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4차 Cumulant를 이용한 Matrix Pencil Method

(Matrix Pencil Method using Fourth Order Cumulant)

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요 약

Array 신호처리에서 복소 지수함수의 합으로 구성된 신호의 파라미터를 추정하는데 고차 통계를 이용할 수 있다. 본 논문에서는 4차 cumulant 를 이용한 고차 Matrix Pencil method(MPM)를 제안하였다. 4차 cumulant는 Gaussian 잡음를 억제할 수 있기 때문에, MPM의 응답은 기존의 방법에 비하여 더 좋은 잡음 면역을 가지고 있다. 본 논문에서는 높은 정확성을 가지는 MPM의 모든 장점을 유지하면서 성공적으로 고차 MPM을 공식화하였다. 그리고 Numerical simulation을 통해서 본 논문에서 제안된 4차 cumulant를 이용한 방법이 Gaussian 잡음환경에서 더 우수한 DOA 분해능을 가지고 있음을 증명하였다.

Abstract

In array signal processing, high order statistics can be used to estimate parameters from signal of sums of complex exponential. This paper presents a high order Matrix Pencil method(MPM) using the fourth order cumulant. Since the fourth order cumulant can suppress the Gaussian noise, the response of MPM has better noise immunity than the conventional approaches. We successfully formulate the high order MPM with all the benefits of MPM along with higher accuracy. In the numerical simulations we demonstrated that the proposed method with forth order cumulant has better resolution to find degree of arrival(DOA) in the presence of the Gaussian noise.

Keywords : Cumulant, Matrix Pencil Method, DOA estimation, high order statistics

I. Introduction

In array signal processing, most high resolution eigen-structure algorithms, such as MUSIC, root-MUSIC and ESPRIT[1], for direction of arrival(DOA) estimation of multiple source have been developed using the second-order statistics of the received array signals. That is, these methods make use of input covariance matrix, and assuming that the covariance matrix of the additive sensor noise is diagonal or can be estimated accurately[1]. The Matrix Pencil method (MPM) can be used when the received signals of arrays are approximated by sums of complex exponentials[2][3]. This method has a lower variance of the estimates of the parameters of interest than a polynomial method, that is, it performs better than the polynomial method [4][5], and is also computationally more efficient[2][3].

However, in recent years, higher order cumulant-based methods, such as fourth-order cumulant, have received increasing interest due to their advantage over second-order statistics. If the additive noise sources are spatially colored and Gaussian, they can be suppressed in the fourth-order cumulant domain[6]. In 1991, Y. Hua first applied higher order statistics, third-order moment, to the Matrix Pencil method[7], but the performance of these method was not enough to satisfy. Because, if a random process, such as noise, symmetrically

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distributed, then its third cumulant and moment is equal to zero, for example, Laplace, Uniform, Gaussian distributions are symmetric, but the third-order cumulants of input signals ,which are described as the sums of complex exponentials, are equal to zero as well. On the other hands, the fourth-order cumulants are not equal to zero under the above condition. Hence for such a process we must use the fourth-order cumulant to prevent signal cancellation. Utilizing this property, one can enhance the performance of estimation

In this paper, we applied the fourth-order cumulant to the Matrix Pencil method. By substituting the original array input data to the fourth order cumulant, the Gaussian additive noise in the signals can be suppressed. The average of conventional MPM has been compared to the proposed method. Simulation results demonstrate that the fourth order cumulant method generates better performance in finding DOA than the conventional MPM.

Ⅱ. Matrix pencil method

The sampled signal x(k) is to be modeled by a sum of complex exponentials, i.e.,

$$x(k)=\sum_{i=1}^{M}R_i e^{\omega_i kT_s}=\sum_{i=1}^{M}R_i z_i^{k} \tag{1}$$

where R_i = Residues or complex amplitudes,

ω_i = Angular frequencies,

$e^{\omega_i T_s}=z_i$

for i = 1,2,⋯,M. and we assumed the damping factor is not important.

