has the exponential distribution with the scale parameter $\theta^{\delta}\equiv\beta$. The entropy of the exponential distribution is $\ln\beta+1$.

It is known that if X is a random variable with $P[X>0]=1$ and its mean $E(X)=\beta$, then

$$H(X;\beta)\leq \ln\beta+1 \tag{2.8}$$

and among all random variable with densities concentrated on $(0,\infty)$, the exponential distribution maximizes $H(X;\beta)$ with $\ln\beta+1$. From the equation (2.8) we get the following equation;

$$\frac{\exp\{H(X;\beta)\}}{\beta}\leq e.$$

We will use the following test statistic

$$T = \frac{\exp \widehat{H}(X;\beta)}{\widehat{\beta}}. \tag{2.9}$$

T is invariant with respect to the transformation of the location and the scale (see Esteban et al. (2001)). The exact critical values T_α of T at significance level α are defined by the equation

$$P[T \le T_\alpha] = \alpha.$$

We use the estimators (2.4), (2.5), and (2.6) instead of $\widehat{H}(X;\beta)$ in the equation (2.9), then the test statistics can be rewritten as

$$TH = \frac{\exp\{H^{r,s}_{m,n}\}}{\widehat{\beta}}, \tag{2.10}$$

$$TV = \frac{\exp\{V^{r,s}_{m,n}\}}{\widehat{\beta}}, \tag{2.11}$$

and

$$TC = \frac{\exp\{C^{r,s}_{m,n}\}}{\widehat{\beta}}. \tag{2.12}$$

To compare the powers of the proposed goodness-of-fit tests, we use the modified Kolmogorov-Smirnov test and Cramer-von-Mises test for doubly Type-II censored samples which were proposed by Kang et al. (2003).

To compute the powers, natural alternatives to the Weibull distribution are gamma distribution ($\mathrm{Gam}(1,3)$), half logistic distribution ($\mathrm{HL}(1,0)$), Pareto distribution ($\mathrm{Par}(1,0)$) and normal distribution ($\mathrm{N}(3,1)$).

4. Simulated results

We simulate the MSEs of $H^{r,s}_{m,n}$, $V^{r,s}_{m,n}$, and $C^{r,s}_{m,n}$ and compare the powers of TH, TV and TC by Monte Carlo simulation method. The simulation procedure is repeated 10,000 times for $n = 20, 30$ and various censoring. The MSEs of entropy estimators are simulated for the window size $m = 1(1)4$. These values are given in Table 1.

The modified Correa's estimator are generally better than the modified Vasicek estimator and van Es' entropy estimator in sense of the MSE. When censoring number is same, the MSEs of the estimators of the entropy in left censored sample are smaller than those of the estimators in right censored sample.

The powers are determined by generating 10,000 random samples of various censoring, and the window size $m=1$ for each of several alternative distributions. These powers are given in Table 2.

In the complete sample or large sample, test statistics based on sample entropy are more powerful than Cramer-von-Mises test statistic and Kolmogorov-Smirnov test statistic for the chi square distribution with degree of freedom 1.

TH and TV are more powerful than TC for most distributions. When sample size is small, TH is more powerful than the other test statistics for the Pareto distribution. When sample sizes are small and censored samples are large, test statistics based on sample entropy are more powerful than Cramer-von-Mises test statistic and Kolmogorov-Smirnov test statistic for the gamma distribution.

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<**Table 1**> The MSEs of the proposed entropy estimators based on doubly Type-II censored samples.

