구조 최적화를 위한 Metropolis 유전자 알고리즘의 개발과 효율성 평가

Development and Efficiency Evaluation of Metropolis GA for the Structural Optimization

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요 지

모사풀림(SA)의 특징적인 Metropolis 규준을 단순 유전자 알고리즘(SGA)의 재생산 연산과정에 도입함으로써 Metropolis 유전자 알고리즘(MGA)이 개발되고, 구조 설계 최적화에 응용되었다. 이러한 결합을 통하여 MGA는 개체의 다양성을 유지하며, 초기 세대에서 나타날 수 있는 유용한 유전자 정보가 보존될 수 있다. 따라서 이 연구에서 제안된 MGA는, 국부적 최적해로 조기 수렴하게 되는 SGA의 단점과 정밀한 전역적 최적해를 찾기 위해 수없이 많은 계산을 해야 하는 SA의 단점을 극복하게 되었다. 수치예를 통하여 MGA의 성능과 적용성을 재래의 알고리즘들과 비교하고 평가하였다. 특히 MGA의 성능 신뢰성과 강건성을 평가하는 데는 집단 크기와 최대 반복세대수의 효과를 인용하였다. 이론적 고찰과 수치예의 결과로부터, 이 연구에서 개발된 MGA가 효율성과 신뢰성을 갖춘 구조 설계 최적화의 도구로서 평가되었다.

핵심용어 : Metropolis 유전자 알고리즘, 단순 유전자 알고리즘, 모사 풀림, 구조 설계 최적화

Abstract

A Metropolis genetic algorithm (MGA) is developed and applied for the structural design optimization. In MGA, favorable features of Metropolis criterion of simulated annealing (SA) are incorporated in the reproduction operations of simple genetic algorithm (SGA). This way, the MGA maintains the wide varieties of individuals and preserves the potential genetic information of early generations. Consequently, the proposed MGA alleviates the disadvantages of premature convergence to a local optimum in SGA and time consuming computation for the precise global optimum in SA. Performances and applicability of MGA are compared with those of conventional algorithms such as Holland's SGA, Krishnakumar's micro GA, and Kirkpatrick's SA. Typical numerical examples are used to evaluate the computational performances, the favorable features and applicability of MGA. The effects of population sizes and maximum generations are also evaluated for the performance reliability and robustness of MGA. From the theoretical evaluation and numerical experience, it is concluded that the proposed MGA is a reliable and efficient tool for structural design optimization.

keywords : *Metropolis genetic algorithm, simple GA, simulated annealing, structural design optimization*

1. Introduction

Recently, a considerable amount of research effort has been made for the development and application of stochastic search methods in various fields of engineering optimization. Among them, update versions of simulated annealing(SA) and genetic algorithms(GAs) have been advantageously applied for the structural design optimization due to their simplicity in theortical concepts and superiority in numerical searching capability. Both SA and

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GA are, in nature, combinatorial optimization techniques that do not require the continuity or the differentiability of associated functions. Hence, they are very suitable and useful for the optimization problems with discrete design variables.

SA has a favorable advantage of possible convergence to a precise global optimum(Kim *et al*, 2000). However, its major drawback lies in the time consuming computation to reach the neighborhood of global optimum, or slower convergence(Van Laarhoven *et al.*, 1987).

GA has various versions such as simple GA (SGA), micro GA(μGA), hybrid GA, and others. They have been successfully applied in various fields of optimization, due to the desirable features of avoiding undesirable local optima and rapid convergence to near global optimum. However, most versions of GA have common drawbacks, i.e., since they basically search a solution space using random numbers, it is very hard to reach the precise optimum solution. Hence, different optimum solution may be obtained in every trial to get an optimum for the same problem.

Many a researcher has tried to overcome the shortcomings of both SA and GA, but the results are not at all satisfactory in the field of structural optimization(Kim *et al.*, 2002; Lee *et al.*, 1997; Woo *et al.*, 2003; Goldberg, 1989). In order to realize the advantageous features and alleviate the undesirable drawbacks of both SA and GA, the Metropolis criterion of SA may be incorporated in the reproduction operations of GA. Thus, we develop and propose a Metropolis genetic algorithm(MGA) in which both the Metropolis criterion and roulette wheel selection are used in SGA.