The objective is to find the best estimates of M, R_i and z_i from $x(kT_s)$.

Consider a matrix Y (Assume we have N sampled data) and two sub-matrices Y_a, Y_b.

$$Y=\begin{bmatrix} x(0) & x(1) & \cdots & x(L) \\ x(1) & x(2) & \cdots & x(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-L-1) & x(N-L) & \cdots & x(N-1) \end{bmatrix}_{(N-L)\times(L+1)}$$

$$=\begin{bmatrix} \sum_{i=1}^{M}R_i & \sum_{i=1}^{M}R_i z_i & \cdots & \sum_{i=1}^{M}R_i z_i^{L} \\ \sum_{i=1}^{M}R_i z_i & \sum_{i=1}^{M}R_i z_i^{2} & \cdots & \sum_{i=1}^{M}R_i z_i^{L+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{M}R_i z_i^{N-L-1} & \sum_{i=1}^{M}R_i z_i^{N-L} & \cdots & \sum_{i=1}^{M}R_i z_i^{N-1} \end{bmatrix}_{(N-L)\times(L+1)} \tag{2}$$

$$Y_a=\begin{bmatrix} x(0) & x(1) & \cdots & x(L-1) \\ x(1) & x(2) & \cdots & x(L) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-L-1) & x(N-L) & \cdots & x(N-2) \end{bmatrix}_{(N-L)\times L} \tag{3}$$

$$Y_b=\begin{bmatrix} x(1) & x(2) & \cdots & x(L) \\ x(2) & x(3) & \cdots & x(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-L) & x(N-L+1) & \cdots & x(N-1) \end{bmatrix}_{(N-L)\times L} \tag{4}$$

where L is called the pencil parameter, L is chosen in between N/3 to N/2 for efficient noise filtering [4][5]. One can write

$$Y_a=Z_a R_0 Z_b, \tag{5}$$

$$Y_b=Z_a R_0 Z_0 Z_b, \tag{6}$$

where,

$$Z_a=\begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{(N-L-1)} & z_2^{(N-L-1)} & \cdots & z_M^{(N-L-1)} \end{bmatrix}_{(N-L)\times M}$$

$$Z_b=\begin{bmatrix} 1 & z_1 & \cdots & z_1^{(L-1)} \\ 1 & z_2 & \cdots & z_2^{(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & z_M & \cdots & z_M^{(L-1)} \end{bmatrix}_{M\times L}$$

$$Z_0=diag[z_1,z_2,\cdots,z_M],\ R_0=diag[R_1,R_2,\cdots,R_M]$$

Consider the matrix pencil

$$Y_b-\lambda Y_a=Z_a R_0\,[Z_0-\lambda I]Z_b \tag{7}$$

Therefore, $\lambda=z_i$, for i=1,2,⋯,M would be the eigenvalue of the generalized eigenvalue problem,

$$Y_b-\lambda Y_a \tag{8}$$

It can be shown that this is the same as solving the ordinary eigenvalue problem

$$Y_a^+ Y_b - \lambda I \tag{9}$$

where Y_a^+ is the Moore-Penrose Pseudo-inverse of Y_a which is defined by

$$Y_a^+ = (Y_a^H Y_a)^{-1} Y_a^H \tag{10}$$

Once $\lambda = z_i$ are known, the frequency component is computed from

$$\omega_i = \mathrm{Imag}[\ln(z_i)] \tag{11}$$

Since we do not know how many frequency components exist in the signal, the number of estimated frequencies M should be determined using some criteria. Typically the singular values beyond M are equal to zero. The way selecting M is as follows[4]. Consider the singular value σ_c such that

$$\frac{\sigma_c}{\sigma_{\max}} \approx 10^{-p}. \tag{12}$$

where, p is the number of significant decimal digits in the data.