m			1			2			3			4		
n	r	s	$H^{r,s}_{m,n}$	$V^{r,s}_{m,n}$	$C^{r,s}_{m,n}$	$H^{r,s}_{m,n}$	$V^{r,s}_{m,n}$	$C^{r,s}_{m,n}$	$H^{r,s}_{m,n}$	$V^{r,s}_{m,n}$	$C^{r,s}_{m,n}$	$H^{r,s}_{m,n}$	$V^{r,s}_{m,n}$	$C^{r,s}_{m,n}$
20	0	0	0.235	0.119	0.093	0.141	0.164	0.080	0.125	0.191	0.082	0.123	0.203	0.086
	0	1	0.368	0.173	0.134	0.240	0.241	0.122	0.221	0.282	0.126	0.223	0.303	0.130
	0	2	0.490	0.226	0.184	0.338	0.308	0.172	0.318	0.358	0.176	0.323	0.386	0.180
	0	3	0.610	0.280	0.239	0.439	0.374	0.227	0.418	0.431	0.230	0.428	0.464	0.234
	1	0	0.204	0.105	0.091	0.121	0.140	0.074	0.108	0.162	0.074	0.107	0.171	0.076
	2	0	0.176	0.097	0.094	0.104	0.120	0.072	0.095	0.136	0.070	0.094	0.142	0.071
	3	0	0.152	0.091	0.103	0.092	0.104	0.076	0.085	0.115	0.072	0.085	0.118	0.072
	1	1	0.330	0.153	0.122	0.212	0.211	0.107	0.196	0.246	0.110	0.198	0.264	0.113
	1	2	0.448	0.202	0.166	0.306	0.275	0.151	0.289	0.319	0.154	0.295	0.343	0.158
	1	3	0.566	0.255	0.217	0.405	0.339	0.202	0.388	0.390	0.204	0.400	0.420	0.209
	2	1	0.295	0.138	0.114	0.187	0.184	0.096	0.174	0.214	0.097	0.177	0.228	0.100
	2	2	0.410	0.185	0.152	0.277	0.245	0.134	0.263	0.283	0.136	0.271	0.304	0.140
	2	3	0.527	0.235	0.198	0.374	0.307	0.181	0.360	0.352	0.183	0.374	0.378	0.186
	3	1	0.263	0.127	0.112	0.166	0.161	0.090	0.155	0.184	0.090	0.158	0.195	0.091
	3	2	0.374	0.170	0.142	0.252	0.219	0.122	0.240	0.250	0.122	0.248	0.267	0.125
	3	3	0.490	0.218	0.183	0.347	0.279	0.163	0.335	0.317	0.164	0.351	0.340	0.167
30	0	0	0.177	0.071	0.061	0.094	0.117	0.051	0.079	0.150	0.053	0.075	0.169	0.056
	0	1	0.263	0.098	0.084	0.154	0.165	0.076	0.134	0.209	0.078	0.130	0.237	0.082
	0	2	0.341	0.126	0.113	0.212	0.206	0.105	0.189	0.258	0.108	0.185	0.291	0.112
	0	3	0.417	0.153	0.145	0.272	0.245	0.137	0.246	0.304	0.140	0.243	0.341	0.145
	1	0	0.157	0.064	0.059	0.082	0.102	0.048	0.069	0.131	0.048	0.065	0.148	0.050
	2	0	0.138	0.059	0.060	0.071	0.090	0.046	0.060	0.115	0.045	0.058	0.129	0.046
	3	0	0.122	0.059	0.063	0.063	0.080	0.047	0.054	0.100	0.044	0.052	0.112	0.044
	1	1	0.239	0.088	0.077	0.137	0.147	0.067	0.119	0.187	0.069	0.115	0.212	0.072
	1	2	0.314	0.113	0.102	0.192	0.186	0.093	0.171	0.234	0.096	0.168	0.265	0.099
	1	3	0.388	0.140	0.132	0.250	0.223	0.123	0.226	0.278	0.126	0.224	0.313	0.129
	2	1	0.215	0.080	0.072	0.121	0.130	0.061	0.105	0.167	0.061	0.102	0.190	0.064
	2	2	0.288	0.102	0.093	0.174	0.167	0.083	0.155	0.212	0.085	0.152	0.240	0.088
	2	3	0.360	0.127	0.120	0.229	0.204	0.111	0.207	0.254	0.113	0.206	0.287	0.116
	3	1	0.195	0.076	0.069	0.108	0.117	0.056	0.094	0.149	0.056	0.091	0.169	0.058
	3	2	0.266	0.096	0.087	0.158	0.153	0.075	0.140	0.192	0.076	0.138	0.217	0.078
	3	3	0.336	0.120	0.112	0.211	0.187	0.100	0.191	0.233	0.101	0.190	0.262	0.104

<Table 2> Powers of the goodness-of-fit tests based on the modified entropy estimators.

