스피어 디코더에서 초기 반지름을 결정하는 두 가지 방법에 대한 비교 연구

Comparison of Two Methods for Determining Initial Radius in the Sphere Decoder

전은성*, 김요한*, 김동구*

Eun-Sung Jeon*, Yo-Han Kim*, and Dong-Ku Kim*

요 약

스피어 디코더의 초기 반지름 결정 문제는 비트 오율 (bit error rate)과 복잡도에 있어서 많은 영향을 미친다. 이런 초기 반지름은 채널의 통계적 특성을 고려함으로 설정되거나, MMSE 결정 값을 이용하여 설정할 수 있다. 채널의 통계적 특성을 이용한 방법은 초기 반지름이 송신 신호에 해당하는 격자점을 매우 높은 확률로 포함한다. MMSE 결정 값을 이용하는 방법은 먼저 수신 신호에서 MMSE 연 판정 부호(soft output information)을 얻은 후, 경 판정(hard decision)을 내린 다음, 수신 신호 공간에서 경 판정 부호에 해당하는 격자점을 찾는다. 그리고 수신 신호와 경 판정 부호에 해당하는 격자점 사이의 유클리디안 거리(Euclidean distance)를 초기 반지름으로 설정한다. 본 논문에서는 채널의 통계적 특성을 이용한 방법에 있어서 기존의 복잡한 수식에 비해 간단한 새로운 식을 유도하고, MMSE 결정값을 이용한 방법과 비교 연구 하였다. 비교를 위해 'Tightness'라는 새로운 측도를 이용하였다. 전산 실험 결과, 낮은 SNR 영역과 중간 정도의 SNR 영역에서는 MMSE를 이용한 방법의 더 많이 디코딩 복잡도 감소를 보였고, 높은 SNR 영역에서는 채널의 통계적 특성을 이용한 방법이 더 낮은 디코딩 복잡도를 보였다.

Abstract

The initial radius of sphere decoder has great effect on the bit error rate performance and computational complexity. Until now, it has been determined either by considering the statistical property of channel or by using of MMSE solution. The initial radius obtained by using statistical property of channel includes the lattice point corresponding to the transmit signal vector with very high probability. The method using MMSE solution first calculates out the MMSE solution of the received signal, then maps the hard decision of this solution into the received signal space, and finally the distance between the mapped point and the received signal is selected as the initial radius of the sphere decoding. In this paper, we derive a simple equation for initial radius selection which uses statistical property of channel and compare it with the method using MMSE solution. To compare two methods we define new metric 'Tightness'. Through the simulation, we observe that in low and moderate SNR region, the method using MMSE solution provides more complexity reduction for decoding while in high SNR region, the method using channel statistics is better.

Key words : sphere decoder, initial radius, minimum mean-squared error(MMSE)

* 연세대학교 전기전자공학(Electrical and Electronic Eng., Yonsei University)

· 제1저자 (First Author) : 전은성

· 접수일자 : 2006년 10월 24일

I. INTRODUCTION

MIMO systems has attracted great attention due to increasing the capacity [1]. Although there are many kinds of detection schemes in MIMO systems, Maximum-likelihood (ML) is the optimum in terms of bit error rate performance. The main problem in ML detection, however, is its huge complexity which increases exponentially when the number of transmit antenna and/or constellation size of signal is increased. However, sphere decoder has moderate complexity while achieving ML performance [2]. Since the concept of sphere decoder was introduced in [3], various versions of sphere decoders have been discussed for more reduction of its complexity [4]-[5]. If the sphere radius is too small, the closest lattice point may exist outside of the radius. In this case, the decoder either restarts its new search with larger radius or declares decoding failure. On the other hand, if the sphere radius is set too large, sphere decoder has to search over too many lattice points and this increases the computational complexity. However, the initial radius determination is still a problem.

Until now, it has been known that the initial radius can be determined either by using statistical property of channel or using the MMSE solution. In this paper, we derive a simple equation for initial radius selection which uses of statistical property of channel and compare it with the method using MMSE solution. To compare two methods we define new metric 'tightness'. Thorough the simulation, we observe that in low and moderate SNR region, the method using MMSE solution provides more complexity reduction to sphere decoder while in high SNR region, the method using channel statistics is better.

