Z-R Relationships for a Weather Radar in the Eastern Coast of Northeastern Brazil

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Abstract**—**A disdrometer has been used to determine *Z-R* relationships for the weather radar, which is unique coastal radar operating regularly in western tropical south Atlantic. Rainfall rates were divided into the stratiform rain and the convective rain on the basis of 10 mm h^{-1}. The *Z-R* relationship for the stratiform class was similar to the general one since the convective clouds did not developed and two classes of the rain rate were mixed.

Keywords**—**Z-R relationship, disdrometer, weather radar

I. INTRODUCTION

Droplets of natural clouds and precipitation satisfy the condition of Rayleigh Approximation, namely diameter much smaller than the wavelength, for wavelengths used in weather radars (Sauvageot, 1992). If it is hypothesized that radars can monitor certain atmospheric phenomena, then the radar reflectivity factor (*Z*) can be related to physical quantities of these phenomena. Many works were performed relating rainfall rates (*R*) and *Z*, resulting in a general function of the form:

$$Z = aR^b \qquad (1)$$

where Z is given in mm ^{6} m $^{-3}$, R in mm h^{-1} and a and b are coefficients which depend on the raindrop number distribution $N(D)$ as function of the drops diameter (D).

Several methods have been proposed to establishing the *Z-R* relationship. One of them requires a disdrometer for measuring a set of $N(D)$ (Joss and Waldvogel, 1967, 1969; Campistron, et al., 1987). In others, Z and R are measured simultaneously and independently by radar (Z) and rain gage network (R). Theoretical values of a and b coefficient can be either computed using the $N(D)$ distributions or taken from literature with certain restrictions. The objective of this work was to determine the *a* and *b* coefficients, with in situ measurements of the required variables, for the weather radar installed in the eastern coast of northeastern Brazil, the only coastal radar operating regularly in western tropical south Atlantic.

The study was performed in the Campus A. C. Simões, Universidade Federal de Alagoas (UFAL), in Maceió, Alagoas, Brazil (9°3′17.24″S, 35°46′54.84″W), located over a large flat area 104 m above sea level, known as *tabuleiro costeiro* (coastal plateau). The campus is 13 km away from the coast line. A disdrometer RD-69 was used to collect the data. Data were collected in a period of 10 months, from December 2001 to September of 2002, encompassing both the dry and rainy seasons.

II. METHODOLOGY

The equation (1) was used to determine the relationship between rainfall rate (R) and radar reflectivity factor (Z). Natural logarithm is applied to both side of equation (1) resulting in:

$$\ln Z = \ln a + b \ln R \qquad (2)$$

Making Y = ln Z; α = ln a; β = b; X = ln R, a straight-line function is obtained $Y = \alpha + \beta X$. where α e β are the y-axis intercept and the slope, respectively. The coefficients a and b of equation (1) were estimated by linear regression Z versus R.

The list of rainfall events is in the third column of Table 1. The criteria of rainfall duration $T \geq 20$ min and accumulated rain $R_{ac} \geq 10$ mm were used to select special events listed in column 4 of the same Table.

Table 1 Number of Raindrop Size Distribution and events.

Months	RSDs (per minute)	Events	Special events
DEC/2001	250	7	-
JAN/2002	1149	26	3
FEB/2002	831	10	1
MAR/2002	1185	38	1
APR/2002	631	51	-
MAI/2002	2423	33	2
JUN/2002	1311	43	1
JUL/2002	884	19	2
AUG/2002	993	22	-
SEP/2002	709	19	-
Σ	10,366	238	10

III. RESULTS

A. Z-R GENERAL RELATONSHIP

All values of R were considered to establishing the radar general equation. The result was

$$Z = 176.5\, R^{1.29} \quad [r = 0.83] \qquad (3)$$

The regression line obtained with the 10-month data set

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is shown in Figure 1.

The monthly values for the *a* and *b* coefficients are shown in Table 2 together with the number of RSD recorded in each month. Except for July (1.25), the mean monthly *b* coefficients did not differ from each other significantly. The mean monthly a coefficients, however, seems to be divided in 3 classes, with January and February well above 200, a range from 150 to 200 from March to June and below 150 for July to September. This may be tied up with the nature of the rainfall. The maximum values occurred during the strong solar heating months, January and February, and may resulted in high top clouds and convective rainfall whereas the minima were found in the coldest months, when the mean sea level pressure is higher and the trade winds inversion stronger over the region, resulting in lower cloud tops and probably stratiform-like rainfall. The correlation coefficients were high, in the range of 0.74 to 0.94.