In the study, critical procedures of both SA and GA are firstly reviewed. Then, the Metropolis criterion, an essential process of SA, is incorporated in SGA to develop an MGA. Finally, the proposed MGA is applied for the typical numerical examples of structural design optimization and its performances and applicability are evaluated. Numerical examples in hand are, in general, finite dimensional constrained minimization problems, which are symbolically expressed as:

Find a design variable vector $x \in R^n$
to minimize the cost function $f(x)$
subject to the m constraints $gi(x) \leq 0$.

2. Simulated Annealing and Metropolis Criterion

Simulated annealing(SA) is, in nature, an iterative stochastic search technique which simulates solid thermodynamic annealing process to get the lowest energy level. In SA, there are usually two basic iterative loops, i.e., an inner loop of Metropolis criterion or reproduction operation and an outer loop of temperature reduction or new design iteration.

Once the initial values and starting current design x^c for SA are chosen, a random search is used to generate a new design x^N in the inner loop of Metropolis criterion. Metropolis criterion is a stochastic process to reach a thermal equilibrium status of a solid in heat bath(Metropolis *et al*, 1987). For a specified temperature T, reproduction with Metropolis criterion will be continued until the maximum number of inner loop iteration M_{max} is reached. In the inner loop, the increment of cost values between the current design x^c and a new design x^N is first calculated as,

$$\Delta f = f(x^N) - f(x^C) \tag{1}$$

If the cost of new design is smaller, or $\Delta f \leq 0$, then x^N will be naturally accepted as an improved solution. Even in case of $\Delta f > 0$, x^N will be reconsidered and accepted as one of new solution set only if the following Metropolis criterion is met.

$$P_M = \exp(-\Delta f / k_B T) \geq R \tag{2}$$

Here, P_M is acceptance probability, k_B Boltzmann constant(usually taken as 1.0 in practice), T the target temperature specified in the outer loop, and R a random number generated in {0, 1}. In the high temperature or large T, P_M becomes larger and most of new designs are accepted. Due to such a characteristic of the Metropolis criterion, a new design worse than the current one can play an important role to extend its domain of search and possibly escape from an undesirably premature local minimum. Consequently, SA has higher probability to find global optimum than other traditional optimization techniques.

In the outer loop iteration, the reduction of temperature is controlled by a cooling schedule until it becomes very small or $T \approx 0$. If T becomes very small, acceptance probability P_M also becomes smaller and the Metropolis criterion is hardly met. Thus, the global optimum will hardly be missed once it is found. As mentioned above, the temperature T is a key parameter in the Metropolis criterion which controls the acceptance probability. In early stages when T is relatively large, most of new designs tend to be accepted no matter they may be better or worse than current design. As a result, the domain of search is considerably extended. As the outer loop iteration proceeds, T is reduced to very small value and acceptance probability of a new design becomes even smaller. In these stages, the solution is located near the global optimum and the search domain need not be extended any more.

The temperature T is reduced according to the cooling schedule, i.e., the search domain is gradually reduced. Rapid reduction of the temperature may cause the convergence to an undesirable local optimum. Therefore, a geometric cooling schedule is widely used, i.e.,

$$T_{i+1} = \alpha T_i \tag{3}$$

where α is a cooling coefficient (usually, 0.5 ~ 0.99) and T_0 is a prescribed initial temperature.

Hence, SA is a stochastic search method in which a cooling schedule of annealing process is incorporated with basic Metropolis criterion. In other words, an initial design space of random search is gradually reduced to a specific neighborhood of a global optimum without loss of generality.

3. Genetic Algorithms

Genetic algorithm(GA) is primarily based on the principles of natural selection and survival of the fittest in the ecosystem. Simple GA (SGA) is robust and still applied in the fields of structural design optimization(Jin, 2000). However, a critical drawback of SGA lies in the premature convergence to a local optimum if any relatively dominant individual appears in early generation. Such undesirable phenomenon becomes even severer as the population size becomes smaller due to the lack of variety of genetic information. Thus the population size of 30~200 is widely recommended for engineering problems. To alleviate such a drawback, a lot of research effort has been made to improve or modify the SGA. Among them, the micro GA(μGA) is quite promising.