Ⅲ. Proposed matrix pencil method using the fourth-order cumulant

We assume that the received signal of the respective array is used, which is expressed by (1) and is time series, thus it can be described as the following expression

$$x_n(t) = \sum_{i=1}^{M} R_i e^{j\omega_i(t+n\tau)} \tag{13}$$

where n=0,1,2,⋯, N-1, N denotes the number of arrays, and τ is time delay. If the respective input signals have zero mean, the fourth-order cumulant can be estimated from the following:

$$\begin{aligned} C_4 &= Cum\langle x_n(t), x_{n+1}(t), x_{n+2}(t), x_{n+3}(t)\rangle \\ &= E\{x_n(t), x_{n+1}(t), x_{n+2}(t), x_{n+3}(t)\} \\ &\quad - E\{x_n(t), x_{n+1}(t)\}E\{x_{n+2}(t), x_{n+3}(t)\} \\ &\quad - E\{x_n(t), x_{n+2}(t)\}E\{x_{n+1}(t), x_{n+3}t)\} \\ &\quad - E\{x_n(t), x_{n+3}(t)\}E\{x_{n+1}(t), x_{n+2}(t)\}, \end{aligned} \tag{14}$$

where "Cum" is the abbreviation of cumulant, this expression implies that the fourth-order cumulant requires knowledge of all moments up to order 4.

To attain the sum of the complex exponentials related to only time τ, we must exchange $Cum\langle x_n(t), x_{n+1}(t), x_{n+2}(t), x_{n+3}(t)\rangle$ for $Cum\langle x_n^*(t), x_n^*(t), x_n(t), x_{n+1}(t)\rangle$, then (14) can be rewritten by

$$\begin{aligned} C_4 &= Cum\langle x_n^*(t), x_n^*(t), x_n(t), x_{n+1}(t)\rangle \\ &= E\{x_n^*(t), x_n^*(t), x_n(t), x_{n+1}(t)\} \\ &\quad - E\{x_n^*(t), x_n^*(t)\}E\{x_n(t), x_{n+1}(t)\} \\ &\quad - E\{x_n^*(t), x_n(t)\}E\{x_n^*(t), x_{n+1}(t)\} \\ &\quad - E\{x_n^*(t), x_{n+1}(t)\}E\{x_n^*(t), x_n(t)\} \end{aligned} \tag{15}$$

One can develop the above expression (15)

$$\begin{aligned} &Cum\langle x_n^*(t), x_n^*(t), x_n(t), x_{n+1}(t)\rangle \\ &= E\{\sum_{i=1}^{M} R_i e^{-j\omega_i(t+n\tau)} \sum_{i=1}^{M} R_i e^{-j\omega_i(t+n\tau)} \sum_{i=1}^{M} R_i e^{j\omega_i(t+n\tau)} \sum_{i=1}^{M} R_i e^{j\omega_i(t+(n+1)\tau)}\} \\ &- E\{\sum_{i=1}^{M} R_i e^{-j\omega_i(t+n\tau)} \sum_{i=1}^{M} R_i e^{-j\omega_i(t+n\tau)}\} E\{\sum_{i=1}^{M} R_i e^{j\omega_i(t+n\tau)} \sum_{i=1}^{M} R_i e^{j\omega_i(t+(n+1)\tau)}\} \\ &- E\{\sum_{i=1}^{M} R_i e^{-j\omega_i(t+n\tau)} \sum_{i=1}^{M} R_i e^{j\omega_i(t+n\tau)}\} E\{\sum_{i=1}^{M} R_i e^{-j\omega_i(t+n\tau)} \sum_{i=1}^{M} R_i e^{j\omega_i(t+(n+1)\tau)}\} \\ &- E\{\sum_{i=1}^{M} R_i e^{-j\omega_i(t+n\tau)} \sum_{i=1}^{M} R_i e^{j\omega_i(t+(n+1)\tau)}\} E\{\sum_{i=1}^{M} R_i e^{-j\omega_i(t+n\tau)} \sum_{i=1}^{M} R_i e^{j\omega_i(t+n\tau)}\} \end{aligned} \tag{16}$$

The right term remains to only the following expression.

$$C_4(\tau) = Cum\langle x_n^*(t), x_n^*(t), x_n(t), x_{n+1}(t)\rangle = -\sum_{i=1}^{M} R_i^4 e^{j\omega_i\tau} \tag{17}$$

It turns out that the fourth-order cumulant hasthe sum of the complex exponentials, which is related to τ, such as (1).