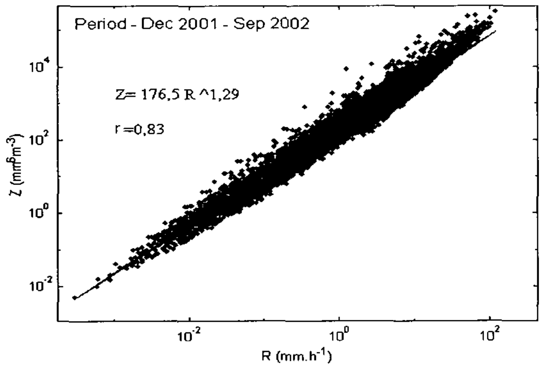

Fig. 1 Regression line *Z-R* for the whole data set composed of 10,366 min samplings equivalent to 172.76 h of rainfall recording.

B. Z-R RELATIONSHIP FOR CONVECTIVE AND STRATIFORM RAINFALL

A threshold rainfall rate of $R < 10$ mm h^{-1} was adopted for stratiform rainfall events. Rain events were considered of convective class with *R* equal to or above that threshold, bearing in mind that due to the regional characteristics, it is difficult to distinguish the 2 types of rainfall. With this simple criterion, employed by other researchers, e. g. Nzeukou et al. (2002), 84% of rainfall events were classified as stratiform rain and the remaining as convective rain.

Table 2 Values for the coefficients *a* and *b* of the *Z-R* monthly equations, number of RSD recorded in each month and the correlation coefficient *r*.

Months	RSDs (min^{-1})	a	b	r
DEC/2001	250	170.9	1.25	0.76
JAN/2002	1149	247.1	1.33	0.90
FEB/2002	831	264	1.36	0.94
MAR/2002	1185	185.1	1.30	0.84
APR/2002	631	182.5	1.31	0.82
MAI/2002	2423	150.5	1.29	0.74
JUN/2002	1311	154.3	1.33	0.90
JUL/2002	884	116.8	1.25	0.84
AUG/2002	993	115.6	1.28	0.75
SEP/2002	709	145.2	1.28	0.75

In Table 3 the RSD monthly numbers as well as the *a* and *b* coefficients resultant from the *Z-R* regression technique were shown. For stratiform class, the coefficient *a* was in the range 134 to 269, being larger in January and February. The coefficient *b* did not vary significantly, remaining in the range 1.22 to 1.38. Again, highest values occurred in January and February. The correlation coefficients were all above 0.66. For the convective class, the monthly values of the coefficient *a* varied widely from 31 to 136 whereas the coefficient *b* values were high, ranging from 1.5 to 1.9. Except for December, the correlation coefficients were all above 0.73 suggesting a good *Z-R* relationship. The *Z-R* regression using the classified data set produced the following relationships and respective correlation coefficients:

$$\text{Stratiform rainfall: } Z = 167.8R^{1.26}; \quad [r = 0.70] \qquad (4)$$

$$\text{Convective rain: } Z = 65.46\,R^{1.69}; \quad [r = 0.84] \qquad (5)$$

It can be seen in Table 3 that the *a* and *b* coefficients values had a low spread when compared to the values of the overall stratiform data equation (equation (4)). For tropical Western Africa, Nzeukou et al. (2002) found slightly different results using the same criterion. The difference may be due to the fact that these authors used a longer data set, with a 4 years sampling period. The difference may be attributed also to the difficulty to establishing a precise threshold to distinguishing the 2 classes of rainfall. In the eastern coast of NEB, the clouds are advected within the southeast trade winds field from the Atlantic. Cloud development, thus, results

Table 3 *RSD* and *Z-R* monthly coefficient and respective correlation coefficients for stratiform and convective rain classes.

Months	RSDs (min^{-1})			Z-R Relations and Correlation Coefficients (r)			
	Total	Stratiform	Convective	Stratiform		Convective	
DEC/2001	250	238	12	$Z = 170.0\,R^{1.25}$	$r = 0.66$	$Z = 132.5\,R^{1.5}$	$r = 0.51$
JAN/2002	1149	1018	131	$Z = 251.8\,R^{1.34}$	$r = 0.69$	$Z = 119.3\,R^{1.5}$	$r = 0.92$
FEB/2002	831	728	103	$Z = 269.0\,R^{1.38}$	$r = 0.76$	$Z = 87.2\,R^{1.6}$	$r = 0.93$
MAR/2002	1185	995	190	$Z = 169.3\,R^{1.25}$	$r = 0.80$	$Z = 84.9\,R^{1.6}$	$r = 0.87$
APR/2002	631	496	135	$Z = 161.4\,R^{1.26}$	$r = 0.85$	$Z = 54.8\,R^{1.7}$	$r = 0.79$
MAI/2002	2423	1979	444	$Z = 143.4\,R^{1.26}$	$r = 0.73$	$Z = 50.4\,R^{1.6}$	$r = 0.80$
JUN/2002	1311	953	358	$Z = 150.9\,R^{1.28}$	$r = 0.70$	$Z = 136.3\,R^{1.4}$	$r = 0.92$
JUL/2002	884	801	83	$Z = 152.0\,R^{1.22}$	$r = 0.84$	$Z = 48.4\,R^{1.7}$	$r = 0.88$
AUG/2002	993	863	130	$Z = 145.0\,R^{1.24}$	$r = 0.76$	$Z = 31.3\,R^{1.9}$	$r = 0.75$
SEP/2002	709	647	62	$Z = 134.7\,R^{1.24}$	$r = 0.82$	$Z = 54.3\,R^{1.8}$	$r = 0.73$