A μGA was proposed by Krishnakumar (1989), in which the population size and amount of computation were successfully reduced without loss of variety of potential genetic information. Like SGA, μGA also uses tournament selection and one point crossover as reproduction and crossover operator, respectively. However, crossover probability is usually taken as 1.0 so as to surely exchange the genetic information between selected individuals. Mutation is no more necessary since μGA uses the wide variety of genetic information through a restarting option. The elitism is favorably used in μGA in order to keep the best individual in a generation.

4. Metropolis Genetic Algorithm

In the conventional GAs such as SGA and μGA, the reproduction operation is performed primarily based on the individual fitness. Hence, any individual with low fitness tends to be deleted from the mating pool even if it may have potential genetic information. Therefore, Metropolis criterion should be applied just before the reproduction operation, as the fitness value of each individual should be evaluated first for the reproduction.

Other genetic operations such as one point crossover, simple mutation, and elitism can be also effectively used in MGA as they are in SGA or μGA. If Metropolis criterion is combined to any genetic operation other than reproduction, its favorable features may not be highlighted or even useless. For example, the number of function evaluation will be doubled if Metropolis criterion is combined to crossover or mutation. If Metropolis criterion is used before or after elitism, the procedure will be exactly same as the conventional GAs.

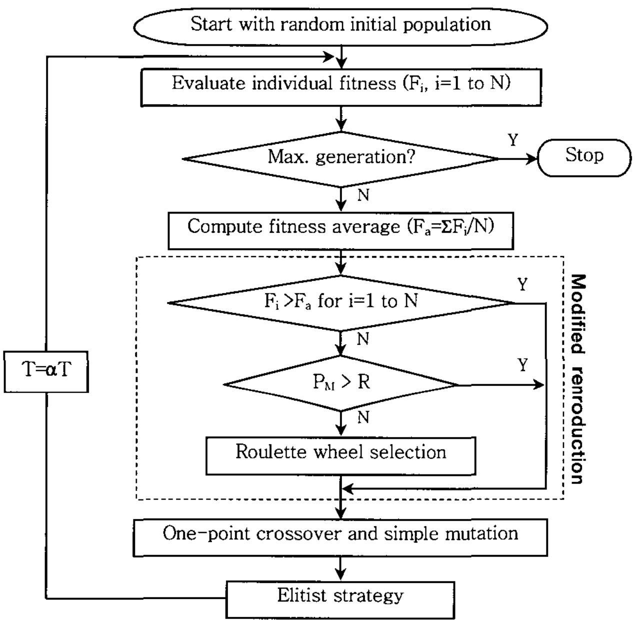

Fig. 1 Flow chart of Metropolis genetic algorithm.

Therefore, in the study, Metropolis criterion is combined to the reproduction operator of SGA in addition to a roulette wheel selection to develop MGA as shown in Fig. 1. At early generations, Metropolis criterion in MGA enables an individual with low fitness (i.e., less than average fitness F_a) to be possibly accepted so that the genetic varieties may be maintained in the population without losing the potential genetic information. It also prevents the convergence to a premature local optimum through the expansion of search space. In other words, an acceptance probability of lower fitness individual is made very high at early generations (i.e., when the temperature parameter T is large enough) and the resulting solution space can be considerably extended. Thus, a global optimum may not be missed and premature local optima can be avoided as well. At near final generations, T becomes very small, the Metropolis acceptance probability of low fitness individual will be considerably reduced, and the reproduction operation like roulette wheel selection will be mainly used. Consequently, the precise convergence to a global optimum is assured within the minimal number of generations in MGA.

Typical features and favorable aspects of proposed MGA are presented and discussed in the theoretical point of view:

(1) Metropolis criterion of SA is combined with the roulette wheel selection of SGA reproduction operator for the purpose of keeping the potential genetic information at early generations. Due to the high temperature (T) or high acceptance probability in Metropolis criterion, random search is performed in a wider solution space at early generations. At later generations, as the temperature or the acceptance probability of lower fitness individual decreases, a precise global optimum can be expected due to the narrow band of solution space.

(2) The convergence in early generations of MGA may be slower than that of conven-

tional GAs. This is due to the process of Metropolis criterion in the reproduction operation. However, in the later generations, convergence in MGA would be stable and even faster as the elite individuals are kept in the population.