H_1: Gam(1, 3)								H_1: HL(1, 0)							
n	r	s	TH	TC	TV	D	W^2	n	r	s	TH	TC	TV	D	W^2
	0	0	0.97	0.64	0.92	1.00	1.00		0	0	0.41	0.43	0.38	0.31	0.34
	0	1	0.84	0.44	0.75	1.00	1.00		0	1	0.40	0.44	0.38	0.43	0.47
	0	2	0.58	0.29	0.51	0.92	0.85		0	2	0.42	0.46	0.40	0.48	0.51
	0	3	0.33	0.18	0.30	0.49	0.07		0	3	0.43	0.46	0.42	0.50	0.54
	1	0	0.95	0.83	0.93	0.98	0.85		1	0	0.40	0.43	0.37	0.45	0.50
	2	0	0.95	0.90	0.90	0.00	0.00		2	0	0.36	0.39	0.33	0.51	0.54
	3	0	0.92	0.90	0.88	0.00	0.00		3	0	0.31	0.33	0.29	0.55	0.56
	1	1	0.85	0.66	0.78	0.24	0.00		1	1	0.40	0.43	0.38	0.50	0.53
10	1	2	0.65	0.48	0.58	0.00	0.00	10	1	2	0.40	0.43	0.38	0.53	0.55
	1	3	0.43	0.33	0.38	0.00	0.00		1	3	0.41	0.43	0.39	0.54	0.57
	2	1	0.84	0.75	0.77	0.00	0.00		2	1	0.38	0.40	0.35	0.53	0.56
	2	2	0.64	0.56	0.59	0.00	0.00		2	2	0.38	0.40	0.36	0.56	0.58
	2	3	0.46	0.39	0.41	0.00	0.00		2	3	0.39	0.40	0.38	0.57	0.61
	3	1	0.80	0.75	0.72	0.00	0.00		3	1	0.32	0.34	0.32	0.55	0.59
	3	2	0.60	0.55	0.54	0.00	0.00		3	2	0.34	0.37	0.33	0.57	0.61
	3	3	0.43	0.41	0.38	0.00	0.00		3	3	0.36	0.38	0.35	0.61	0.65
	0	0	1.00	0.66	1.00	1.00	1.00		0	0	0.72	0.58	0.62	0.47	0.51
	0	1	1.00	0.55	1.00	1.00	1.00		0	1	0.65	0.58	0.58	0.60	0.63
	0	2	1.00	0.46	0.99	1.00	1.00		0	2	0.64	0.58	0.57	0.66	0.66
	0	3	0.99	0.38	0.97	1.00	1.00		0	3	0.63	0.58	0.57	0.68	0.68
	1	0	1.00	0.87	1.00	1.00	1.00		1	0	0.70	0.62	0.60	0.60	0.65
	2	0	1.00	0.95	1.00	1.00	1.00		2	0	0.68	0.64	0.59	0.69	0.71
	3	0	1.00	0.98	1.00	1.00	0.95		3	0	0.66	0.63	0.57	0.72	0.73
	1	1	1.00	0.80	1.00	1.00	1.00		1	1	0.67	0.60	0.58	0.67	0.69
20	1	2	1.00	0.72	0.99	1.00	1.00	20	1	2	0.64	0.60	0.57	0.70	0.70
	1	3	0.99	0.63	0.98	1.00	0.99		1	3	0.63	0.59	0.56	0.70	0.70
	2	1	1.00	0.91	1.00	1.00	1.00		2	1	0.64	0.62	0.57	0.71	0.71
	2	2	1.00	0.85	0.99	0.99	0.50		2	2	0.62	0.61	0.55	0.73	0.72
	2	3	0.99	0.79	0.98	0.74	0.00		2	3	0.62	0.60	0.55	0.73	0.72
	3	1	1.00	0.95	1.00	0.87	0.00		3	1	0.63	0.62	0.56	0.74	0.74
	3	2	1.00	0.91	0.99	0.01	0.00		3	2	0.61	0.60	0.55	0.75	0.74
	3	3	0.99	0.86	0.98	0.00	0.00		3	3	0.61	0.59	0.54	0.75	0.74

<Table 2> Continued.