from the low level convergence of the moisture flux near to or on the coast and the associate convective cells are usually imbedded in large stratus layers producing a sort of 'mixed rainfall'. In such circumstances, the coefficients estimates tend towards the stratiform rain ones. The results of stratiform and convective regressions were depicted in Figures 2 and 3, respectively.

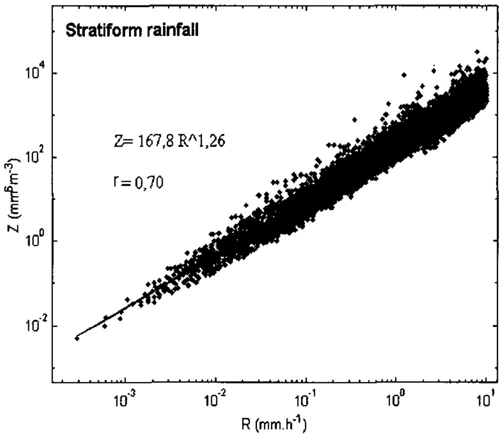

Fig. 2 Regression line *Z-R* for stratiform rainfall with R< 10 mm.h^{-1}. The size of the recorded samples was 8,718 min or 145.3h.

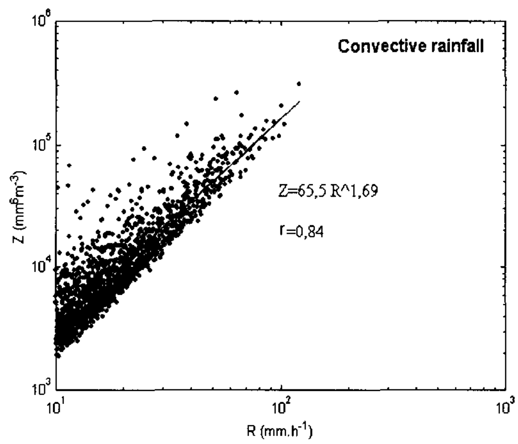

Fig. 3 Regression line *Z-R* for convective rainfall with *R* ≥10 mm h $^{-1}$. The size of the recorded samples was 1,648 min or 27.46h.

C. Z-R RELATIONSHIP FOR SPECIAL RAINFALL EVENTS

In this item, the purpose was to evaluate the *Z-R* relationships for special rain events $T \geq 20$ min long and accumulated rain $R_a \geq 10$ mm selected from the sampling record. The rain event accumulation and duration, together with the results for *a* and *b* coefficient and respective correlation coefficients, were shown in Table 4. The coefficient *a* values were all higher than 135 whereas the coefficient *b* values did not vary significantly, being similar to the general *Z-R* equation (3). The correlation coefficient values were high, in the range of 0.82 to 0.97. From this Table, three events were selected that represented the summer season (December to February), the transition from dry to wet season (April-May) and the wet season (June-July).

The event of January 22nd was associated with an upper troposphere cyclonic vortex (see Molion and Bernardo 2002), systems that were frequent during that particular month. This event had duration of 144 min (2.4 h) but accumulated only 12.19 mm rain. The following equation represented this event that had a high correlation coefficient of 0.92.

$$Z = 136.5\, R^{1.29} \tag{6}$$

The rainfall rate *R* as function of time was depicted in Figure 4. It is apparent that convective cells produced *R* well above 10 mm h^{-1} but of short duration, no longer than 13 min. Examples are the peaks at about 0420 LST and 0900 LST. Examples of stratiform rainfall rates, with $R < 10$ mm.h^{-1}, are between 0340 LST and 0415 LST.

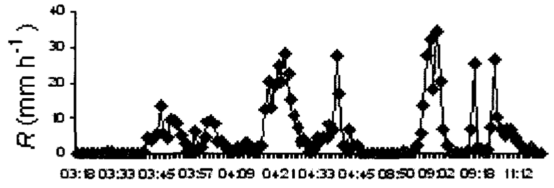

Fig. 4 Rainfall rate (*R*) versus time of duration (*T*) of the January 22nd event.