(3) In the study, the algebraic mean of fitness values(F_a) is used in the Metropolis criterion of selection. In case of the selection level higher than F_a, the variety of genetic information in the solution space would be consistently preserved and tremendous number of generations would be necessary to reach a global optimum. On the other hand, the MGA with lower selection level than F_a would virtually make no difference with SGA as the acceptance probability becomes very low even at early generations. Thus, the algebraic mean F_a is adequately used in the Metropolis criterion of selection.

(4) The number of function evaluations in a generation of MGA is the same as that of SGA. This is because the fitness values of all the individuals in the population are evaluated in both algorithms.

(5) MGA searches a solution with a group of individuals in a population, while SA does with individual by individual. Thus, it is expected that the search capability of MGA is superior to that of SA, especially for the large scale problems.

Analytic discussions would be investigated and realized through a class of numerical examples. Efficiency and reliability of MGA is compared with those of conventional algorithms such as SGA, μGA, and SA.

5. Numerical Examples

5.1 Parameters for the competitive algorithms

In order to evaluate the performance of the MGA, a few numerical examples of structural design optimization are investigated. The algorithms of comparison's purpose are SGA, μGA, and SA. Initial input parameters for each algorithm are listed in Tables 1 and 2(Fan *et al*, 2000).

Table 1 Design optimization parameters used for SA in numerical examples

SA parameters	3-bar truss	Composite breakwater
Initial temperature	100	1000
Final temperature	0.0001	0.0001
Cooling schedule	Geometric	Geometric
Cooling coefficient	0.95	0.99
Maximum inner loops	30	50
Boltzmann constant	1.0	1.0

Table 2 Genetic operators used for SGA, μGA and MGA

GA parameters	SGA	OGA	MGA
Reproduction	Roulette wheel	Tournament	Metropolis criterion & Roulette wheel
Crossover(Rate)	One-point (0.85)	One-point (1.0)	One-point (0.85)
Mutation(Rate)	Simple mutation (0.01)		Simple mutation (0.01)
Elitism	Yes	Yes	Yes

5.2 Formulation of design optimization problems

Generally, SA is suited for the unconstrained minimization problems, and GA for the unconstrained maximization problems, respectively. However, most structural optimization problems are the constrained minimization problems as presented in the introduction and they need be transformed into unconstrained minimization problems. For this purpose SUMT exterior penalty function method can be effectively used.

$$\phi(\mathrm{x}) = f(\mathrm{x}) + t\sum_{i=1}^{m}\left[g_i(\mathrm{x}) + \left|g_i(\mathrm{x})\right|\right]^2 \tag{4}$$

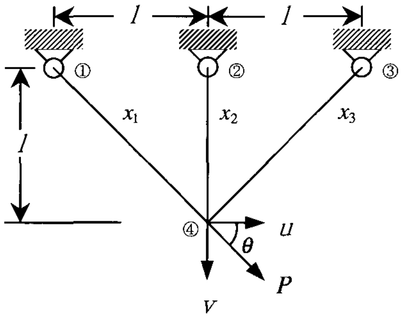

Fig. 2 Three bar-truss.

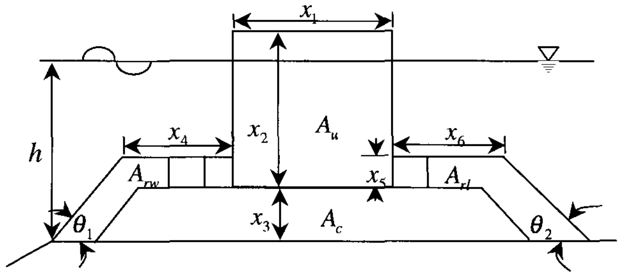

Fig. 3 Definition of design variables for composite breakwater.

Here, $\phi(x)$ is a transformed objective function to be minimized, $f(x)$ is the objective function, t is a penalty parameter, and $g_i(x)$ are the constraint functions, respectively. Again it is transformed into an unconstrained maximization problem which is suitable for GAs by defining a fitness function;

$$F(\mathrm{x}) = K - \log[\phi(\mathrm{x})] \tag{5}$$

Here, $F(x)$ is the fitness function to be maximized and K is arbitrary large number. In this study, the parameters t and K are set as t = 10^5 and K = 15, respectively.