Accordingly, we can write the fourth-order cumulant matrix C in the following.

$$\begin{aligned} C &= Cum\left\langle x_0^*(t), x_0^*(t), x_0(t) \begin{bmatrix} x_0(t) & x_1(t) & \cdots & x_L(t) \\ x_1(t) & x_2(t) & \cdots & x_{L+1}(t) \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-L-1}(t) & x_{N-L}(t) & \cdots & x_{N-1}(t) \end{bmatrix} \right\rangle \\ &= \begin{bmatrix} C_4(0) & C_4(1) & \cdots & C_4(L) \\ C_4(1) & C_4(2) & \cdots & C_4(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ C_4(N-L-1) & C_4(N-L) & \cdots & C_4(N-1) \end{bmatrix} \end{aligned} \tag{18}$$

Thus, we can now apply this matrix to the Matrix Pencil method [4][5].

The sampled signal $C_4(k)$ is to be modeled by a sum of complex exponentials, i.e.,

$$C_4(k) = -\sum_{i=1}^{M} R_i^4 e^{j\omega_i k \tau_s} = -\sum_{i=1}^{M} R_i^4 z_i^k \tag{19}$$

where R_i^4 = Residues or complex amplitudes,

ω_i = Angular frequencies,

$e^{\omega_i \tau_s} = z_i$

for i = 1,2,⋯,M. and we assumed the damping factor is not important.

The objective is to find the best estimates of M, R_i and z_i from $C_4(k)$.

Consider a matrix C (Assume we have N sampled data) and two sub-matrices C_a, C_b.

$$C = \begin{bmatrix} C_4(0) & C_4(1) & \cdots & C_4(L) \\ C_4(1) & C_4(2) & \cdots & C_4(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ C_4(N-L-1) & C_4(N-L) & \cdots & C_4(N-1) \end{bmatrix}_{(N-L)\times(L+1)}$$

$$= \begin{bmatrix} \sum_{i=1}^{M} R_i^4 & \sum_{i=1}^{M} R_i^4 z_i & \cdots & \sum_{i=1}^{M} R_i^4 z_i^{L} \\ \sum_{i=1}^{M} R_i^4 z_i & \sum_{i=1}^{M} R_i^4 z_i^2 & \cdots & \sum_{i=1}^{M} R_i^4 z_i^{L+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{M} R_i^4 z_i^{N-L-1} & \sum_{i=1}^{M} R_i^4 z_i^{N-L} & \cdots & \sum_{i=1}^{M} R_i^4 z_i^{N-1} \end{bmatrix}_{(N-L)\times(L+1)} \tag{20}$$

$$C_a = \begin{bmatrix} C_4(0) & C_4(1) & \cdots & C_4(L-1) \\ C_4(1) & C_4(2) & \cdots & C_4(L) \\ \vdots & \vdots & \ddots & \vdots \\ C_4(N-L-1) & C_4(N-L) & \cdots & C_4(N-2) \end{bmatrix}_{(N-L)\times L} \tag{21}$$

$$C_b = \begin{bmatrix} C_4(1) & C_4(1) & \cdots & C_4(L) \\ C_4(2) & C_4(2) & \cdots & C_4(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ C_4(N-L) & C_4(N-L) & \cdots & C_4(N-1) \end{bmatrix}_{(N-L)\times L} \tag{22}$$

One can write

$$C_a = Z_a R_0 Z_b \tag{23}$$

$$C_b = Z_a R_0 Z_0 Z_b \tag{24}$$

where $R_0 = diag\left[R_1^4, R_2^4, \cdots, R_M^4\right]$.