This paper is organized as follow. In Section II, the system model is presented. In Section III, we show two methods and suggest a new metric 'tightness'. Section IV presents numerical simulation results for comparison.

II. SYSTEM MODEL

We consider MIMO systems comprising n_T transmit antennas and n_R receive antennas. Received signal of the MIMO system can be represented as

$$\tilde{\mathbf{r}} = \widetilde{\mathbf{M}}\,\tilde{\mathbf{u}} + \tilde{\mathbf{w}}, \quad (1)$$

where $\tilde{\mathbf{u}} = [\widetilde{u_1}, \widetilde{u_2}, \dots, \widetilde{u_{n_T}}]$ is transmit signal vector, where each component is independently drawn from a complex constellation such as QAM. $\tilde{\mathbf{r}}$ is received signal vector and $\widetilde{\mathbf{M}}$ is $n_R \times n_T$ channel matrix. $\tilde{\mathbf{w}}$ is complex Gaussian noise vector of which element has zero mean and variance of $2\sigma^2$.

This complex MIMO system can be transformed to real equivalent equation as

$$\mathbf{r} = \mathbf{M}\,\mathbf{u} + \mathbf{w}, \quad (2)$$

where

$$\mathbf{r} = [Re(\tilde{\mathbf{r}})^T \quad Im(\tilde{\mathbf{r}})^T]^T, \quad (3)$$

$$\mathbf{M} = \begin{matrix} Re(\widetilde{\mathbf{M}})^T & -Im(\widetilde{\mathbf{M}})^T \\ Re(\widetilde{\mathbf{M}})^T & Im(\widetilde{\mathbf{M}})^T \end{matrix}, \quad (4)$$

$$\mathbf{u} = [Re(\tilde{\mathbf{u}})^T \quad Im(\tilde{\mathbf{u}})^T]^T, \quad (5)$$

$$\mathbf{w} = [Re(\tilde{\mathbf{w}})^T \quad Im(\tilde{\mathbf{w}})^T]^T. \quad (6)$$

A superscript T denotes the transpose of the vector. The sphere decoder finds the closet lattice point from $\mathbf{r}$ inside the hyper-sphere radius R in $\mathbb{R}^{2n_R}$ as follows

$$\| \mathbf{r} - \mathbf{M}\mathbf{u} \|^2 < R^2 \quad (7)$$

Ⅲ. INITIAL RADIUS SETTING FOR SPHERE DECODER

3-1 Initial radius selection based on MMSE solution

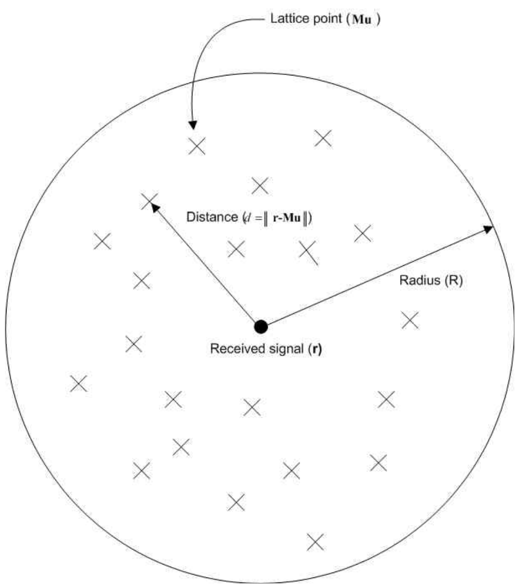

그림 1. 수신 신호와 격자점 사이의 거리에 관한 모델
Fig. 1. Model of distance between received signal $\mathbf{r}$ and lattice point $\mathbf{M}\mathbf{u}$.

A method using MMSE solution of received signal is suggested in [6]. This method consists of the following steps.