Two example structures are formulated for the optimum design herein; a symmetric 3 bar truss and a mass concrete type composite breakwater(Ryu *et al,* 1994; Seo *et al,* 1992).

5.2.1 Three bar truss

A symmetric 3 bar truss is shown in Fig. 2. Cross sectional areas of the members are the design variables. The constraints are based on member crushing, member buckling, failure by excessive deflection of node 4, and failure by resonance when the fundamental natural frequency of the structure is below a given threshold. They are summarized in Table 3.

The cost function to be minimized is the total volume of the structure;

$$f(\mathrm{x}) = l(2\sqrt{2}x_1 + x_2) \tag{6}$$

Table 3 Design constraints for the 3-bar truss problem

Stress	$\{P_u / x_1 + P_v /(x_1 + \sqrt{2}x_2)\} / \sqrt{2} / \sigma_{1a} - 1.0 \le 0$; $\{\sqrt{2}P_v /(x_1 + \sqrt{2}x_2)\} / \sigma_{2a} - 1.0 \le 0$
Deflection	$\{(\sqrt{2}lP_u / E) / x_1\} / \Delta_{ua} - 1.0 \le 0$; $\{(\sqrt{2}lP_v / E) /(x_1 + \sqrt{2}x_2)\} / \Delta_{va} - 1.0 \le 0$
Frequency	$\{-3Ex_1 /(\rho l^2(4x_1 + \sqrt{2}x_2))\} /(2\pi\omega_0)^2 + 1.0 \le 0$
Buckling	$(1/\sqrt{2})\{P_v /(x_1 + \sqrt{2}x_2) - P_u / x_1\}\{2l^2 /(\pi^2 E\beta x_1)\} - 1.0 \le 0$
Minimum area	$-x_1 / x_{min} + 1.0 \le 0$; $-x_2 / x_{min} + 1.0 \le 0$
Symbols	Pu = horizontal component; Pv = vertical component; σ_{1a}, σ_{2a} = allowable stress E = Young's modulus; Δ_{ua}, Δ_{va} = allowable displacement; ρ = material mass density; β = constant

Here, x_i is the area of the i th member and l is defined as shown in Fig. 2.

5.2.2 Composite breakwater

A composite breakwater consists of upright structure(A_u) and foundation or rubble mound which is again composed of core (A_c) and revetments (A_{rl}, A_{rw}) as shown in Fig. 3. In the figure, 6 design variables are defined as the dimension of structural components. The design constraints are based on sliding, overturning, stability, bearing capacity of structural components, etc. as summarized in Table 4.

The cost function is the construction cost of unit length of the breakwater, which is expressed as weighted sum of structural components with the weighting factors or unit costs (c_u, c_c, c_{rw}, c_{rl});

Table 4 Design constraints for the composite breakwater problem

Sliding	$-(\mu_1 W_e)/(1.2P)+1.0 \le 0$
Overturning	$(M_u - W_t)/(1.2M_p)+1.0 \le 0$
Bearing capacity	$P_e/50-1.0 \le 0$; $P'/q_a - 1.0 \le 0$
Sliding of underwater structure	$-\mu_2 F/1.2+1.0 \le 0$
Additional constraints	$-(x_2+x_3)/(h+0.6H_{1/3})+1.0 \le 0$; $-x_3/1.5+1.0 \le 0$ $-x_4/(2R_x)+1.0 \le 0$; $(x_3+x_5)/h-1.0 \le 0$ $-x_6/5+1.0 \le 0$; $-x_1 \le 0$, $-x_2 \le 0$, $-x_5 \le 0$
Stability of rubble mound	$(1/R_x R_z x_5)\{H_{1/3}/N_s(S_r-1)\}^3-1.0=0$
Symbols	$\mu 1$, $\mu 2$ = friction coefficient; P' = Terazghi's equation; q_a = allowable bearing capacity $H_{1/3}$ = significant wave height; R_x, R_z = length and width of foot protection block N_s = safety factor; $S_r = \gamma_r/W_0$; h = water depth