Consider the matrix pencil

$$C_b - \lambda_c C_a = Z_a R_0 \left[Z_0 - \lambda_c I\right] Z_b \tag{25}$$

Therefore, $\lambda_c = z_i$, for i=1,2,⋯,M would be the eigenvalue of the generalized eigenvalue problem,

$$C_b - \lambda_c C_a \tag{26}$$

It can be shown that this is the same as solving the ordinary eigenvalue problem,

$$C_a^+ C_b - \lambda_c I \tag{27}$$

where C_a^+ is the Moore-Penrose Pseudo-inverse of C_a which is define by

$$C_a^+ = (C_a^H C_a)^{-1} C_a^H \tag{28}$$

Once $\lambda_c = z_i$ are known, the frequency component is computed from

$$\omega_i = \text{Imag}[\ln(z_i)] \tag{29}$$

Ⅳ. Numerical Simulations

For the First example, consider a signal of unit amplitude arriving from $\beta_1 = \pi/5$, i.e., $u = e^{(j\beta_1 t_{i,j})} + N_{i,j}; i = 0,1,2,\ldots,99,\ j=0,1,2,\ldots,10$, where $N_{i,j}$ denote the noise with Gaussian distributions and β is $\frac{d}{\lambda} d\cos\theta$, d is distance between arrays, θ is the DOA, λ is the wave length of signals. Fig. 1 shows that as the SNR increase the error in the estimation of the fourth-order cumulant decreases much more than that of the conventional MPM. The pencil parameter, L, was equal to 5 with N=10 and the size of the pencil matrix was 5 by 5.

For the second example, consider a signal of unit amplitude arriving from $\beta_1 = \pi/3$ and $\beta_2 = \pi/3 + \beta_d$, i.e., $u = e^{(j\beta_1 t_{i,j})} + e^{(j\beta_2 t_{i,j})} + N_{i,j}; i = 0,1,2,\ldots,99,\ j=0,1,2,\ldots,60$. The frequency error in the estimation was defined as

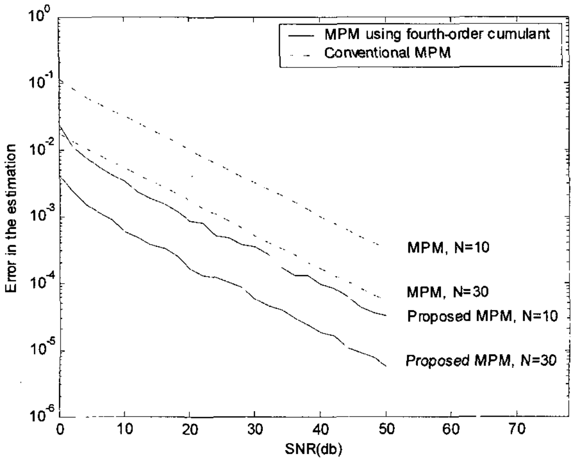

그림 1. 어레이 개수에 따른 SNR 및 에러 추정
Fig. 1. Error in the estimations with different number of arrays.

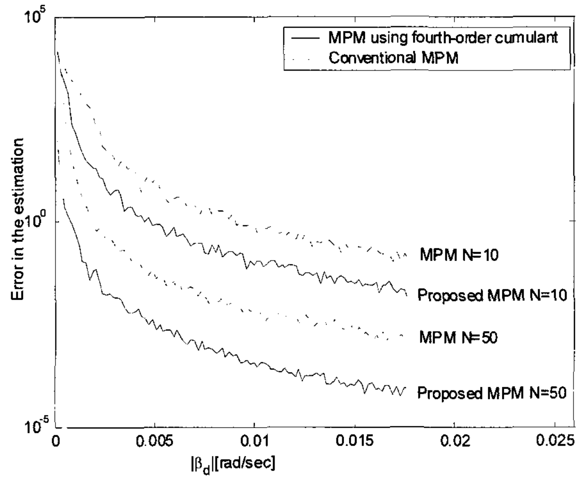

그림 3. 어레이 개수에 따른 에러 추정
Fig. 3. Errors in the estimations with different number of arrays.