H_1: Par(1, 0)								H_1: N(3, 1)							
n	r	s	TH	TC	TV	D	W^2	n	r	s	TH	TC	TV	D	W^2
10	0	0	0.96	0.40	0.82	0.20	0.14	10	0	0	1.00	0.70	0.99	1.00	0.99
	0	1	0.63	0.23	0.51	0.16	0.08		0	1	0.98	0.60	0.96	1.00	0.99
	0	2	0.33	0.14	0.29	0.08	0.04		0	2	0.91	0.51	0.87	0.98	0.97
	0	3	0.18	0.09	0.17	0.05	0.03		0	3	0.76	0.42	0.73	0.85	0.54
	1	0	0.98	0.60	0.90	1.00	0.25		1	0	1.00	0.93	1.00	1.00	1.00
	2	0	0.98	0.75	0.92	1.00	0.25		2	0	1.00	0.99	1.00	0.00	0.00
	3	0	0.98	0.83	0.93	1.00	0.28		3	0	1.00	1.00	1.00	0.00	0.00
	1	1	0.76	0.41	0.65	0.27	0.08		1	1	0.99	0.87	0.99	0.81	0.00
	1	2	0.49	0.29	0.42	0.08	0.04		1	2	0.96	0.80	0.94	0.00	0.00
	1	3	0.28	0.19	0.26	0.04	0.03		1	3	0.86	0.70	0.81	0.00	0.00
	2	1	0.80	0.54	0.70	0.36	0.08		2	1	1.00	0.96	0.99	0.00	0.00
	2	2	0.53	0.38	0.47	0.09	0.05		2	2	0.96	0.90	0.94	0.00	0.00
	2	3	0.34	0.28	0.31	0.05	0.04		2	3	0.86	0.80	0.82	0.00	0.00
	3	1	0.80	0.63	0.72	0.48	0.09		3	1	0.99	0.98	0.98	0.00	0.00
	3	2	0.54	0.45	0.49	0.09	0.05		3	2	0.95	0.92	0.91	0.00	0.00
	3	3	0.34	0.33	0.31	0.05	0.04		3	3	0.81	0.78	0.77	0.00	0.00
20	0	0	1.00	0.48	1.00	1.00	0.94	20	0	0	1.00	0.60	1.00	1.00	0.97
	0	1	1.00	0.31	0.98	1.00	0.82		0	1	1.00	0.55	1.00	1.00	0.97
	0	2	0.99	0.22	0.91	1.00	0.44		0	2	1.00	0.50	1.00	1.00	0.97
	0	3	0.92	0.16	0.76	0.95	0.14		0	3	1.00	0.46	1.00	1.00	0.97
	1	0	1.00	0.66	1.00	1.00	1.00		1	0	1.00	0.89	1.00	1.00	1.00
	2	0	1.00	0.78	1.00	1.00	1.00		2	0	1.00	0.97	1.00	1.00	1.00
	3	0	1.00	0.87	1.00	1.00	1.00		3	0	1.00	0.99	1.00	1.00	1.00
	1	1	1.00	0.52	0.99	1.00	0.85		1	1	1.00	0.85	1.00	1.00	1.00
	1	2	1.00	0.40	0.95	1.00	0.32		1	2	1.00	0.81	1.00	1.00	1.00
	1	3	0.96	0.32	0.85	0.99	0.10		1	3	1.00	0.78	1.00	1.00	1.00
	2	1	1.00	0.65	1.00	1.00	0.73		2	1	1.00	0.96	1.00	1.00	1.00
	2	2	1.00	0.54	0.97	1.00	0.23		2	2	1.00	0.94	1.00	1.00	1.00
	2	3	0.98	0.45	0.90	1.00	0.09		2	3	1.00	0.92	1.00	0.99	0.00
	3	1	1.00	0.77	1.00	1.00	0.58		3	1	1.00	0.99	1.00	1.00	0.08
	3	2	1.00	0.67	0.98	1.00	0.20		3	2	1.00	0.98	1.00	0.25	0.00
	3	3	0.99	0.57	0.92	1.00	0.09		3	3	1.00	0.97	1.00	0.00	0.00

[recieved date : Jan. 2006, accepted date : Feb. 2006]