$$f(\mathrm{x}) = (c_u A_u + c_c A_c + c_{rw} A_{rw} + c_{rl} A_{rl}) \tag{7}$$

Here, the slopes of rubble mound, θ_1 and θ_2, are kept constant, and the cross sectional areas of components are expressed as follows:

$$\begin{aligned} A_u &= x_1 x_2 \\ A_c &= 0.5x_3^2(\cot\theta_1+\cot\theta_2)+x_3[x_1+x_4+x_6+x_5 \\ &\quad (\cot\theta_1+\cot\theta_2-\csc\theta_1-\csc\theta_2)] \\ A_{rw} &= x_5(x_4+0.5x_5\cot\theta_1+x_3\csc\theta_1) \\ A_{rl} &= x_5(x_6+0.5x_5\cot\theta_2+x_3\csc\theta_2) \end{aligned} \tag{8}$$

5.3 Numerical results

5.3.1 Three bar truss

The initial data for the 3 bar truss and the parameters for GAs are listed in Table 5 and 6. The results are presented in Table 7, where N_{opt} is the number of function evaluations to reach the optimum. Initial design for SA is taken as $(7.7, 3.9)\times 10^3 \mathrm{m}^2$, which is one of the local optima found with SGA. As shown in the table, MGA yields the best solution with the minimal N_{opt}. MGA and SA give the exactly same optimum. However, the amount of computation in MGA is much less than that in SA, i.e., less than 20%. This shows the efficiency of MGA.

Table 5 Input data for design optimization of 3 bar truss

Input data	Value
Allowable stresses	(34450, 137800)kN/m^2
Allowable displacements	$(0.127\times10^{-3}, 0.127\times10^{-3})$m
Young's modulus	6.89×10^7kN/m^2
Unit weight	0.69kN/m^3
Load	17.8kN
Angle	45°
Lower bound on design variables	$(64.52\times10^{-6}, 64.52\times10^{-6})$m^2
Upper bound on design variables	$(64.52\times10^{-3}, 64.52\times10^{-3})$m^2
Lower limit on frequency	2500Hz

Table 6 Parameters for 3 bar truss

Parameter	SGA	μGA	MGA
Population size	10	5	10
Maximum generation	200	200	200
String length	15	15	15

Table 7 Results of 3 bar truss example with SGA, μGA, MGA, and SA

Output data	SGA	μGA	MGA	SA
x_1(m^2)	4.17×10^{-3}	3.96×10^{-3}	**4.10×10^{-3}**	4.10×10^{-3}
x_2(m^2)	1.73×10^{-3}	2.52×10^{-3}	**2.00×10^{-3}**	2.00×10^{-3}
Violation	0.00	0.00	**0.00**	0.00
Cost(m^3)	3.47×10^{-4}	3.49×10^{-4}	**3.46×10^{-4}**	3.46×10^{-4}
N_{opt}	950	860	**650**	3341

The values of optima in Table 7 are the best results obtained out of 100 trials. In general, GAs may yield a different optimum in each trial. Thus, the global optimum is assured from the results of 100 trials. The distribution of optima in 100 trials is shown in Fig. 4. As shown in the figure, the probabilities to obtain the true global optimum(P_{opt}) are 100% in MGA, 91% in SGA, and 43% in μGA, respectively. This shows the global convergence property and reliability of MGA.

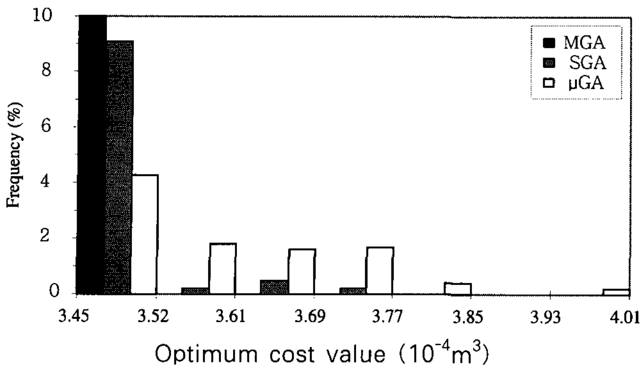

Fig. 4 Distribution of obtained optima for 3-bar truss problem with 100 trials.