The MMSE solution of received signal is firstly calculated out :

$$\mathbf{u}_{mmse} = \left(\mathbf{M}^H \mathbf{M} + \frac{1}{SNR}\mathbf{I} \right)^{-1} \mathbf{M}^H \mathbf{r} \quad (8)$$

And by hard mapping of soft output, we get $\hat{\mathbf{u}}_{mmse}$. Mapping $\hat{\mathbf{u}}_{mmse}$ into the received signal space $\mathbb{R}^{2n_R}$ gives :

$$\hat{\mathbf{r}}_{mmse} = \mathbf{M}\hat{\mathbf{u}}_{mmse} \quad (9)$$

Finally, we choose the distance between $\mathbf{r}$ and $\hat{\mathbf{r}}_{mmse}$. The initial radius for the sphere decoder is given by

$$R = \| \mathbf{r} - \hat{\mathbf{r}}_{mmse} \| . \quad (10)$$

This initial radius not only guarantees no decoding failure since there is at least one lattice point $\hat{\mathbf{u}}_{mmse}$ in the searched sphere. In addition it is statistically shortest. However, it requires the pre-processing in which gets MMSE soft output, hard mapping of $\hat{\mathbf{u}}_{mmse}$ and calculate out $\hat{\mathbf{r}}_{mmse}$ for each time the sphere decoder performs the decoding process.

3-2 Initial radius selection based on probabilistic property of channel

Generally the initial radius has been set according to the suggestion in [2]; the initial radius $r^2 = \alpha m \sigma^2$ may be chosen with α satisfying the equation given by

$$\int_0^{\alpha m} \frac{\lambda^{\frac{m}{2}-1}}{\Gamma(\frac{m}{2})} e^{-\lambda} d\lambda = 0.99 \ , \quad (11)$$

where m is the dimension of the sphere. This equation is not a closed form. So, we derive simple equation for setting initial radius.

In Fig.1, if we define transmit signal vector as $\mathbf{u}$ and the squared Euclidean distance between lattice point $\mathbf{M}\mathbf{u}$ and received signal $\mathbf{r}$ as D, the random variable $D = \| \mathbf{r} - \mathbf{M}\mathbf{u} \|^2 = \| \mathbf{w} \|^2$ has central χ^2- distribution with $2n_R$ degree of freedom since each elements of $\mathbf{w}$ is Gaussian random variable with zero mean and variance σ^2. To guarantee no decoding failure, the initial radius drawn from the received signal should include all the possible lattice points. Since the random variable D has even number of degree of freedom, the initial radius can be set as

$$Pr\{D \le R^2\} = 1 - e^{-\frac{R^2}{2\sigma^2}} \sum_{k=0}^{n_R-1} \frac{1}{k!}\left(\frac{R^2}{2\sigma^2}\right)^k = 1-\epsilon \quad (12)$$

where $\epsilon \ll 1$. When we have variable transform of $x = R^2/2\sigma^2$ and $U(x) = Pr\{D \le R^2\}$, we can write Eq.(12) as

$$U(x) = 1 - e^{-x} \sum_{k=0}^{n_R-1} \frac{1}{k!} x^k = 1 - \epsilon \ . \quad (13)$$

The solution of Eq.(13) can be obtained by numerical method. Fig.2 shows the plot of Eq.(13) for $\epsilon = 10^{-4}$. If we set $x = \hat{x}$ that satisfies Eq.(13), we can determine the initial radius as

$$R^2 = \hat{x} 2\sigma^2. \quad (14)$$

Table.1 shows $\hat{x}$ that approximately satisfies the Eq.(13) for various transmit antenna configurations.

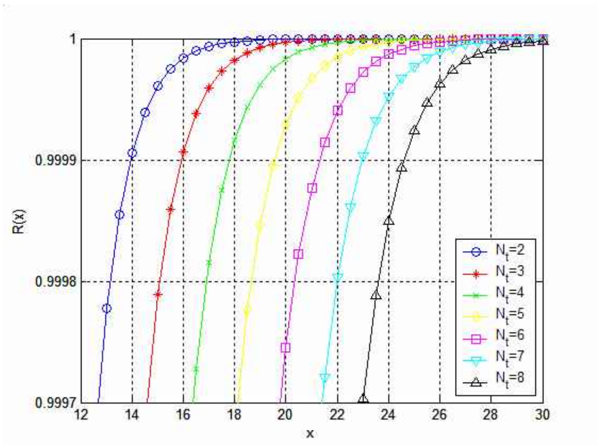

그림 2. 송신 안테나 개수에 대한 수식(13)의 그래프
Fig. 2. Plot of Eq.(13) for various transmit antenna configurations.