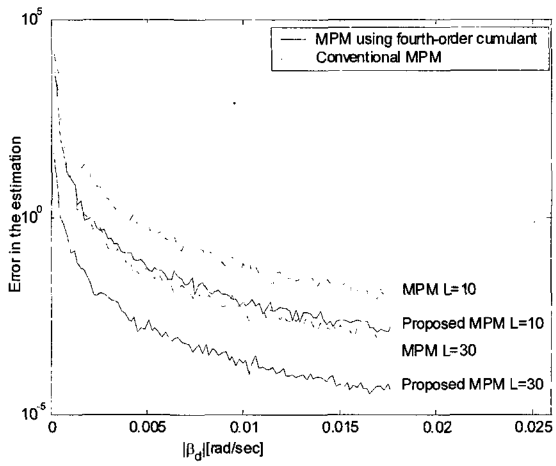

그림 2. Pencil matrix 의 크기 L에 따른 에러 추정
Fig. 2. Errors in the estimations with different L and the size of pencil matrix.

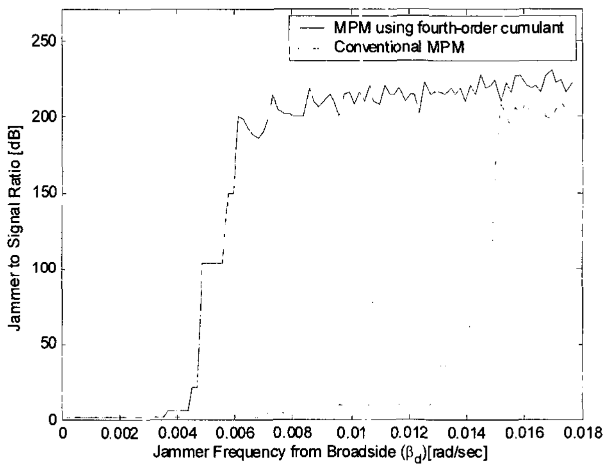

그림 4. 40dB SNR인 노이즈 환경에서 신호에 대한 Jammer의 비
Fig. 4. Jammer to signal with 40dB SNR.

$\|\beta-\beta_{est}\|_2/\|\beta_2-\beta_1\|_2$, where β_{est} is estimated frequency of β_1 and β_2 and $\beta=[\beta_1\beta_2]^T$. Fig. 2 and Fig. 3 show the Monte Carlo simulation of the angle, β_d, versus the error in the estimation. In Fig. 2, the pencil parameter, L, was equal to 10 and 30, and the size of pencil matrix was 10 by 10 and 30 by 30. In Fig. 3, the pencil parameter, N, was equal to 10 and 50. The error performance of the MPM using fourth-order cumulant was proved in comparison with conventional MPM.

For the third example, consider a signal of unit amplitude arriving from $\beta_1=\pi/2$ and a jammer arriving at $\beta_2=\pi/2+\beta_d$ with amplitude J_{mag}, i.e., $u=e^{(j\beta_1 t_{i,j})}+J_{mag}e^{(j\beta_2 t_{i,j})}+N_{i,j}; i=0,1,2,...,99,$ j=0,1,2,...,50.
Fig.4 is to find jammer strength that is going to produce an output error of 1% in the estimation of the signal strength. We can observe that the fourth-order cumulant gives higher resolution than the MPM.

V. Conclusion

In this paper, the performance of the proposed Matrix Pencil method using the fourth-order cumulant and the conventional Matrix Pencil method for DOA estimation was compared. Simulation results show the proposed method has better performance than the conventional MPM in terms of SNR, as well as resolution. We can observe that the fourth-order cumulant can successfully suppress the Gaussian noise, however, since fourth-order cumulant is higher order, the computational load of the proposed method is more increased than MPM.

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