On May 26th, as seen in satellite images, 2 convective cloud clusters, associated with traveling disturbances in the southeast trade wind field, developed at the coast due to enhanced moisture flux convergence. The event lasted 444 min (7.4 h) and accumulated 74.32 mm rain. The equation (7) shows the *Z-R* relationship for this event, which had a correlation coefficient of 0.93.

$$Z = 142.6\, R^{1.32} \tag{7}$$

The evolution of the rainfall rate (*R*) with time was shown in Figure 5. It can be seen that convective rain dominated the event, which most of the rain occurring during the nighttime. At this time, the land-breeze forms due to the inverted land-sea temperature gradient and this enhances moisture flux convergence and favors convective development.

Finally, the event of July 26th, representing the wet or winter season, was analyzed and its *Z-R* relationship was given below (equation (8)), with a correlation coefficient of 0.96. This event lasted 104 min (1.73 h) and accumulated 14.05 mm rain.

$$Z = 142.3\, R^{1.36} \tag{8}$$

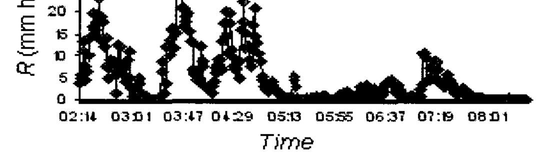

Fig. 5 Rainfall rate (*R*) versus time of duration (*T*) of the May 26th event.

The time variation *R* was shown in Figure 6. Convective rainfall rates of about 40 mm h^{-1} near 2120 LST and 50 mm h^{-1}, at about 0030 LST can be seen in this Figure. The event was also associated with traveling disturbances in the southeast trade winds field, as seen in satellite images.

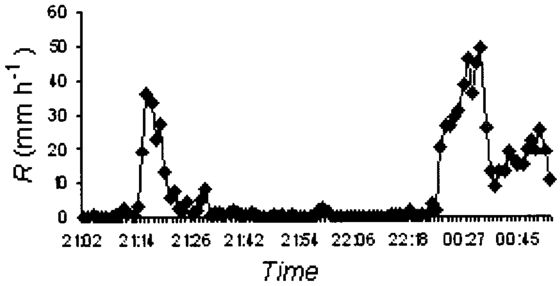

Fig. 6 Rainfall rate (R) versus time of duration (T) of the 26th July event.

IV. CONCLUSIONS

For determining the *Z-R* relationship for the weather radar in the eastern coast of northeastern Brazil, the recorded rainfall rates were divided into 2 classes using the same the criteria as described in Nzeukou et al. (2002), namely, for stratiform rainfall rates $R < 10$ mm h^{-1} and for convective rain $R \geq 10$ mm h^{-1}. For the stratiform class, the values obtained for the a coefficient were in the range of 134 to 269 whereas the values for the coefficient *b* ranged 1.22 to 1.38, as opposed to the convective class where a varied from 31 to 136 and *b* from 1.4 to 1.9. The stratiform values were in accordance with values found in the literature, the convective ones, were not. The *Z-R* general relationship had coefficients similar to the ones of stratiform class. However, this does not mean that stratiform rainfall rates predominated during the study period. The weather systems, that prevail during the wet season (April to July), are traveling disturbances in the SE trade winds field and convective cells, imbedded in the disturbances, form and develop as they approach the coast and the convergence of the moisture flux is enhanced. Thus, it is possible that most of the rainfall rates were mixed classes, with rain coming from both stratiform and convective clouds, the latter not well developed. A larger data set covering a larger period, that would include climate inter-annual variability, would be necessary to resolve these questions. A larger data period would also allow for *Z-R* relationships comparison within the same season, which was not the case in the present work.

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Table 4. Special rainfall events accumulation and duration, results for *a* and *b* coefficients and respective correlation coefficients during the study period, with $T \geq 20$ minutes.

Events	Date	Hour	Duration (min)	R_{ac} (mm)	Coefficients		
					a	b	r
1	4/JAN	15:47 - 16:58	64	14.85	234.10	1.36	0.93
2	10/JAN	23:33 - 00:40	68	13.99	215.90	1.35	0.90
3	22/JAN	03:18 - 11:22	144	12.19	136.50	1.29	0.92
4	14/FEB	19:24 - 03:07	411	30.05	360.78	1.34	0.97
5	18/MAR	03:02 - 10:12	232	36.93	172.79	1.33	0.94
6	15, 16/MAI	20:03 - 10:37	444	74.32	142.64	1.32	0.95
7	26/MAI	02:14 - 08:38	368	29.37	135.03	1.27	0.93
8	9/JUN	04:53 - 05:57	51	10.0	188.76	1.34	0.92
9	10/JUL	07:56 - 15:59	449	13.28	171.65	1.19	0.82
10	26/JUL	21:02 - 00:52	104	14.05	142.35	1.36	0.96

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