표 1. $\epsilon = 10^{-4}$에 대한 $\hat{x}$의 근사 값
Table 1. Approximate value of $\hat{x}$ for $\epsilon = 10^{-4}$.

$\epsilon = 10^{-4}$							
# of Transmit antenna	2	3	4	5	6	7	8
$\hat{x}$	14	16	18	20	21	23	25

3-3 Metric for comparison

Even though the closest lattice point exists inside the searched sphere, too large initial radius increases the decoding complexity. So we should set the initial radius efficiently. In other words, the initial radius satisfies followings; (i) it should include the closest lattice point and (ii) it should be as small as possible. So we define “tightness” of initial radius as

$$T = E\{R - \| \mathbf{r} - \mathbf{M}\mathbf{u} \| \}, \quad (15)$$

where R is always greater than $\| \mathbf{r} - \mathbf{M}\mathbf{u} \|$ and $E\{\cdot\}$ is the statistical expectation. Smaller value of T means tighter radius and it makes more efficient lattice point search. In Section IV, we compare two initial radius selection methods by using this metric.

IV. SIMULATION RESULTS

We perform simulation employing $n_T = n_R = 4$ and 16QAM modulation with an average symbol energy of 10. Fig.3 shows the analytically obtained initial radius and received signal samples versus the SNR for $\epsilon = 10^{-4}$. The dots are samples of lattice points and the solid line is initial radius. It shows that the initial radius obtained by proposed simple Eq.(14) covers the lattice points with very high probability. Fig.4 compares the expected ‘tightness’ defined in Eq.(14) between initial radius from MMSE solution and that from the proposed Eq.(14). Fig.5 shows the computational complexity of the sphere decoder. This graph is consistent to the Fig.4. So the proposed metric is reasonable to compare the efficiency of initial radius selection method. Through these two graphs, it is observed that initial radius from the MMSE solution enables sphere decoder to perform more efficient lattice search in low and moderate SNR region. In high SNR region, however, the initial radius derived from the probabilistic property of channel is better.

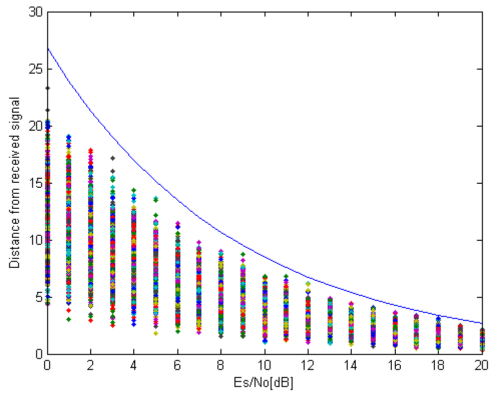

그림 3. $\epsilon = 10^{-4}$ 에서 최소 근접 격자점과 수신 신호 사이의 거리의 신호 대 잡음 비에 대한 그래프

Fig. 3. Euclidean distance between closest lattice points and received signal with the analytically obtained initial radius versus SNR for $\epsilon = 10^{-4}$.

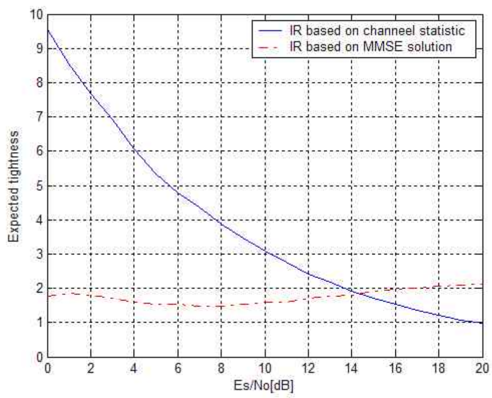

그림 4. MMSE 결정 값과 채널의 통계적 특성을 이용한 방법에서 평균 tightness를 이용한 비교

Fig. 4. Comparison of expected tightness between initial radius based on channel statistics and that based on MMSE solution.