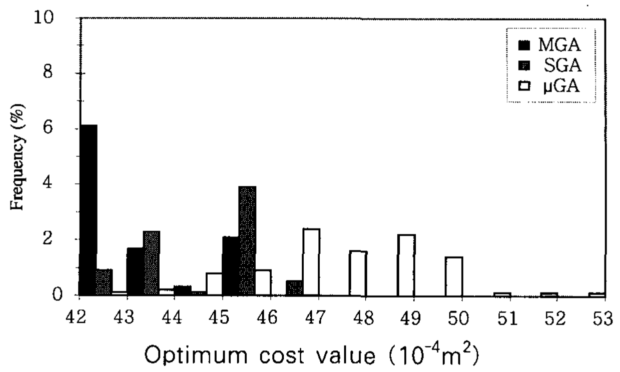

Fig. 5 Distribution of obtained optima for composite breakwater problem with 100 trials.

5.3.2 Composite breakwater

Input data for the design of composite breakwater are listed in Table 8. All the weighting factors (c_u, c_c, c_{rw}, c_{rl}) of the cost function are set to 1.0 in the numerical evaluation. Design variables are defined for the GAs in Table 9. In Table 10, optimization parameters for GAs are presented. Initial starting design for SA is (18, 10, 8, 12, 3, 12) m. The best results of 100 trials are summarized in Table 11, where P_{opt} is the probability (or, frequency out of 100 trials) to obtain the best optima shown in the corresponding column. The distribution of optima in 100 trials is shown in Fig. 5. The probability to obtain the best solution(P_{opt}) is 62% in MGA, 9% in SGA, and only 1% in μGA, respectively.

Table 8 Input data for design optimization of composite breakwater

Input data	Value	Input data	Value	Input data	Value
H_0'	6.30m	γ_u	3.20t/m^3	μ_1	0.60
$T_{1/3}$	11.4sec	γ_r	2.65t/m^3	μ_2	0.80
β	0.26rad	γ_c'	1.00t/m^3	N_r	6.80
h	10.1m	γ_g'	1.00t/m^3	N_q	9.00
i	0.01	θ_1	0.46rad	R_x	4.00m
w_0	1.03t/m^3	θ_2	0.59rad	R_z	2.00m

MGA needs less amount of computation than SA for the same quality of solutions. Comparing other GAs, MGA yields the best solution with the highest probability to obtain it. Hence, the efficiency and robustness of MGA is again verified.

Table 9 Definition of design variables for composite breakwater

Design variable	Min.	Max.	Resolution	String length
x_1(m)	10	20	0.01	10
x_2(m)	1	15	0.01	12
x_3(m)	1	10	0.01	10
x_4(m)	1	15	0.01	12
x_5(m)	0.1	5	0.01	15
x_6(m)	1	15	0.01	12

Table 10 Parameters for composite breakwater

Parameter	SGA	μGA	MGA
Population size	60	5	60
Maximum generation	2000	2000	2000

Table 11 Results of composite breakwater example with SGA, μGA, MGA, and SA

Output data	SGA	μGA	MGA	SA
x_1(m)	15..92	15.43	**16.10**	15.87
x_2(m)	6.91	6.57	**6.94**	6.96
x_3(m)	6.66	7.10	**6.63**	6.88
x_4(m)	8.02	8.03	**8.04**	8.30
x_5(m)	1.00	0.99	**1.00**	0.99
x_6(m)	5.50	5.50	**5.11**	5.44
Violation	0.00	0.00	**0.00**	0.00
Cost(m^2)	422.67	429.26	**421.00**	421.00
N_{opt}	117960	4722	**117240**	139427
P_{opt}	9	1	**62**	-

5.4 Performance of MGA

A parametric study is performed to investigate the effects of algorithm parameters in the numerical performances of GAs. The population size and the maximum generations are used as parameters to evaluate the parametric dependency of SGA, μGA, and MGA. Numerical example of composite breakwater problem is again used for the evaluation. All other input data are same as before, i.e., as shown in Tables 2, 4, 8, and 9. The results are obtained with 100 trials of each algorithm.

5.4.1 Effect of population size

A number of population sizes are used as shown in Table 12. For μGA, the only population size of 5 is used. The maximum generations are fixed as 2000 for all the cases.