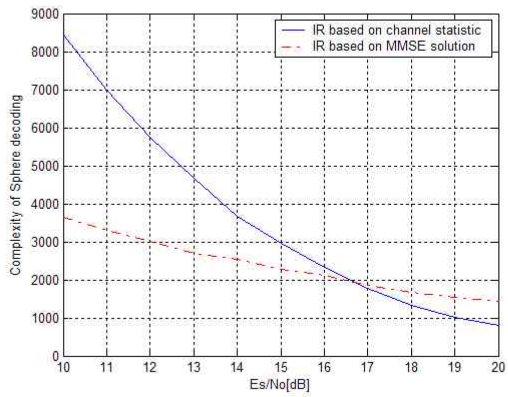

그림 5. 연산의 복잡도 비교

Fig. 5. Comparison of computational complexity of sphere decoding.

V. CONCLUSION

Selection of initial radius in the sphere decoder has critical problem that has great effect on the SER performance and complexity. Initial radius can be determined either by considering the probabilistic property of channel or using of MMSE solution. The initial radius based on the channel characteristic includes all possible lattice points with very high probability. Although it gives simple equation as Eq.(14), it does not provide the sphere decoder the efficient lattice point search in low and moderate. In high SNR region, however, the initial radius obtained by Eq.(14) provides more complexity reduction to the sphere decoder.

REFERENCE

[1] I.E.Telatar, "Capacity of multiple-antenna Gaussian channel," *European Trans. on Telecommunications,* vol. 10, pp. 585-595, Nov. 1999.

[2] B. Hassibi and H. Vikalo, "On the sphere decoding algorithm, part I : Expected complexity," *IEEE Trans. Signal Process*, vol. 53 no. 8, pp. 2806-2818, Aug. 2005.

[3] E. Viterbo and J. Boutros, "Universal Lattice Code Decoder for Fading Channels," *IEEE Transaction on*

Information Theory, vol. 45, no. 5, pp. 1639-1642, July 1999.

[4] A. Chan and I. LEE, "A new reduced-complexity sphere decoding for multiple antenna systems," *Proc. ICC*, vol. 1, pp. 460-464, New York, Apr. 28-May 2 2002.

[5] M. O. Damen, H. EI Gamal, and G. Caire, "On maximum-likelihood detection and the search for the closest lattice point," *IEEE Transactions on Information Thoery*, vol. 49, pp 2389-2402, Oct. 2003.

[6] Q. Liu and L. Yang, "A novel method for initial radius selection of sphere decoding," *Proc. VTC2004*. vol. 2, pp. 1280-1283, Sep. 2004.

[7] J.G. Proakis, *Digital Communications*. New York, NY:MaGraw-Hill, 1995.

전 은 성 (全恩成)

2005년 2월 : 연세대학교 전기전자공학부(공학사)

2005년 3월 ~ 현재 : 연세대학교 전기전자 공학과 석사과정

관심분야 : MIMO 수신기 알고리즘, sphere decoder.

김 요 한

2002년 2월 : 연세대학교 전기전자공학부(공학사)

2004년 2월 : 연세대학교 전기전자공학과(공학석사)

2005년 3월 ~ 현재: 연세대학교 전기전자공학과 박사과정

관심분야 : Multi-user MIMO, 정보이론, 스케줄링.

김 동 구 (金東九)

1983년 2월 : 한국항공대학교 통신공학과 졸업(공학사)

1985년 : U.S.C. Dept. of Electrical Engineering(공학석사)

1992년 : U.S.C. Dept. of Electrical Engineering(공학박사)

1999년 ~ 2006년 : 연세대학교 공과대학 전기전자 공학과 부교수

2006년 ~ 현재 : 연세대학교 공과대학 전기전자 공학과 교수

관심분야 : CDMA 이동통신, 다중 접속 기술, 변조 및 채널 부호화 기술, 스케줄링/MAC 기술, UWB, Binary CDMA