Table 13 shows the range of optima obtained in 100 trials. As the population size increases the bandwidth of optima locations decreases in both SGA and MGA. However, the bandwidths in MGA are consistently narrower and the lower limits are also consistently less in MGA. In Table 14, the frequencies of obtained optima are summarized according to the range of optima and population size. As the population size increases the probability in MGA consistently becomes higher while SGA does not. Moreover, the probability in MGA is much higher than others.

From the results shown in Tables 13 and 14, it is verified that MGA is the stable and efficient algorithm.

Table 12 Population size as a parameter for SGA, μGA, and MGA

Parameter	SGA	MGA	μGA
Population size	40, 60, 80, 100, 120		5
Maximum generation	2000		

5.4.2 Effect of maximum generations

Two values of the maximum generations, 3000 and 5000, are used for the evaluation's purpose. Here, the population size is set to 100. Table 15 shows the range of obtained optima according to the maximum generations in each algorithm. Range of optimum values with MGA is the lowest and the bandwidths are smallest. As shown in Table 16, the optima are improved as the maximum generations increase. Among the GAs, the frequencies in MGA are consistently highest.

Therefore, it is proved that MGA is the robust and excellent algorithm.

Table 13 Effect of population size on the range of optima obtained in 100 trials

Population size		40	60	80	100	120
Obtained optima: Lower Upper (Range)	SGA	423.1 490.4(67.3)	422.7 487.5(64.8)	422.4 481.2(58.8)	422.1 499.9(59.8)	423.4 478.7(55.3)
	μGA	429.3 520.7(91.4)				
	MGA	**423.1 471.1(48.0)**	**421.0 469.8(48.8)**	**421.1 468.8(47.7)**	**421.2 465.9(44.7)**	**421.1 467.1(46.0)**

Table 14 Frequencies of obtaining optima depending on the range of optima and population size

Range of optima		[420, 430]					[420, 470]				
Population size		40	60	80	100	120	40	60	80	100	120
P_{opt} in the range	SGA	5	3	11	7	15	86	73	94	91	94
	μGA	1	1	1	1	1	37	37	37	37	37
	MGA	**19**	**24**	**25**	**35**	**46**	**99**	**100**	**100**	**100**	**100**

Table 15 Effect of maximum generations on the range of optima obtained in 100 trials

Algorithms	SGA		μGA		MGA	
Max generations	3000	5000	3000	5000	3000	5000
Range of obtained optima	423.0 490.5 (67.5)	422.3 465.3 (43.0)	439.1 530.2 (91.1)	424.0 488.8 (64.8)	**420.8 447.8 (27.0)**	**420.8 444.0 (23.2)**

Table 16 Frequencies of obtaining optima depending on the range of optima and maximum generations

Range of optima		[420, 430]		[420, 450]	
Max. generations		3000	5000	3000	5000
P_{opt} in the range	SGA	8	33	17	74
	μGA	1	11	2	34
	MGA	**38**	**100**	**53**	**100**

6. Summary and Conclusions

In the application of genetic algorithms, the individual varieties in a population should be maintained to assure the global optimality. Any favorable genetic information inherent in early generations should never be lost during the design iterations. By keeping desirable potential genetic information, undesirable premature convergence to local minima or time consuming computation to get a precise global optimum can be avoided. To this end, the Metropolis selection criterion of SA is incorporated in the reproduction process of SGA to develop an MGA.

The resulting MGA is proposed for the structural design optimization and its behavior and performances are evaluated using typical structural design examples. Applicability and robustness of MGA are evaluated through a series of numerical investigations with the parameters such as population size and maximum number of generations. The results are compared with those of SA, SGA, and μGA, and the following conclusions are drawn:

1) The results of numerical examples show the superior applicability of the proposed MGA to other stochastic algorithms.
2) A combinatory reproduction operation with the Metropolis criterion and roulette wheel selection enhances the convergence property of MGA, i.e., excellent capability to get a precise global optimum and faster convergence. Hence, the performance of the proposed MGA is more efficient than other single algorithm.
3) Through a parametric study with population sizes and maximum generations, the proposed MGA is shown to be robust and reliable when compared with other versions of GA.
4) It is also concluded that the proposed MGA can also be effectively applied for the real world problems of structural design optimization.

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